

# Essay on Symplectic Geometry

The essay should not be longer than 10 pages and should be typed on computer. Moreover, it should contain the following:

1. Define what a symplectic manifold is and give examples. In particular, show that the cotangent space  $T^*M$  of any manifold  $M$  is in a natural way a symplectic manifold.
2. State and prove the Theorem of Darboux (which says that any symplectic manifold of dimension  $2n$  is locally isomorphic to the symplectic manifold  $T^*\mathbb{R}^n = \mathbb{R}^n \times (\mathbb{R}^n)^*$ .)
3. Define what a Lagrangian submanifold of a symplectic manifold is. Give examples of Lagrangian submanifolds of  $T^*M$ : When is the image of a section of  $T^*M$  a Lagrangian submanifold of  $T^*M$ ? What about the (so-called) conormal bundle inside  $T^*M$  of a submanifold  $X \subset M$ ?
4. What is the Poisson bracket of a symplectic manifold?

## References

- [1] Agrikola, I. and Friedrich, T.: *Global analysis : differential forms in analysis, geometry and physics*, Chapter 7, Transl. by A. Nestke. American Mathematical Society, 2002.
- [2] Cannas da Silva, A.: *Lectures on Symplectic Geometry*, Springer, 2008.  
Also available at: <https://people.math.ethz.ch/acannas/Papers/lsg.pdf>
- [3] Michor, P.: *Topics in Differential Geometry*, Chapter VII, American Mathematical Society, 2008.