Exercises Week 11

① (Eckmann-Hilton argument)

- let A be a set with two monoid structures $(A, \times, e)$ & $(A, \bullet, e)$ such that $(a \bullet b) \times (c \bullet d) = (a \times c) \bullet (b \times d)$.
- Prove that $\bullet = \times$ & that the monoid is commutative.

This is why the higher homotopy groups are commutative.

② Recall that for G a group & K a field the group ring $K[G] = \{ \lambda_1 g_1 + \dots + \lambda_n g_n : \lambda_i \in K, g_i \in G \}$

As mentioned in the notes, a KG-module is a representation of G: a K-vector space V & group homomorphism

$$G \longrightarrow GL(V,V).$$

- Let $G = S_3$, the symmetric group, $K = \mathbb{R}$. Can you describe a $\mathbb{R}[S_3]$-module str. on $\mathbb{R}^3$.

• let $D_8 = \langle a, b : a^4 = b^2 = 1, b^{-1}ab = a^{-1} \rangle$ be dihedral group w/ 8 ell's. These elements describe the symmetries of the square, generated by a rotation & a reflection.
Can you describe a $\mathbb{R}[G]$-module str. on $\mathbb{R}^2$?

③ As an algebraic functor, we know that $U : Rng \longrightarrow Grp$ must have a left adjoint.
- Can you describe it explicitly?