

## Exercises 13

- ① By a projective cover of an obj.  $A$  we mean  $A' \xrightarrow{p_A} A$  with  $p_A$  surj. &  $A'$  projective? Does the assignment  $A \mapsto A'$  give a functor?
- ② Show that each vector space is both projective & injective.
- ③ Show  $\mathbb{Z}_n$  not an inj  $\mathbb{Z}$ -module for  $n > 1$ .
- ④ Prove that  $\mathbb{Q}$  is not a projective  $\mathbb{Z}$ -module. (Hint: if it was it would be a submodule of a free module - even a retract. Find a property of  $\mathbb{Q}$  not held by free  $\mathbb{Z}$ -modules.)
- ⑤ Use Baer criterion to show that for  $A$  an abelian group &  $a \in A \exists f: A \rightarrow \mathbb{Q}/\mathbb{Z}$  s.t.  $f(a) \neq 0$ . (We say that  $\mathbb{Q}/\mathbb{Z}$  is a cogenerator in the cat. of abelian groups.)

Hint: consider cyclic  
group  $\langle a \rangle \subseteq A$ .