

## Exercise 5

① Let  $PX = \{U : U \subseteq X\}$  be the set of subsets of  $X$ .  
Extend  $P$  to a functor  $\text{Set} \xrightarrow{P} \text{Set}$   
& describe a natural transformation  $1 \Rightarrow P$ .

② Consider the adjoint functors  
 $\text{Vect} \xrightleftharpoons{F} \text{Set}$

Describe a natural transformation  $\varepsilon : F \circ U \Rightarrow \text{Id}$ .

③ Consider the forgetful functor

$$U : \text{Rng} \longrightarrow \text{Set}$$

It has a left adjoint. Can you describe it explicitly?

④ - Let  $A, B$  be categories. The product category  $A \times B$  has objects  $(a, b)$  where  $a \in A, b \in B$  & morphisms  $(a, b) \xrightarrow{(f, g)} (c, d)$  where  $f : a \rightarrow c$  &  $g : b \rightarrow d$ .

- Let  $F, G : A \rightrightarrows B$  be functors, and  $J = \{0 \rightarrow 1\}$ .

Show that natural transformations  $F \Rightarrow G$  are the same thing as functors

$$H : J \times A \rightarrow B$$

such that  
the diagram

commutes.

