

$$G_{23} G_{13} G_{12} A = R \Rightarrow A = G_{12}^* G_{13}^* G_{23}^* R = (G_{23} G_{13} G_{12})^* R = QR$$

$$G_{12} = \begin{pmatrix} G & 0 \\ 0 & 1 \end{pmatrix}$$

$$G = G_{23} G_{13} G_{12}$$

$$\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$GA = R$$

$$A = G^* R \quad Q = G^*$$

$$\det(A) = \det(Q \cdot R) = \det(Q) \cdot \det(R)$$

$$1 = \det I = \det(Q^* Q) = \det Q^* \cdot \det Q$$

Q -komplexní \Rightarrow jaký je vztah mezi $\det Q$ a $\det Q^*$?

Q - reálná, ortog.

$$\det Q^T = \det Q$$

$$\Rightarrow 1 = |\det Q|^2 \Rightarrow$$

$$\det Q = \pm 1$$

$$Q^* = \overline{Q^T} \Rightarrow \det Q^* = \det \overline{Q^T} = \overline{\det Q^T} = \overline{\det Q} \quad - \text{pro lib. kompleksni matrici}$$

Q-unitarni

$$1 = \det I = \det Q^* Q = \overline{\det Q} \cdot \det Q \Rightarrow |\det Q| = 1 \quad \det Q^* = \frac{1}{\det Q}$$

$$|\det A| = |\det Q| \cdot |\det R| = |\det R| = \prod_{i=1}^n |r_{ii}|$$

QR algorithmus

$$A = A_0 = Q_1 R_1 \rightarrow A_1 = R_1 Q_1$$

$$A_1 = Q_2 R_2 \rightarrow A_2 = R_2 Q_2$$

\vdots

$$A_{s-1} = Q_s R_s \rightarrow A_s = R_s Q_s$$

A_s, A -podobné

$$R_1 = Q_1^* A_0 \Rightarrow A_1 = Q_1^* A Q_1$$

$$R_2 = Q_2^* A_1 \Rightarrow A_2 = Q_2^* A_1 Q_2 = Q_2^* Q_1^* A Q_1 Q_2$$

A_2, A -podobné

A_1, A -podobné