

HOMEWORK 1 – 2019

Exercise 1. Show that $SS^k = S^{k+1}$. *Hint:* Consider the map $f: S^k \times I \rightarrow S^{k+1}$, where

$$f(x, t) = ((\sqrt{1 - (2t - 1)^2})x, 2t - 1)$$

Exercise 2. Suppose that the inclusion $A \hookrightarrow X$ is a cofibration and consider a map $f: A \rightarrow Y$. Prove that the inclusion $Y \hookrightarrow X \cup_f Y$ is also a cofibration.

Exercise 3. Prove: If X is a Hausdorff space, then its diagonal

$$\Delta = \{(x, x) \in X \times X\}$$

is a closed subspace of $X \times X$.

Exercise 4. Let X be a Hausdorff space and $A \subseteq X$ be its retract. Prove that A is closed. *Hint:* Use the previous exercise and the map $X \rightarrow X \times X: x \mapsto (x, r(x))$ where $r: X \rightarrow A$ is a retraction.

Exercise 5. The consequence of the previous exercise is: If X is Hausdorff and $A \hookrightarrow X$ is a cofibration, then A is closed. *Hint:* $X \times I$ is Hausdorff.