

## HOMEWORK 7 – 2019

**Exercise 1.** Let  $f : X \rightarrow Y$  be a constant map. Prove that  $f_* : H_n(X) \rightarrow H_n(Y)$  and  $f^* : H^n(Y) \rightarrow H^n(X)$  are zero maps for  $n \geq 1$ . (Hint: One can do it from the definition, but much easier is to factor  $f$  as a composition of suitable two maps and use the fact that  $H_*$  and  $H^*$  are a functor and a cofunctor, respectively.)

**Exercise 2.** Let the cohomology rings of the spaces  $X$  and  $Y$  are the following

$$H^*(X) \cong \mathbb{Z}[x]/\langle x^n \rangle, \quad H^*(Y) \cong \mathbb{Z}[y]/\langle y^m \rangle$$

where  $x \in H^1(X)$  and  $y \in H^1(Y)$ . Prove that

$$H^*(X \vee Y) \cong \mathbb{Z}[u, v]/\langle u^n, v^m, uv \rangle.$$