

## HOMEWORK 8 – 2019

**Exercise 1.** Let  $f : M \rightarrow N$  be a map between two oriented compact manifolds of dimension  $n$  with fundamental classes  $[M]$  and  $[N]$ , respectively. We say that  $f$  has degree  $d$  if

$$f_*([M]) = d[N].$$

Prove that for every oriented compact manifold  $M$  of dimension  $n$  there is a map  $f : M \rightarrow S^n$  of degree 1. (Hint: Find a geometric prescription and use the definition of the fundamental class.)

**Exercise 2.** Use cup product and  $\mathbb{Z}_2$  coefficients to show that  $\mathbb{R}P^3$  is not homotopy equivalent to  $\mathbb{R}P^2 \vee S^3$ .