HOMEWORK 2 – 2020

Exercise 1. Given the short exact sequence of Abelian groups

$$0 \longrightarrow A \xrightarrow{f} B \xrightarrow{g} C \longrightarrow 0$$

the following conditions are equivalent:

(1) There exists $p\colon B \to A$ such that $p \circ f = id_A$.
(2) There exists $q\colon C \to B$ such that $g \circ q = id_C$.
(3) There are $p\colon B \to A$ and $q\colon C \to B$ such that $f \circ q + q \circ g = id_B$.

Prove that (2) $\Rightarrow$ (1) and (3).

Exercise 2. For the short exact sequence od chain complexes

$$0 \longrightarrow A_* \xrightarrow{f} B_* \xrightarrow{g} C_* \longrightarrow 0$$

there is a long exact sequence of homology groups

$$\ldots \longrightarrow H_{n+1}(C_*) \xrightarrow{\partial_*} H_n(A_*) \xrightarrow{f_*} H_n(B_*) \xrightarrow{g_*} H_n(C_*) \xrightarrow{\partial_*} H_{n-1}(A_*) \longrightarrow \ldots$$

with the connecting homomorphism ∂_* defined by the prescription

$$\partial_*([c]) = [a], \quad \text{where} \;\; \partial c = 0,\; f(a) = \partial b,\; g(b) = c.$$

(1) Prove the exactness in $H_n(A_*)$.
(2) Prove the exactness in $H_n(B_*)$.
(3) Prove the exactness in $H_n(C_*)$.