HOMEWORK 6 – 2020

Exercise 1. Construct a CW-complex X with the following homology groups:

$$H_n(X) = \begin{cases} \mathbb{Z}, & \text{for } n = 0; \\ \mathbb{Z} \oplus \mathbb{Z}_1 0, & \text{for } n = 2020; \\ \mathbb{Z}_4, & \text{for } n = 2021; \\ 0, & \text{otherwise.} \end{cases}$$

Exercise 2. Let $f : X \to Y$ be a constant map. Prove that $f_* : H_n(X) \to H_n(Y)$ and $f^* : H^n(Y) \to H^n(X)$ are zero maps for $n \geq 1$. (Hint: One can do it from the definition, but much easier is to factor f as a composition of suitable two maps and use the fact that H_* and H^* are a functor and a cofunctor, respectively.)

Exercise 3. Let the cohomology rings of the spaces X and Y are the following

$$H^*(X) \cong \mathbb{Z}[x]/\langle x^n \rangle, \quad H^*(Y) \cong \mathbb{Z}[y]/\langle y^m \rangle$$

where $x \in H^1(X)$ and $y \in H^1(Y)$. Prove that

$$H^*(X \vee Y) \cong \mathbb{Z}[u,v]/\langle u^n, v^m, uv \rangle.$$