HOMEWORK 7 – 2020

Exercise 1. Let $f : M \to N$ be a map between two oriented compact manifolds of dimension n with fundamental classes $[M]$ and $[N]$, respectively. We say that f has degree d if

$$f_*([M]) = d[N].$$

Prove that for every oriented compact manifold M of dimension n there is a map $f : M \to S^n$ of degree 1. (Hint: Find a geometric prescription and use the definition of the fundamental class via local orientations.)

Exercise 2. Use cup product and $\mathbb{Z}_2$ coefficients to show that $\mathbb{R}P^3$ is not homotopy equivalent to $\mathbb{R}P^2 \vee S^3$.