SYSTÉMY LIN. DIF. ROVNIC 1. ŘÁDU

$$\mathbf{x}(t+1) = A(t)\mathbf{x}(t) + \mathbf{b}(t), \qquad \mathbf{x}(t_0) = \mathbf{x}_0$$

a) $\mathbf{b}(t) \equiv 0 \quad \to \quad \mathbf{x}(t) = \left(\prod_{k=t_0}^{t-1} A(k)\right) \cdot \mathbf{c}$

speciálně pro konstantní matici A

$$\mathbf{x}(t) = A^{t-t_0} \cdot \mathbf{c} = A^{t-t_0}\mathbf{x}_0 \quad \Big| \quad \mathbf{x}(t_0) = A^0 \cdot \mathbf{c} = \mathbf{x}_0$$

b) obecněji $\mathbf{x}(t) = \left(\prod_{k=t_0}^{t-1} A(k)\right) \cdot \mathbf{x}_0 + \sum_{k=t_0}^{t-1} \left(\prod_{l=k+1}^{t-1} A(l)\right) \cdot \mathbf{b}(k)$

→ budeme potřebovat umocňovat matici A

↳ Jordanův kanonický tvar $\quad A = P \cdot J \cdot P^{-1}$

$$A^2 = PJ\cancel{P^{-1}}\cancel{P}JP^{-1} = PJ^2P^{-1}$$

$$A^t = PJ^tP^{-1}$$

$$J = \begin{pmatrix} J_1 & & 0 \\ 0 & J_2 & \\ 0 & 0 & \ddots \end{pmatrix} \quad ; \quad J = \begin{pmatrix} \lambda & 1 & \\ 0 & \lambda & \\ & & \ddots \end{pmatrix}$$

$P = (v_1, v_2 \dots v_k) \leftarrow$ vlastní vektory

$$J_1 = \lambda \qquad J_1^t = \lambda^t$$

$$J_2 = \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix} \quad J_2^t = \begin{pmatrix} \lambda^t & t \cdot \lambda^{t-1} \\ 0 & \lambda^t \end{pmatrix}$$

$$\vdots$$

$\boxed{\text{str. 74 DiskrMod}}$

pr.

$x(t+1) = \underbrace{\begin{pmatrix} 2 & -1 \\ 0 & 4 \end{pmatrix}}_{A} x(t) \quad ; \quad x(0) = \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$A^t = P \cdot J^t \cdot P^{-1} \quad ; \quad J^t = \begin{pmatrix} 2^t & 0 \\ 0 & 4^t \end{pmatrix}$

$x(t) = A^t \cdot x(0)$

$(2-\lambda)(4-\lambda) \rightarrow \begin{matrix} \lambda_1 = 2 \\ \lambda_2 = 4 \end{matrix}$

$v_1: \begin{pmatrix} 0 & -1 \\ \cancel{0} & \cancel{2} \end{pmatrix} \quad v_1 = (1;0)^T$

$v_2: \begin{pmatrix} -2 & -1 \\ \cancel{0} & \cancel{0} \end{pmatrix} \quad v_2 = (1;-2)$

$P = \begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix}$

$P^{-1} = \frac{-1}{2} \cdot \begin{pmatrix} -2 & -0 \\ -1 & 1 \end{pmatrix}^T = -\frac{1}{2} \begin{pmatrix} -2 & -1 \\ 0 & 1 \end{pmatrix}$

$-\frac{1}{2}\begin{pmatrix} 1 & 1 \\ 0 & -2 \end{pmatrix} \cdot \begin{pmatrix} 2^t & 0 \\ 0 & 4^t \end{pmatrix} \cdot \begin{pmatrix} -2 & -1 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 \\ 2 \end{pmatrix}$

$= -\cancel{\frac{1}{2}} \begin{pmatrix} 2^t & 4^t \\ 0 & -2 \cdot 4^t \end{pmatrix} \begin{pmatrix} \cancel{-4}\ 2 \\ \cancel{2}\ 1 \end{pmatrix} = \begin{pmatrix} 2 \cdot 2^t - 4^t \\ 2 \cdot 4^t \end{pmatrix}$

$$\begin{pmatrix} 1 & 3 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}^{-1} = \frac{1}{1}\begin{pmatrix} 1 & 0 & 0 \\ -3 & 1 & 0 \\ 1 & -2 & 1 \end{pmatrix}^{T} = \begin{pmatrix} 1 & -3 & 1 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}$$

pr. $A = \begin{pmatrix} 1 & -5 \\ 1 & -1 \end{pmatrix}$ | $-(1-\lambda)(1+\lambda)+5 = -1+\lambda-\lambda+\lambda^2+5$

$= \lambda^2+4 \rightarrow \lambda = \pm 2i = 2\cdot\left(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2}\right)$

$$A-\lambda E = \begin{pmatrix} 1-2i & -5 \\ 1 & -1-2i \end{pmatrix} \sim \begin{pmatrix} 1-2i & -5 \\ \cancel{1-2i} & \cancel{-5} \end{pmatrix}$$

$v_1 = (5;\ 1-2i)$

$v_2 = (5;\ 1+2i)$

$J = \begin{pmatrix} 2i & 0 \\ 0 & -2i \end{pmatrix}$

$$x(t) = (2i)^t \cdot \begin{pmatrix} 5 \\ 1-2i \end{pmatrix} = 2^t \Big(\underbrace{\cos\frac{t\pi}{2}}_{c} + i\underbrace{\sin\frac{t\pi}{2}}_{s}\Big) \cdot \begin{pmatrix} 5 \\ 1-2i \end{pmatrix}$$

$P = \begin{pmatrix} 5 & 5 \\ 1-2i & 1+2i \end{pmatrix}$

$$= 2^t \begin{pmatrix} 5\cdot c + i\cdot 5s \\ c - i2c + is + 2s \end{pmatrix} = 2^t \cdot \begin{pmatrix} 5c \\ c+2s \end{pmatrix} + 2^t i \begin{pmatrix} 5s \\ s-2c \end{pmatrix}$$

$$x(t) = 2^t \begin{pmatrix} 5c \\ c+2s \end{pmatrix} \cdot A + 2^t \begin{pmatrix} 5s \\ s-2c \end{pmatrix} \cdot B \qquad A, B \in \mathbb{R}$$

$$A = \begin{pmatrix} 4 & 1 & 2 \\ 0 & 2 & -4 \\ 0 & 1 & 6 \end{pmatrix} \qquad |A-\lambda E| = \begin{vmatrix} 4-\lambda & 1 & 2 \\ 0 & 2-\lambda & -4 \\ 0 & 1 & 6-\lambda \end{vmatrix} = \dots \Rightarrow \lambda_1 = \lambda_2 = \lambda_3 = 4$$

$$\begin{pmatrix} 0 & 1 & 2 \\ \cancel{0} & \cancel{-2} & \cancel{-4} \\ \cancel{0} & \cancel{1} & \cancel{2} \end{pmatrix} \qquad \begin{aligned} v_1 &= (1; -2; 1) \\ v_2 &= (0; -2; 1) \end{aligned} \qquad P = \begin{pmatrix} v_1 & v_3 & v_2 \end{pmatrix} \quad J = \left(\begin{array}{cc|c} 4 & 1 & 0 \\ 0 & 4 & 0 \\ \hline 0 & 0 & 4 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 0 & 1 & 2 & 1 \\ \cancel{0} & \cancel{-2} & \cancel{-4} & \cancel{2} \\ \cancel{0} & \cancel{1} & \cancel{2} & \cancel{1} \end{array}\right) \qquad \begin{aligned} v_3 &= (0; 3; -1) \\ \tilde{v}_3 &= (0; -1; 1) \end{aligned}$$

$$\begin{array}{cc} v_1 & v_2 \\ \uparrow ? & ?\uparrow \\ v_3 & v_3 \end{array} \qquad \left(\begin{array}{ccc|c} 0 & 1 & 2 & 0 \\ \cancel{0} & \cancel{-2} & \cancel{-4} & \cancel{-2} \\ 0 & 1 & 2 & 1 \end{array}\right)$$

NEMÁ ŘEŠENÍ

$$P = \begin{pmatrix} 1 & 0 & 0 \\ -2 & 3 & -2 \\ 1 & -1 & 1 \end{pmatrix} \qquad P^{-1} = \frac{1}{1} \begin{pmatrix} 1 & 0 & -1 \\ 0 & 1 & -\cancel{3}1 \\ 0 & +2 & 3 \end{pmatrix}^T$$

$$|P| = 1$$

$$x(t) = \underbrace{P J^t}_{\text{OBECNÉ}} P^{-1} x_0 = \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ -2 & 3 & -2 \\ 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} 4^t & t \cdot 4^{t-1} & 0 \\ 0 & 4^t & 0 \\ 0 & 0 & 4^t \end{pmatrix}}_{\text{FUND. ŘEŠENÍ } \Phi} \underbrace{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ -1 & 1 & 3 \end{pmatrix}}_{k} x_0$$

$$\Phi(t) = \begin{pmatrix} 4^t & t \cdot 4^{t-1} & 0 \\ -2 \cdot 4^t & -2 \cdot t\, 4^{t-1} + 3 \cdot 4^t & -2 \cdot 4^t \\ 4^t & t \cdot 4^{t-1} - 4^t & 4^t \end{pmatrix} \cdot k$$

$$\boxed{x(t) = P J^t P^{-1} \cdot x(0)}$$

$P \cdot J^t$ – báze (obecné ř.)

$$\begin{pmatrix} 4^t \\ -2\cdot 4^t \\ 4^t \end{pmatrix}\cdot k_1 + \begin{pmatrix} t\cdot 4^{t-1} \\ -2\cdot t\cdot 4^{t-1} + 3\cdot 4^t \\ t\cdot 4^{t-1} - 4^t \end{pmatrix} k_2 + \begin{pmatrix} 0 \\ -2\cdot 4^t \\ 4^t \end{pmatrix}\cdot k_3$$

$$= 4^t \underbrace{\begin{pmatrix} 1 \\ -2 \\ 1 \end{pmatrix}}_{v_1} k_1 + 4^t \underbrace{\begin{pmatrix} \frac{t}{4} + 0 \\ -\frac{t}{2} + 3 \\ \frac{t}{4} - 1 \end{pmatrix}}_{\frac{1}{4} v_1 + v_3} k_2 + 4^t \underbrace{\begin{pmatrix} 0 \\ -2 \\ 1 \end{pmatrix}}_{v_2} k_3$$

$$J = \begin{pmatrix} \lambda_1 & 0 & 0 \\ 0 & \lambda_2 & 0 \\ 0 & 0 & \lambda_3 \end{pmatrix} \qquad P = \begin{pmatrix} v_1 & v_2 & v_3 \end{pmatrix}$$

$$x(t) = \lambda_1^t \cdot v_1 \cdot k_1 + \lambda_2^t \cdot v_2 \cdot k_2 + \lambda_3^t \cdot v_3 \cdot k_3$$