Pr: $x(t+2)-5x(t+1)+6x(t) = (1+t)\cdot 1^t$

$$L(\lambda) = \lambda^2 - 5\lambda + 6 = (\lambda-2)(\lambda-3)$$

$$P(\lambda) = (\lambda-1)^2$$

$$L(\lambda)\cdot P(\lambda) = (\lambda-1)^2(\lambda-2)(\lambda-3)$$

$$x_H(t) = A\, 2^t + B\cdot 3^t$$

$$x(t) = \underbrace{A\cdot 2^t + B\cdot 3^t}_{x_H} + \underbrace{(C+Dt)\cdot 1^t}_{x_P}$$

$$C + D(t+2) - 5(C + D(t+1)) + 6\cdot(C+Dt) = 1+t$$

$$C + Dt + 2D - 5C - 5Dt - 5D + 6C + 6Dt = 1 + t$$

$t:\ 2D = 1$ $\qquad D = 1/2$

$t^0:\ 2C - 3D = 1$ $\qquad C = \frac{5}{4}$

$$x(t) = A\cdot 2^t + B\cdot 3^t + \frac{5}{4} + \frac{t}{2}$$

$A, B \in \mathbb{R}$

Pr. $x(t+2)-5x(t+1)+4x(t)=4^t-t^2 \rightsquigarrow P_1(\lambda)=(\lambda-4)$

$P_2(\lambda)=(\lambda-1)^3$

$L(\lambda)=\lambda^2-5\lambda+4=(\lambda-1)(\lambda-4)$

$x_H(t)=A+4^t\cdot B$

$L(\lambda)\cdot P_1(\lambda)\cdot P_2(\lambda)=(\lambda-1)^4\cdot(\lambda-4)^2 \quad x(t)=A+\underline{Bt}+\underline{Ct^2}+\underline{Dt^3}+(E+\underline{Ft})4^t$

a) $B(t+2)-5B(t+1)+4Bt+C(t+2)^2-5C(t+1)^2+4C(t)^2+$

$+D(t+2)^3-5D(t+1)^3+4Dt^3=-t^2 \quad \to B,C,D$

b) $F(t+2)\cdot 4^{t+2}-5F(t+1)\cdot 4^{t+1}+4Ft\cdot 4^t=4^t \quad \to F$

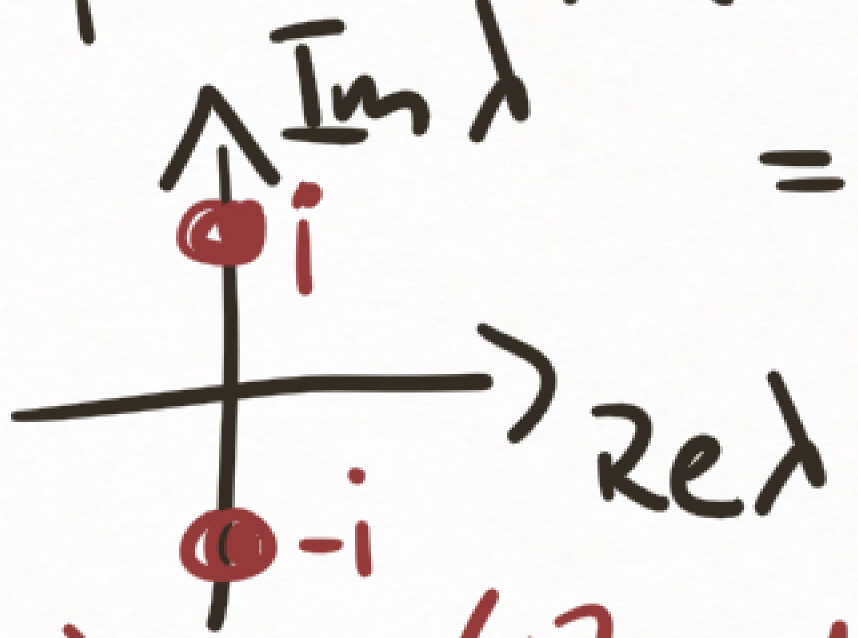
Pr.: $x(t+2) - x(t) = t\cdot\cos\left(\frac{\pi t}{2}\right)$

$$L(\lambda) = \lambda^2 - 1 = (\lambda+1)(\lambda-1)$$

$$x_H(t) = A + (-1)^t\cdot B$$

$$x(t) = A + B(-1)^t + (C+Dt)\cdot\cos\frac{\pi t}{2} + (E+Ft)\cdot\sin\frac{\pi t}{2}$$

víme, že $1\cdot\left(\cos\frac{\pi}{2} \pm i\sin\frac{\pi}{2}\right) = \pm i$



$$P(\lambda) = (\lambda^2+1)^2$$

$$(C+Dt+2D)\cdot\underbrace{\cos\frac{\pi t+2\pi}{2}}_{\cos\left(\frac{\pi t}{2}+\pi\right) = -\cos\frac{\pi t}{2}} - (C+Dt)\cdot\cos\frac{\pi t}{2} + (E+Ft+2F)\cdot\underbrace{\sin\frac{\pi t+2\pi}{2}}_{\sin\left(\frac{\pi t}{2}+\pi\right) = -\sin\frac{\pi t}{2}} - (E+Ft)\sin\frac{\pi t}{2}$$

$$\cos\frac{\pi t}{2}:\ \left(C+2D\right)\cdot\left(-\cancel{\cos\frac{\pi t}{2}}\right) - C\cdot\cancel{\cos\frac{\pi t}{2}} = 0 \rightarrow -2C-2D=0$$

$$C=-D$$

$$t\cdot\cos\frac{\pi t}{2}:\ -D-D=1 \qquad 2D=-1$$

$$\sin\frac{\pi t}{2}:\ -E-2F-E=0 \qquad E=-F$$

$$t\cdot\sin\frac{\pi t}{2}:\ -F-F=0 \qquad F=0$$

$$F=0 \qquad E=0 \qquad D=-1/2 \qquad C=1/2$$

$$x(t)=A+B(-1)^t+1/2\,(1-t)\cdot\cos\frac{\pi t}{2}$$

Ricattiho rovnice

- $x(t+1)\cdot x(t) + p(t)x(t+1) + q(t)x(t) = 0$

<u>subst</u>: $x(t) = \frac{1}{z(t)}$ $\rightarrow$ $\frac{1}{z(t+1)\cdot z(t)} + \frac{p(t)}{z(t+1)} + \frac{q(t)}{z(t)} = 0$

podm. $x(t) \neq 0$ $z(t) \neq 0$

$1 + p(t)z(t) + q(t)z(t+1) = 0$

jinak $x(t) = 0$ je řeš.

- $x(t+1) = \frac{a(t)x(t) + b(t)}{c(t)x(t) + d(t)}$ $\rightarrow$ $c(t)x(t) + d(t) = \frac{z(t+1)}{z(t)}$

př. Pielouova logistická rovnice

$$x(t+1) = \frac{a x(t)}{1 + b x(t)} \qquad z(t) = \frac{1}{x(t)} \qquad \boxed{x(t) \equiv 0}$$

$$\frac{1}{z(t+1)} = \frac{\frac{a}{z(t)}}{1 + \frac{b}{z(t)}} \iff \frac{z(t)+b}{\cancel{z(t)} \cdot z(t+1)} = \frac{a}{\cancel{z(t)}}$$

$$a \cdot z(t+1) = z(t) + b$$

$$z(t+1) = \frac{z(t)}{a} + \frac{b}{a}$$

$$z(t) = z(0) \cdot \prod_{k=0}^{t-1} \frac{1}{a} + \sum_{k=0}^{t-1} \frac{b}{a} \cdot \prod_{l=k+1}^{t-1} \frac{1}{a}$$

$$= z(0) \cdot \left(\frac{1}{a} \right)^t + \sum_{k=0}^{t-1} \frac{b}{a} \left(\frac{1}{a} \right)^{t-k-1}$$

a) $a=1 \to z(0) + b \sum_{k=0}^{t-1} 1 = z(0) + bt$ $\to x(t) = \frac{1}{z(t)}$

b) $a \neq 1 \to z(0) \cdot \left(\frac{1}{a} \right)^t + \frac{b}{a} \cdot \left(\frac{1}{a} \right)^{t-1} \sum_{k=0}^{t-1} a^k$

$$= z(0) \cdot a^{-t} + b \cdot a^{-t} \frac{a^t - 1}{a-1}$$

$$= \begin{cases} x(t) = \dfrac{1}{\frac{1}{x(0)} + b(t)} \\ x(t) = \dfrac{1}{z(t)} \end{cases}$$

Pr:

$$x(t+1)=\frac{2x(t)+3}{3x(t)+2}$$

$$3x(t)+2=\frac{z(t+1)}{z(t)}$$

$$x(t)=\frac{1}{3}\left(\frac{z(t+1)}{z(t)}-2\right)$$

$$\frac{1}{\cancel{3}}\,\frac{z(t+2)-2z(t+1)}{\cancel{z(t+1)}}=\frac{\frac{2}{\cancel{3}}\left(\frac{z(t+1)-2z(t)}{z(t)}\right)+\cancel{3}\,9}{\frac{\cancel{z(t+1)}}{z(t)}}\quad /\cdot 3$$

$$z(t+2)-2z(t+1)=2z(t+1)-4z(t)+9z(t)$$

$$z(t+2)-4z(t+1)-5z(t)=0$$

$$z(t+2) - 4z(t+1) - 5z(t) = 0$$

$$L(\lambda) = \lambda^2 - 4\lambda - 5 = 0 \iff (\lambda - 5)(\lambda + 1) = 0$$

$$z(t) = A \cdot 5^t + B(-1)^t$$

$$x(t) = \frac{1}{3}\left(\frac{z(t+1)}{z(t)} - 2\right) = \frac{1}{3}\left(\frac{A \cdot 5^{t+1} + B(-1)^{t+1}}{A \cdot 5^t + B(-1)^t} - 2\right) =$$

$$= \frac{1}{3}\left(\frac{5\frac{A}{B}5^t + (-1)^{t+1}}{\frac{A}{B}5^t + (-1)^t} - 2\right) \qquad \frac{A}{B} = C \in R$$

DÚ: $$x(t+1) = \frac{2x(t)+4}{x(t)-1}$$

$$x(t+1) = \frac{2x(t)}{x(t)+3}$$