

(2p)(1)

$$|\hat{x}_{n+1} - x_{n+1}| = |\hat{g}(\hat{x}_n) - g(x_n)|$$

$$\leq \underbrace{|g'(\hat{x}_n) - g'(x_n)|}_{\leq \varepsilon} + |g(\hat{x}_n) - g(x_n)| \leq q|\hat{x}_n - x_n|$$

$$\leq \varepsilon + q|\hat{x}_n - x_n| \leq \varepsilon + q(\varepsilon + q|\hat{x}_{n-1} - x_{n-1}|)$$

$$\dots \leq \varepsilon + \varepsilon q + \varepsilon q^2 + \dots + q^{n+1} \underbrace{(\hat{x}_0 - x_0)}$$

$$\leq \frac{\varepsilon}{1-q}$$

Tedy platí

$$\underline{|\hat{x}_{n+1} - x_{n+1}| \leq \frac{\varepsilon}{1-q}}$$

(viz věta o modifikovaném operátoru.

Dále

$$|f - \hat{f}| \leq \frac{\varepsilon}{1-q}$$