13.3. (b) Vzájemná poloha rovin BDE a AFH

σ_1 $\quad\quad$ σ_2

$\sigma_1: B + t_1 \overrightarrow{BD} + t_2 \overrightarrow{BE}$

$[1,0,0] + t_1(-1,1,0) + t_2(-1,0,1)$

norm. vektor: $n = (1,1,1)$

$x_1 + x_2 + x_3 = d$

$1 + 0 + 0 = d$

$\boxed{x_1 + x_2 + x_3 = 1}$

$\sigma_2: A + s_1 \overrightarrow{AF} + s_2 \overrightarrow{AH}$

$[0,0,0] + s_1(1,0,1) + s_2(0,1,1)$

norm. vektor: $n = (1,1,-1)$

$\boxed{x_1 + x_2 - x_3 = 0}$

Závěr: roviny jsou různoběžné

Určíme $p := \sigma_1 \cap \sigma_2$ přímka

$x_1 + x_2 + x_3 = 1$

$x_1 + x_2 - x_3 = 0$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & 1 & -1 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & -2 & -1 \end{array}\right)$$

$x_1 + x_2 + x_3 = 1$

$x_1 + v + \frac{1}{2} = 1$

$$\boxed{\begin{aligned} x_2 &= v \\ x_3 &= \frac{1}{2} \\ x_1 &= 1/2 - v \end{aligned}}$$

$v \in \mathbb{R}$

$(x_1, x_2, x_3) = \left(\frac{1}{2} - v, v, \frac{1}{2}\right)$

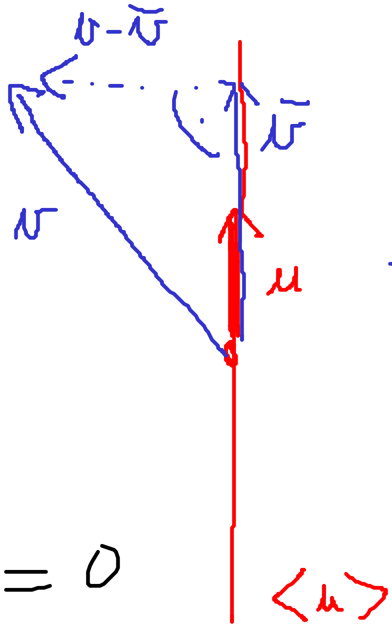
$= \left[\frac{1}{2}, 0, \frac{1}{2}\right] + v\,(-1, 1, 0)$ parametrický popis přímky p

Př: Co musí platit pro souřadnici y bodu $A = [1, y, 0]$, aby jeho vzdálenost od roviny $\tau: 3x - 2y - 6z = 0$ byla rovna 5.

- $O = [0, 0, 0] \in \tau$

$u := \overrightarrow{OA} = (1, y, 0)$

Určíme kolmý průmět vektoru v do normálového směru $u = (3,-2,-6)$.



$\bar{v} = a\cdot u\,,\quad a\in\mathbb{R}$

$\|\bar{v}\| = \|a\cdot u\| =$

$= |a|\cdot\|u\| = |a|\cdot\sqrt{9+4+36}$

$= 7\,|a|$

Má platit: $\|v\| = 7|a| = 5$

$|a| = \frac{5}{7}$

$a = \pm\frac{5}{7}$

$v - \bar{v} \perp u$

$\langle v - a\cdot u, u\rangle = 0$

$\langle v, u\rangle - a\underbrace{\langle u, u\rangle} = 0$

$(3-2y) - a\cdot 49 = 0$

• $a = \frac{5}{7} \Rightarrow 3-2y-5\cdot 7 = 0$

$-2y = 32\,,\quad \boxed{y = -16}$

• $a = -\frac{5}{7} \Rightarrow 3-2y+5\cdot 7 = 0$

$-2y = -38\,,\quad \boxed{y = 19}$

• Jiné řešení: najdeme roviny α_1, α_2 rovnoběžné s ρ ve vzdálenosti 5.

$\alpha_i: 3x - 2y - 6z = d_i, \quad i \in \{1,2\}$

$n = (3, -2, -6) \qquad \|n\| = \sqrt{9+4+36} = 7$

$\Rightarrow \pm \frac{5}{7} n$ má délku 5

$B_1 := [0,0,0] + \frac{5}{7} n \in \alpha_1$

$B_2 := [0,0,0] - \frac{5}{7} n \in \alpha_2$

kde $B_1 = \left[\frac{15}{7}, -\frac{10}{7}, -\frac{30}{7}\right]$

$B_2 = \left[-\frac{15}{7}, \frac{10}{7}, \frac{30}{7}\right]$

$B_1 \in \alpha_1 \Rightarrow 3 \cdot \frac{15}{7} - 2 \cdot \frac{10}{7} - 6 \cdot \frac{30}{7} = d_1$

$\frac{1}{7}(45 - 20 - 180) = d_1$

$\boxed{\frac{155}{7} = d_1}$

$B_2 \in \alpha_2 \Rightarrow \boxed{d_2 = -\frac{155}{7}}$

$\alpha_1: 3x - 2y - 6z = \frac{155}{7}$

$\alpha_2: 3x - 2y - 6z = -\frac{155}{7}$

$A = [1, y, 0] \in \alpha_1$

$3 - 2y = -\frac{155}{7}$ . . . . . .

$\underline{Pr2}: \quad \sigma: x + y + z + 1 = 0$

$$p: \frac{x-1}{2} = \frac{y+1}{1} = \frac{1-z}{1}$$

$p:$

$\frac{x-1}{2} = y+1$	$x - 1 = 2y + 2$
$\frac{x-1}{2} = 1 - z$	$x - 1 = 2 - 2z$

$$\boxed{\begin{array}{l} x - 2y = 3 \\ x + 2z = 3 \end{array}}$$

$\sigma \cap p:$

$$\begin{array}{l} x + y + z = -1 \\ x - 2y = 3 \\ x + 2z = 3 \end{array}$$

$$\left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 1 & -2 & 0 & 3 \\ 1 & 0 & 2 & 3 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & -3 & -1 & 2 \\ 0 & -1 & 1 & 2 \end{array}\right) (\cdot 3)$$

$$\sim \left(\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 1 & -1 & -2 \\ \cancel{0} & \cancel{0} & \cancel{-4} & \cancel{-4} \\ 0 & 0 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 1 \end{array}\right)$$

$x_1 = 1$

$x_2 = -1$

$x_3 = 1$

Závěr: $p \cap \tau = \{A\}$,

kde $A = [1, -1, 1]$

různoběžné