51. (ii) ρ relace na $A = \mathbb{R}^2$

$\rho \subseteq \mathbb{R}^2 \times \mathbb{R}^2$

$(x,y)\,\rho\,(u,v) \iff y - v = 2(x-u)$

$y - 2x = v - 2u$

↳ lehce se ukáže, že to je ekvivalence

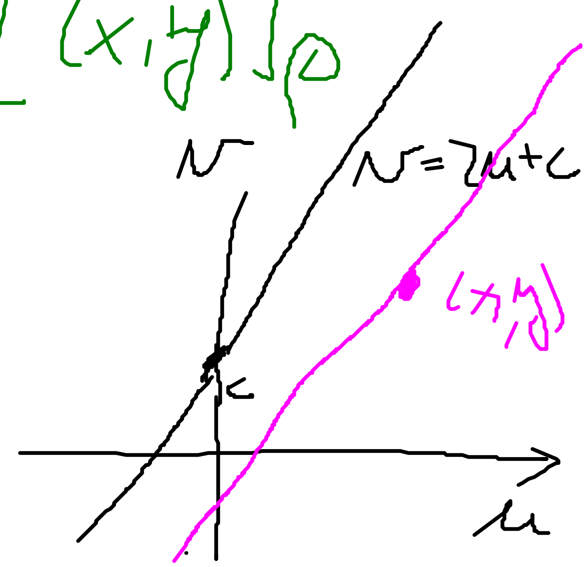
$(x,y) \in \mathbb{R}^2$ lib.

$[(x,y)]_\rho = \{(u,v) \in \mathbb{R}^2 \mid (x,y)\,\rho\,(u,v)\}$

(x,y) pevně daný, $y - 2x = c \in \mathbb{R}$

$\leadsto$ všechny $(u,v) \in [(x,y)]_\rho$ splňují $v - 2u = c$

$v = 2u + c$



Zamera: $\mathbb{R}^2 = \bigcup_{c \in \mathbb{R}} P_c$, kjer

$$P_c = \{(u,v) \in \mathbb{R}^2 \mid v = 2u + c\}$$

4.5 $\mathbb{Z} \setminus \{0\} = \mathbb{Z}_+ \cup \mathbb{Z}_-$

$\rho \subseteq \mathbb{Z} \times \mathbb{Z}$

Pr: relacija ρ na $\mathbb{Z}$

(a) $(x,y) \in \rho \iff 2 \mid (x^2 - y^2)$

$x \rho y$

- ρ je ekvivalenca
- $(x,y) \in \rho \iff 2 \mid (x+y)(x-y)$

$\iff x, y$ imata isto parnost

$[x]_\rho = \{y \in \mathbb{Z} \mid x, y \text{ imata isto parnost}\}$

$\mathbb{Z} = \{\text{soda števila}\} \cup \{\text{liha števila}\}$

$= [0]_\rho \cup [1]_\rho$

5.2 (vi)

$$x \rho y \Leftrightarrow (x = y) \vee (2 \nmid x \wedge 2 \mid y) \vee$$
$$\vee (2 \mid x+y \wedge x < y)$$

- ρ je antisymetrická;

Predpokladajme, že $x \rho y$ a zároveň $y \rho x$

$\Updownarrow$ $\Updownarrow$

$x = y$
nebo $2 \nmid x \wedge 2 \mid y$
nebo $2 \mid x+y \wedge x < y$

$x = y$
nebo $2 \nmid y \wedge 2 \mid x$
nebo $2 \mid x+y \wedge y < x$

$\Downarrow$

$x = y$