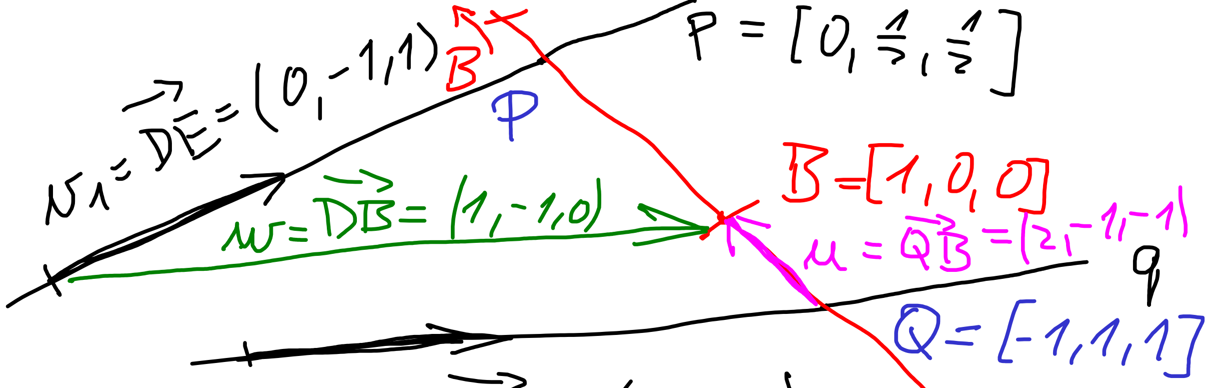
13.1 (a)

$[0,1,0] = D$



$[1,1,1] = G \qquad v_2 = \vec{GH} = (-1,0,0)$

$\boxed{\text{NB: } P = Q + \tfrac{1}{2} u}$

r

Uvažme rovinu α určenou přímkou p a bodem B. Pak $\underbrace{Q = \alpha \cap q}$

$\alpha: D + t v_1 + s w$

$$Q = D + t v_1 + s w = G + r v_2, \quad t, s, r \in \mathbb{R}$$

$$t v_1 + s w - r v_2 = G - D = (1,0,1)$$

$$\begin{array}{ccc} t & s & r \end{array}$$
$$\left(\begin{array}{ccc|c} 0 & 1 & 1 & 1 \\ -1 & -1 & 0 & 0 \\ 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 0 & -1 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|c} 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & -1 \\ 0 & 0 & 1 & 2 \end{array}\right)$$

$$Q = G + r v_2 = [1,1,1] + 2(-1,0,0) = [-1,1,1] \qquad \begin{array}{|l} t = 1 \\ s = -1 \\ r = 2 \end{array}$$

$P = p \cap r$

$$P = D + t v_1 = Q + s u \qquad t, s \in \mathbb{R}$$

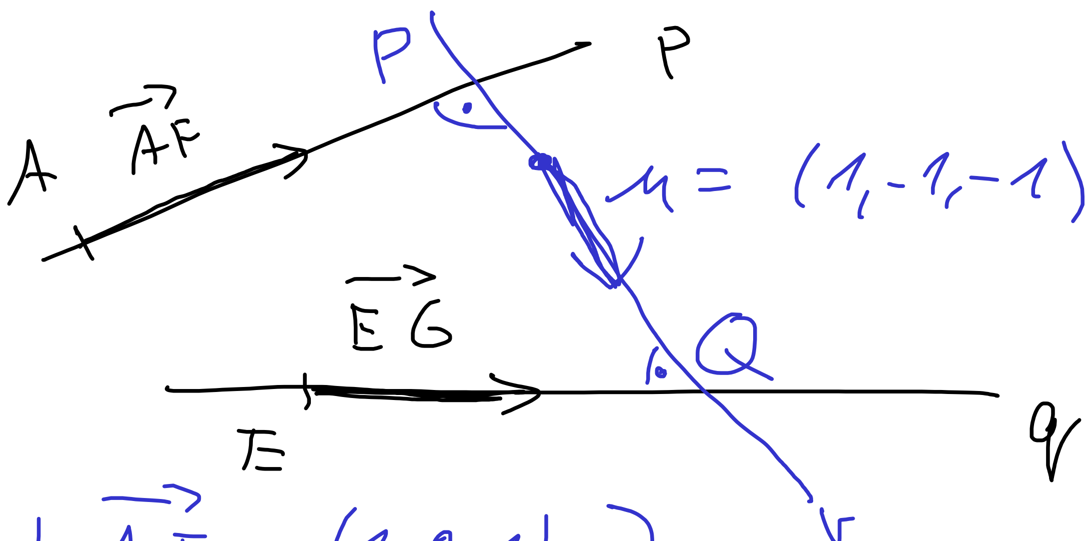
$$t v_1 - s u = Q - D = (-1,0,1)$$

$$\begin{array}{cc} t & s \\ \end{array}$$
$$\left(\begin{array}{cc|c} 0 & -2 & -1 \\ -1 & 1 & 0 \\ 1 & 1 & 1 \end{array}\right) \sim \left(\begin{array}{cc|c} 1 & 1 & 1 \\ 0 & -2 & -1 \\ \cancel{0} & \cancel{-2} & \cancel{-1} \end{array}\right) \qquad \begin{array}{l} s = \frac{1}{2} \\ t = \frac{1}{2} \end{array}$$

$$P = D + t u_1 = [0,1,0] + \tfrac{1}{2}(0,-1,1) = [0, \tfrac{1}{2}, \tfrac{1}{2}]$$

(c) Určete vzdálenost přímek AF a EG.

① řešení ⇝ hledáme příčku kolmou na AF i EG



$$\left.\begin{array}{l} u \perp \overrightarrow{AF} = (1,0,1) \\ u \perp \overrightarrow{EG} = (1,1,0) \end{array}\right\} (1,-1,-1) = u$$

$$u = (a,b,c) \perp (1,0,1)$$
$$\langle (a,b,c), (1,0,1) \rangle = a + c = 0$$
$$\langle (a,b,c), (1,1,0) \rangle = a + b = 0$$

Uvažme rovinu α určenou

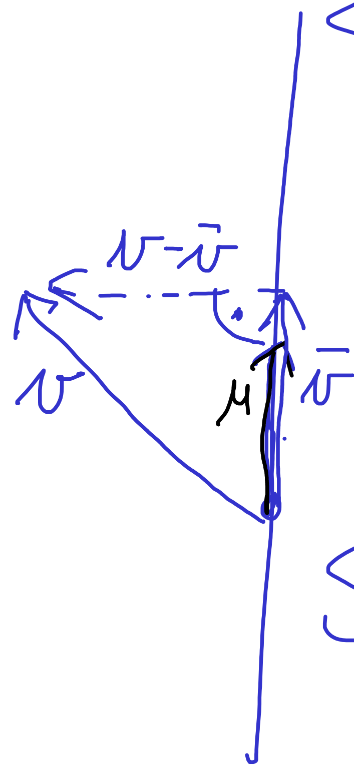
přímkou p a vektorem u

$\alpha: A + t\overrightarrow{AF} + s u$ . . . . .

(2) průmětem do ortogonálního doplňku

• ortogonální doplněk p a q je vekt. podp. kolmý na p, q $\Rightarrow$ je $\langle u \rangle$

• vezmeme lib. spojnici přímek p, q a určíme její kolmý průmět do $\langle u \rangle$



$v =: \overrightarrow{AE} = (0,0,1)$

$\underline{u = (1,-1,-1)}$

$\bar{v} = a u$

$v - \bar{v} \perp u$

$\langle v - a u, u \rangle = 0$

$\underbrace{\langle v, u \rangle}_{-1} - a \langle u, u \rangle = 0$

$-1 - a \cdot 3 = 0$

$a = -\frac{1}{3}$

$\bar{v} = -\frac{1}{3} u$

$= -\frac{1}{3}(1,-1,-1)$

$\|\bar{v}\| = \|-\frac{1}{3}(1,-1,-1)\| =$

$= \frac{1}{3}\|(1,-1,-1)\| = \frac{1}{3}\sqrt{1+1+1} = \underline{\underline{\frac{\sqrt{3}}{3}}}$

13.2: Určete vzdálenost bodu $x = [1,3,0,1]$ od podprostoru

$$\rho: \underbrace{[1,0,0,1]}_{A} + v\underbrace{(1,1,0,1)}_{v_1} + s\underbrace{(1,0,1,-1)}_{v_2} + t\underbrace{(2,1,2,0)}_{v_3}$$

② řešení – průmět do OG-doplňku

Určíme vektory $u \perp \rho$, tj.

$u \perp v_i, \quad i = 1,2,3$

$u = (x_1, x_2, x_3, x_4)$

$\langle u, v_1 \rangle = x_1 + x_2 + x_4 = 0$

$\langle u, v_2 \rangle = x_1 + x_3 - x_4 = 0$

$\langle u, v_3 \rangle = 2x_1 + x_2 + 2x_3 = 0$

$$\begin{pmatrix} 1 & 1 & 0 & 1 \\ 1 & 0 & 1 & -1 \\ 2 & 1 & 2 & 0 \end{pmatrix} \sim \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & -1 & 1 & -2 \\ 0 & -1 & 2 & -2 \end{pmatrix} \sim \begin{array}{c} \begin{matrix} x_1 & x_2 & x_3 & x_4 \end{matrix} \\ \begin{pmatrix} 1 & 1 & 0 & 1 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 1 & 0 \end{pmatrix} \end{array}$$

$x_1 + x_2 + x_4 = 0$ $\qquad x_2 = -2p$ $\qquad \Big|\ x_4 = p$

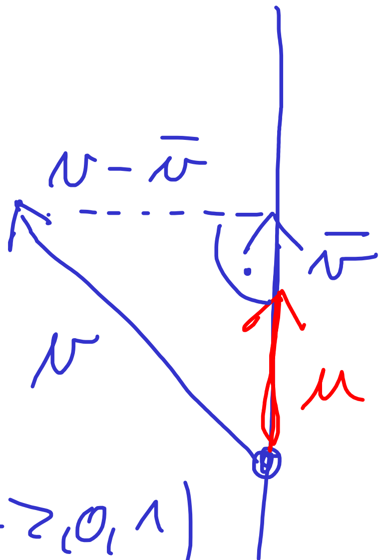
$x_1 - 2p + p = 0$ $\qquad x_1 = p$ $\qquad \Big|\ x_3 = 0$

$u = p(1,-2,0,1) \quad | \; p \in \mathbb{R}$

$$\boxed{u := (1,-2,0,1)}$$

Dále spočteme kolmý průmět $v := \overrightarrow{Ax} = (0,-3,0,0)$ do $\langle u \rangle$.

$\bar{v} = \boxed{a \cdot u}$



$v - \bar{v} \perp u$

$\langle v - a \cdot u, u \rangle = 0$

$\underbrace{\langle v, u \rangle}_{6} - a \underbrace{\langle u, u \rangle}_{(1+4+1)} = 0$

$6 - a(1+4+1) = 0$

$6 - 6a = 0 \quad a = 1$

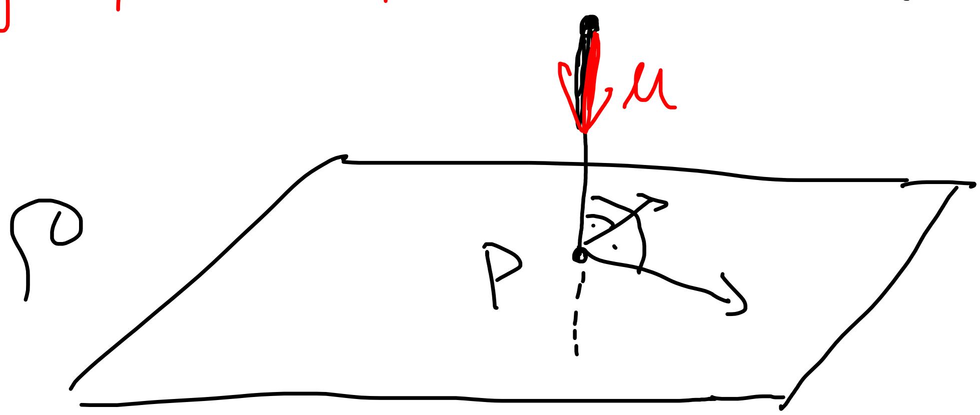
$\bar{v} = u = (1,-2,0,1)$

Hledaná vzdálenost je $\|\bar{v}\| = \|(1,-2,0,1)\| =$

$= \sqrt{1+4+1} = \sqrt{6}$

(1) Jiný postup

X $\quad \|\overrightarrow{XP}\|$ hledaná vzdálenost



$P = X + tu \in \rho$

$\quad = A + s_1 v_1 + s_2 v_2 + s_3 v_3$

13.3 (a) Vzájemná poloha rovin

BEG a ACH
α $\qquad$ β

$\alpha: B + v_1 \overrightarrow{BE} + v_2 \overrightarrow{BG}$

$[1,0,0] + v_1 (-1,0,1) + v_2 (0,1,1)$

norm. vektor $n = (1,-1,1)$

rovnice $\quad x_1 - x_2 + x_3 + d = 0$

$1 - 0 + 0 + d = 0, \; d = -1$

$\boxed{x_1 - x_2 + x_3 = 1}$

$\beta: A + s_1 \overrightarrow{AC} + s_2 \overrightarrow{AH}$

$[0,0,0] + s_1 (1,1,0) + s_2 (0,1,1)$

norm. vektor $n = (1,-1,1)$

$\boxed{x_1 - x_2 + x_3 = 0}$

závěr: rovnoběžné roviny

13.4 (a) odchylka AG a BD

$v_1 = \overrightarrow{AG} = (1,1,1)$

$v_2 = \overrightarrow{BD} = (-1,1,0)$

$$\cos\varphi = \frac{|\langle v_1, v_2\rangle|}{\|v_1\|\cdot\|v_2\|} = 0, \quad \varphi = \frac{\pi}{2}$$

(b) AF a AH

$$v_1 = \overrightarrow{AF} = (1,0,1)$$

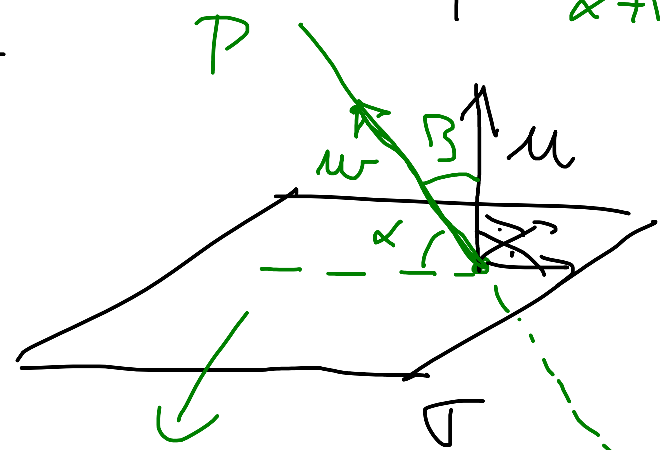
$$v_2 = \overrightarrow{AH} = (0,1,1)$$

$$\cos\varphi = \frac{|\langle v_1, v_2\rangle|}{\|v_1\|\cdot\|v_2\|} = \frac{1}{\sqrt{2}\cdot\sqrt{2}} = \frac{1}{2}$$

$$\varphi = \frac{\pi}{3}$$

(c) odchylka přímky $\underbrace{CG}_{p}$ a roviny $\underbrace{BDE}_{\sigma}$

$\alpha + \beta = \frac{\pi}{2}$



$$\sigma: B + t_1 \overrightarrow{BD} + t_2 \overrightarrow{BE}$$

$\overrightarrow{BD} \parallel (-1,1,0) = v_1$, $\quad \overrightarrow{BE} \parallel (-1,0,1) = v_2$

kolmý průmět p do σ

$u \perp v_1$

$u \perp v_2$

$u = (1,1,1)$

$$w = \overrightarrow{CG} = (0,0,1)$$

$$\cos\beta = \frac{|\langle u, w\rangle|}{\|u\|\cdot\|w\|} = \frac{1}{\sqrt{3}\cdot 1} = \frac{\sqrt{3}}{3}$$

$\beta = \arccos \frac{\sqrt{3}}{3}$

Hledaná odchylka $\alpha = \frac{\pi}{2} - \arccos \frac{\sqrt{3}}{3}$

(d) odchylka rovin

AFG a BDE $\rightarrow$ norm. vektor

$n_1 = (1,1,1)$

$\vec{AF} = (1,0,1)$

$\vec{AG} = (1,1,1)$

n_2 kolmý na

$n_2 = (1,0,-1)$

$$\cos\varphi = \frac{|\langle n_1, n_2\rangle|}{\|n_1\|\cdot\|n_2\|} = 0, \quad \varphi = \frac{\pi}{2}$$

13.5 Určete objem čtyřstěnu $ABCE$

$$V = \frac{1}{6}\left|\det\begin{pmatrix}\vec{AB}\\ \vec{AC}\\ \vec{AE}\end{pmatrix}\right| =$$

$$= \frac{1}{6}\left|\det\begin{pmatrix}1 & 0 & 0\\ 1 & 1 & 0\\ 0 & 0 & 1\end{pmatrix}\right| = \underline{\underline{\frac{1}{6}}}$$