4.1. $A = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}$ $\quad a_1 = b_1 = c_1 = 1$

$$A^2 = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^3 = A \cdot A^2 = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 3 & 6 \\ 0 & 1 & 3 \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^n = \begin{pmatrix} 1 & a_n & b_n \\ 0 & 1 & c_n \\ 0 & 0 & 1 \end{pmatrix}$$

$$\begin{pmatrix} 1 & * & * \\ 0 & 1 & * \\ 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & * & * \\ 0 & 1 & * \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & \cdot & \cdot \\ 0 & 1 & \cdot \\ 0 & 0 & 1 \end{pmatrix}$$

$$A^{n+1} = A \cdot A^n = \begin{pmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & a_n & b_n \\ 0 & 1 & c_n \\ 0 & 0 & 1 \end{pmatrix} =$$

$$\| \quad \begin{pmatrix} 1 & a_{n+1} & b_{n+1} \\ 0 & 1 & c_{n+1} \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & a_n+1 & 1+b_n+c_n \\ 0 & 1 & 1+c_n \\ 0 & 0 & 1 \end{pmatrix} =$$

$$a_{n+1} = a_n + 1$$

$$c_{n+1} = c_n + 1 \qquad \text{kde } a_1 = c_1 = 1$$

$$\Rightarrow \boxed{a_n = n = c_n}$$

$$A^n = \begin{pmatrix} 1 & n & b_n \\ 0 & 1 & n \\ 0 & 0 & 1 \end{pmatrix}$$

$$b_{n+1} = 1 + b_n + c_n$$

$$b_{n+1} = b_n + n + 1$$

$$b_{n+1} = b_{n-1} + n + (n+1)$$

$$= b_{n-2} + (n-1) + n + (n+1)$$

$$\vdots$$

$$= 1 + 2 + \dots + (n-1) + n + (n+1)$$

$$b_n = 1 + 2 + \dots + (n-1) + n = \frac{1}{2} n(n+1)$$

$$A^n = \begin{pmatrix} 1 & n & \frac{1}{2} n(n+1) \\ 0 & 1 & n \\ 0 & 0 & 1 \end{pmatrix}$$

4.2 $p: [2,0] + t(3,2), \quad t \in \mathbb{R}$

$q: [-1,2] + s(1,3), \quad s \in \mathbb{R}$

Průsečík: $A = p \cap q$

$A = [2,0] + t(3,2) = [-1,2] + s(1,3)$

$$\begin{aligned} x &= 2 + 3t = -1 + s \\ y &= 2t = 2 + 3s \end{aligned}$$

$$3t - s = -3$$
$$2t - 3s = 2$$

$\frac{2}{3} - 3 = \frac{-7}{3}$

$$-\frac{2}{3} \quad \begin{array}{cc|c} t & s & \\ \end{array} \left(\begin{array}{cc|c} 3 & -1 & -3 \\ 2 & -3 & 2 \end{array}\right) \sim \left(\begin{array}{cc|c} 3 & -1 & -3 \\ 0 & -\frac{7}{3} & 4 \end{array}\right)$$

$$-7s = 12$$
$$s = -\frac{12}{7}$$

$$3t - \left(-\frac{12}{7}\right) = -3$$
$$t + \frac{4}{7} = -1$$

$t = -1 - \frac{4}{7} = \frac{-7-4}{7}$

$t = -\frac{11}{7}$

$$s = -\frac{12}{7} \qquad t = -\frac{11}{7}$$

$$A = [2,0] - \frac{11}{7}(3,2) = \left[2 - \frac{33}{7}, -\frac{22}{7}\right] = \left[\frac{14-33}{7}, -\frac{22}{7}\right] = \left[-\frac{19}{7}, -\frac{22}{7}\right]$$

$$A = [-1,2] - \frac{12}{7}(1,3) = \dots$$

skalární součin

$\langle (a,b),(c,d) \rangle = ac + bd$

Rovnice přímky p:

$p: [2,0] + t\underbrace{(3,2)}$

směrový vektor v

$v \perp n$

$n = (2,-3)$ kolmý vektor

$p: 2x - 3y + c = 0$

$\langle v, n \rangle = 0$

$\langle (3,2),(2,-3) \rangle = 6 - 6 = 0$

$[2,0] \in p \Rightarrow$

$2 \cdot 2 - 3 \cdot 0 + c = 0$

$4 + c = 0 \Rightarrow c = -4$

$$\boxed{p: 2x - 3y - 4 = 0}$$

4.3 Najděte rovnici přímky p, jež prochází bodem [2,3] a je rovnoběžná s přímkou $x - 3y + 2 = 0$

$p: x - 3y + d = 0, \quad d \in \mathbb{R}$

$[2,3] \in p: \; 2 - 3 \cdot 3 + d = 0$

$d = 7$

$$\boxed{p: x - 3y + 7 = 0}$$

Určete parametrický popis přímky q procházející body $\underbrace{[1,3]}_{A}$ a $\underbrace{[-2,1]}_{B}$.

$\vec{AB} = B - A$

$q: A + t\,\vec{AB}, \quad t \in \mathbb{R}$

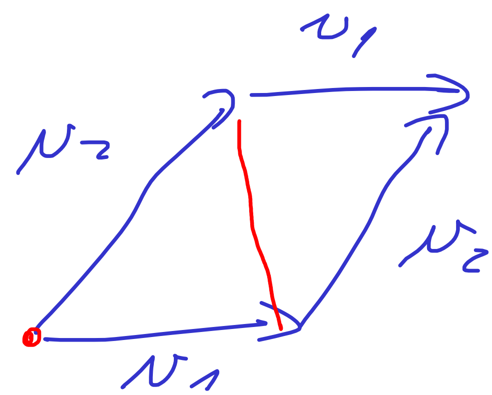
$q: [1,3] + t(-3,-2)$

4 $A = [1,1]$, $B = [3,2]$, $C = [-4,6]$

(i)

$$\det\begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$$

↳ determinant



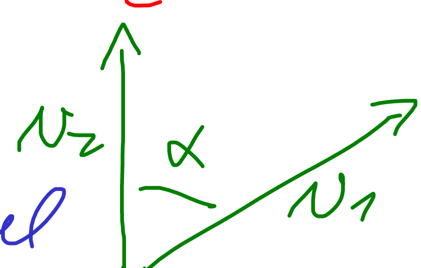
$$S = \frac{1}{2}\left|\det\begin{pmatrix} \vec{AB} \\ \vec{AC} \end{pmatrix}\right|$$

obsah rovnoběžníku je $S = \left|\det\begin{pmatrix} v_1 \\ v_2 \end{pmatrix}\right|$

$$= \frac{1}{2}\left|\det\begin{pmatrix} 2 & 1 \\ -5 & 5 \end{pmatrix}\right|$$

$$= \frac{1}{2}\left(10 - (-5)\right) = \frac{15}{2}$$

(ii) Vnitřní úhly:



$$\cos\alpha = \frac{\langle v_1, v_2 \rangle}{\|v_1\| \cdot \|v_2\|}$$

úhel vektorů

$\vec{AB} = (2,1)$

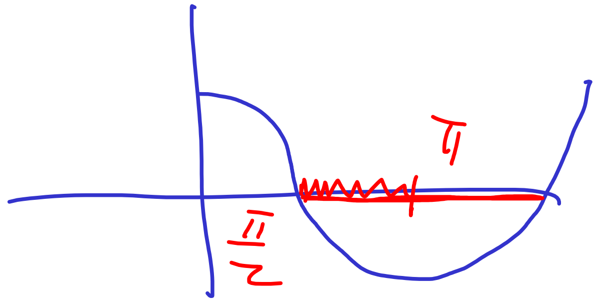
$\vec{AC} = (-5,5)$

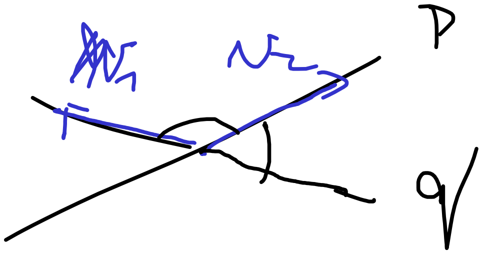
$$\cos\alpha = \frac{\langle (2,1), (-5,5) \rangle}{\|(2,1)\| \cdot \|(-5,5)\|} = \frac{-10+5}{\sqrt{2^2+1} \cdot \sqrt{(-5)^2+(5)^2}} = \frac{-5}{\sqrt{5} \cdot \sqrt{50}} =$$

↓

$\measuredangle BAC$

$$= \frac{-5}{\sqrt{5}\cdot 5\sqrt{2}} = -\frac{1}{\sqrt{10}} = -\frac{\sqrt{10}}{10}$$ (tupý úhel)





úhel přímek

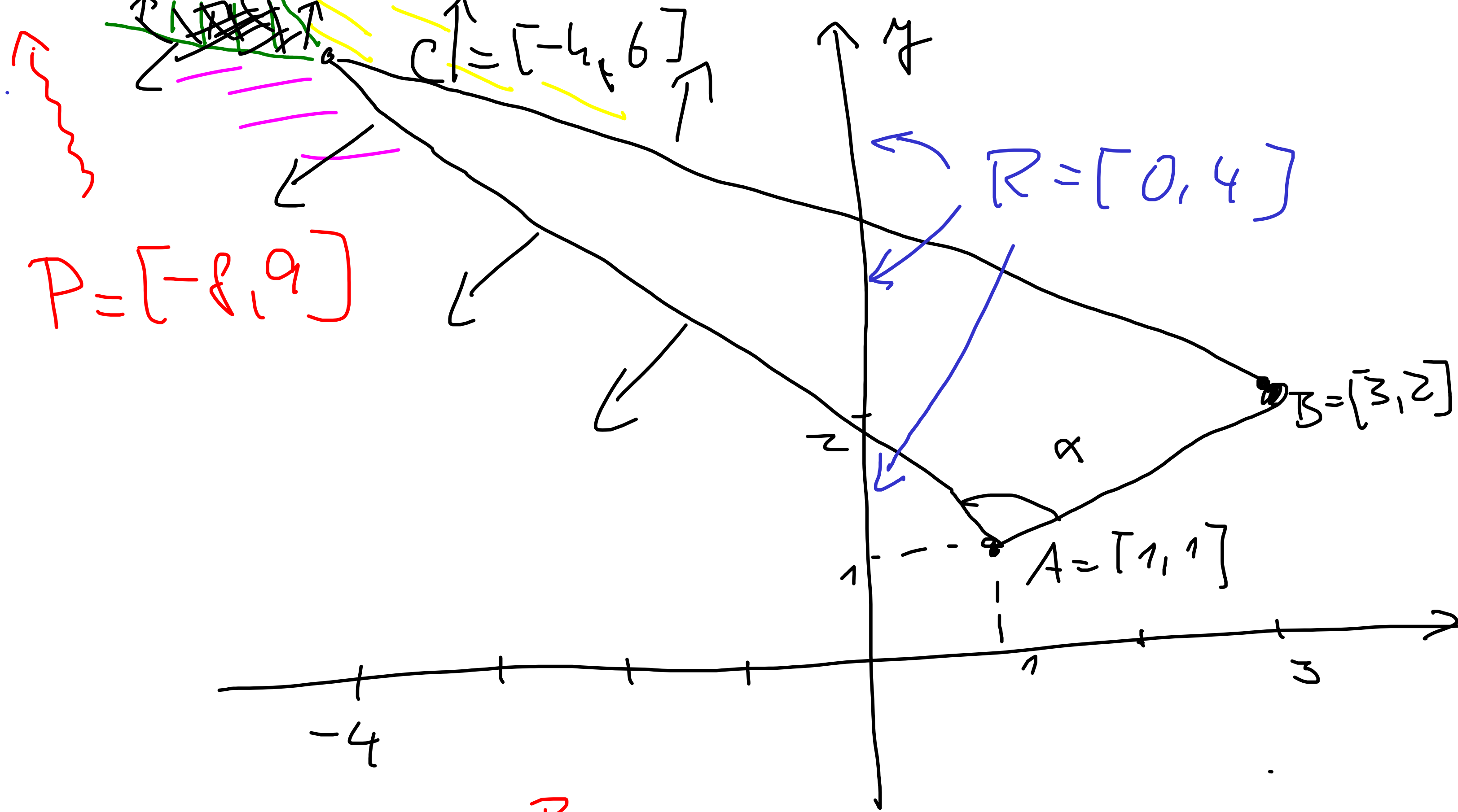
$$\cos\varphi = \frac{|\langle n_1, n_2\rangle|}{\|n_1\|\,\|n_2\|}$$

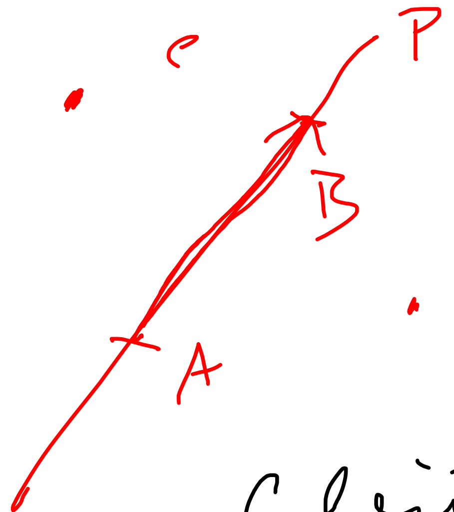
(iii) $R = [0,4]$ leží v trojúhelníku?

Porovnáme R a $\overrightarrow{BC}$:

$$\det\begin{pmatrix}\overrightarrow{BC}\\ \overrightarrow{BR}\end{pmatrix} = \det\begin{pmatrix}-7 & 4\\ -3 & 2\end{pmatrix} = -14-(-12) = -2 < 0$$

$\Rightarrow$ R je napravo od $\overrightarrow{BC}$ $\Rightarrow$ leží mimo trojúhelník





$$\det\begin{pmatrix} AB \\ AC \end{pmatrix}$$

> 0	< 0	$= 0$
C leží nalevo od p	C leží napravo od p	C leží na p

(iv) Které strany [illegible] jsou viditelné z bodu $P = [-8,9]$?

Porovnáme P a $\overrightarrow{AC}$:

$$\det\begin{pmatrix}\vec{AC}\\ AP\end{pmatrix} = \det\begin{pmatrix}-5 & 5\\ -9 & 8\end{pmatrix} =$$

$$= -40 - (-45) = 5 > 0$$

$\Rightarrow$ P je nalevo

Porovnáme P a $\vec{BC}$:

$$\det\begin{pmatrix}\vec{BC}\\ \vec{BP}\end{pmatrix} = \det\begin{pmatrix}-7 & 4\\ -11 & 7\end{pmatrix} = -7^2 - (-44)$$

$$= -49 + 44 < 0$$

P je napravo

Závěr: jsou vidět všechny vrcholy a hrany AC a BC

Připomenutí relací: relace

- A množina $\rho \subseteq A \times A$
- ρ reflexivní, jestliže
  $\forall a \in A: \ (a,a) \in \rho$
- ρ je symetrická, jestliže
  $(a,b) \in \rho \Rightarrow (b,a) \in \rho$
- ρ antisymetrická, jestliže
  $(a,b) \in \rho \wedge (b,a) \in \rho \Rightarrow a = b$
- ρ tranzitivní, jestliže
  $(a,b) \in \rho \wedge (b,c) \in \rho \Rightarrow (a,c) \in \rho$

Relace, která je R, S, T se nazývá ekvivalence.

4.5 $A = \mathbb{Z} \setminus \{0\}$

$$\underbrace{(x,y) \in \rho}_{x \rho y} \Leftrightarrow x \cdot y > 0$$

- $\forall x \in A: (x,x) \in \rho$ neboť $x^2 > 0$
  $\Rightarrow \rho$ je <u>refl.</u>
- $(x,y) \in \rho \Rightarrow x \cdot y > 0 \Rightarrow y \cdot x > 0$
  $\Rightarrow (y,x) \in \rho \Rightarrow \rho$ <u>sym</u>
- $(x,y) \in \rho \wedge (y,z) \in \rho$

$$\Updownarrow \qquad\qquad \Updownarrow$$

$$x \cdot y > 0 \qquad y \cdot z > 0$$

  - x,y kladná $\Rightarrow z$ kladná $\Rightarrow xz > 0$
  - x,y záporná $\Rightarrow z$ záporné $\Rightarrow xz > 0$

Tedy $(x,z) \in \rho \Rightarrow \rho$ je <u>tranz.</u>

$\Rightarrow \rho$ je ekvivalence

Rozklad A na třídy ekvivalence:

$$[a]_\rho = \{b \in A \mid (a,b) \in \rho\}$$

$a \in A$

- $x > 0$: $[x]_\rho = \{y \in \mathbb{Z} \setminus \{0\} \mid x \cdot y > 0\}$
  $= \mathbb{Z}_+$

- $x < 0$: $[x]_\rho = \{y \in \mathbb{Z} \setminus \{0\} \mid x \cdot y > 0\}$
  $= \mathbb{Z}_-$.

$$A = [1]_\rho \cup [-1]_\rho = \mathbb{Z}_+ \cup \mathbb{Z}_-$$

A/ρ má dva prvky