7.2 c) kolmá projekce do roviny $x+y+z=1$

↳ tato rovina není vekt. podprostor
⇝ nejedná se o lineární zobrazení

(x_1, y_1, z_1) t. ž. $x_1+y_1+z_1=1$

(x_2, y_2, z_2) $\qquad x_2+y_2+z_2=1$

$$(x_1+x_2)+(y_1+y_2)+(z_1+z_2)=2$$

↳ tato rovina není uzavřená na součty.

7.2 d) $\varphi: \mathbb{R}^3 \to \mathbb{R}^3$

kolmá projekce do roviny $x+y+z=0$

Chceme $(\varphi)_{\epsilon,\epsilon} = ?$, kde

$$\epsilon = (\underbrace{(1,0,0)}_{\epsilon_1}, \underbrace{(0,1,0)}_{\epsilon_2}, \underbrace{(0,0,1)}_{\epsilon_3})$$

$$(\varphi)_{\epsilon,\epsilon} = \Big((\varphi(\epsilon_1))_{\epsilon}, (\varphi(\epsilon_2))_{\epsilon}, (\varphi(\epsilon_3))_{\epsilon} \Big)$$

– priamy výpočet .....

<u>Iný postup:</u> uvážime vhodnú bázu α t.ž. $(\varphi)_{\alpha,\alpha}$ má "jednoduchý tvar" (v_1, v_2, v_3)

- $x+y+z=0 \rightsquigarrow v_3 = (1,1,1)$ kolmý na rovinu

$$\left.\begin{aligned} v_1 &= (1,-1,0) \\ v_2 &= (0,1,-1) \end{aligned}\right\} \text{ leží v rovine}$$

$$(\varphi)_{\alpha,\alpha} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\|$$

$$\Big((\varphi(v_1))_{\alpha}, (\varphi(v_2))_{\alpha}, (\varphi(v_3))_{\alpha} \Big) =$$

$$= \Big((v_1)_{\alpha}, (v_2)_{\alpha}, (0)_{\alpha} \Big) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\boxed{(\varphi)_{\epsilon,\epsilon} = (id)_{\epsilon,\alpha} \cdot (\varphi)_{\alpha,\alpha} \cdot (id)_{\alpha,\epsilon}}$$

$$(id)_{\epsilon,\alpha} = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{pmatrix}$$

$$\left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ 0 & -1 & 1 & 0 & 0 & 1 \end{array}\right) \sim$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 & 0 \\ \cancel{0} & \cancel{0} & \cancel{3} & \cancel{1} & \cancel{1} & \cancel{1} \\ 0 & 0 & 1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2/3 & -1/3 & -1/3 \\ 0 & 1 & 0 & 1/3 & 1/3 & -2/3 \\ 0 & 0 & 1 & \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{array}\right)$$

-2

$$\underbrace{\begin{pmatrix} 2/3 & -1/3 & -1/3 \\ 1/3 & 1/3 & -2/3 \\ \frac{1}{3} & \frac{1}{3} & \frac{1}{3} \end{pmatrix}}_{(id)_{\alpha,E}}$$

$$(\varphi)_{E,E} = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & -1 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} 2/3 & -1/3 & -1/3 \\ 1/3 & 1/3 & -2/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 0 & -1 & 0 \end{pmatrix} \begin{pmatrix} 2/3 & -1/3 & -1/3 \\ 1/3 & 1/3 & -2/3 \\ 1/3 & 1/3 & 1/3 \end{pmatrix} =$$

$$= \begin{pmatrix} \frac{2}{3} & -\frac{1}{3} & -\frac{1}{3} \\ -\frac{1}{3} & \frac{2}{3} & -\frac{1}{3} \\ -\frac{1}{3} & -\frac{1}{3} & \frac{2}{3} \end{pmatrix} \quad \text{výsledek}$$

e) $\varphi: \mathbb{R}^3 \to \mathbb{R}^3$ rotace kolem osy $(1,1,1)$ o $\frac{2\pi}{3}$

- rotace v rovině o úhel α je dána maticí $\begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix}$ ve std bázi

Volba vhodné báze: chceme bázi $B=(u_1,u_2,u_3)$, kde u_1,u_2 je ON báze roviny $\rho: x+y+z=0$ a $u_3=(1,1,1)$

$\alpha=(v_1,v_2,v_3)$, $v_1=(1,-1,0)$
$v_2=(0,1,-1)$
$v_3=u_3=(1,1,1)$

Nejprve najdeme OG bázi roviny ρ (w_1,w_2):

$w_1:=v_1=(1,-1,0)$

$w_2=v_2+a v_1 \perp v_1$

$\langle v_1,v_2\rangle + a\langle v_1,v_1\rangle = 0$

$-1+2a=0 \Rightarrow a=\frac{1}{2}$

$\Rightarrow w_2=(0,1,-1)+\frac{1}{2}(1,-1,0)$
$=\left(\frac{1}{2},\frac{1}{2},-1\right)$

$\|w_1\|=\sqrt{2}$

$\|w_2\|=\sqrt{\frac{1}{4}+\frac{1}{4}+1}=\sqrt{\frac{3}{2}}$

$u_1=\frac{1}{\sqrt{2}}(1,-1,0)$

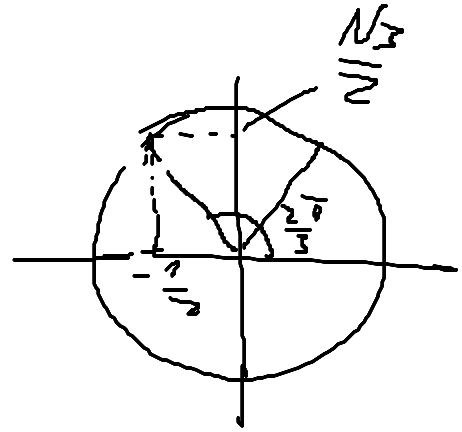
$u_2=\frac{1}{\sqrt{\frac{3}{2}}}\left(\frac{1}{2},\frac{1}{2},-1\right)$

$$\boxed{u_1=\left(\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2},0\right),\quad u_2=\left(\frac{\sqrt{2}}{2\sqrt{3}},\frac{\sqrt{2}}{2\sqrt{3}},-\frac{\sqrt{2}}{\sqrt{3}}\right)}$$

$$(\varphi)_{B,B} = \begin{pmatrix} \cos\frac{2\pi}{3} & -\sin\frac{2\pi}{3} & 0 \\ \sin\frac{2\pi}{3} & \cos\frac{2\pi}{3} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$B = (u_1, u_2, u_3)$

$$(\varphi)_{B,B} = \begin{pmatrix} -\frac{1}{2} & -\frac{\sqrt{3}}{2} & 0 \\ \frac{\sqrt{3}}{2} & -\frac{1}{2} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$



$$(\varphi)_{E,E} = \underbrace{(id)_{E,B}} \cdot (\varphi)_{B,B} \cdot \underbrace{(id)_{B,E}}$$

8.1 $$A = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 2 & 3 & 1 \\ 1 & 0 & -1 & 1 \\ 2 & -3 & 1 & 0 \end{pmatrix}$$

Laplaceův rozvoj

$$\det A = 2 \cdot (-1)^{2+2} \cdot \det \begin{pmatrix} 1 & 0 & 1 \\ 1 & -1 & 1 \\ 2 & 1 & 0 \end{pmatrix} +$$

$$-3\,(-1)^{4+2} \det \begin{pmatrix} 1 & 0 & 1 \\ 0 & 3 & 1 \\ 1 & -1 & 1 \end{pmatrix} =$$

$$= 2 \cdot \left[2(-1)^{3+1} \cdot \det \begin{pmatrix} 0 & 1 \\ -1 & 1 \end{pmatrix} + 1 \cdot (-1)^{3+2} \det \begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix} \right]$$

$$-3\cdot\left[1\,(-1)^{1+1}\cdot\det\begin{pmatrix}3&1\\-1&1\end{pmatrix}+1\cdot(-1)^{3+1}\det\begin{pmatrix}0&1\\3&1\end{pmatrix}\right]$$

$$=2\left(2(0+1)-(1-1)\right)$$

$$-3\left(1\cdot(3+1)+(0-3)\right)$$

$$=2(2-0)-3(4-3)=4-3=\underline{\underline{1}}$$

Determinant pomocí Gaussovy eliminace

$$\det\begin{pmatrix}\boxed{1}&0&0&1\\0&2&3&1\\1&0&-1&1\\2&-3&1&0\end{pmatrix}=\det\begin{pmatrix}1&0&0&1\\0&\boxed{2}&3&1\\0&0&-1&0\\0&-3&1&-2\end{pmatrix}\ \tfrac{3}{2}$$

$$\tfrac{11}{2}\ =\det\begin{pmatrix}1&0&0&1\\0&2&3&1\\0&0&\boxed{-1}&0\\0&0&\tfrac{11}{2}&-\tfrac{1}{2}\end{pmatrix}=\det\begin{pmatrix}1&0&0&1\\0&2&3&1\\0&0&-1&0\\0&0&0&-\tfrac{1}{2}\end{pmatrix}$$

$$=1\cdot2\cdot(-1)\cdot\left(-\tfrac{1}{2}\right)=1$$

Vlastní čísla a vektory matice B

v je vlastní vektor s vlastním číslem λ, jestliže $\underbrace{B}_{\text{čtvercová}} \cdot v = \lambda v$

$v \neq 0$

$$\boxed{\det(B - \lambda E) = 0}$$

pro vlastní čísla $\lambda \in \mathbb{R}$

8.2 Vlastní čísla a vektory matice
$$B = \begin{pmatrix} 1 & 0 & 2 & 0 \\ 0 & 3 & 0 & -2 \\ -3 & 0 & 6 & 0 \\ 0 & 3 & 0 & 8 \end{pmatrix}$$

$$\det \begin{pmatrix} 1-\lambda & 0 & 2 & 0 \\ 0 & 3-\lambda & 0 & -2 \\ -3 & 0 & 6-\lambda & 0 \\ 0 & 3 & 0 & 8-\lambda \end{pmatrix} =$$

$$= (1-\lambda)(-1)^{1+1} \cdot \det \begin{pmatrix} 3-\lambda & 0 & -2 \\ 0 & 6-\lambda & 0 \\ 3 & 0 & 8-\lambda \end{pmatrix}$$

$$+ 2 \cdot (-1)^{1+3} \det \begin{pmatrix} 0 & 3-\lambda & -2 \\ -3 & 0 & 0 \\ 0 & 3 & 8-\lambda \end{pmatrix} =$$

$$= (1-\lambda)(6-\lambda)\cdot(-1)^{2+2}\cdot\det\begin{pmatrix}3-\lambda & -2\\ 3 & 8-\lambda\end{pmatrix}$$

$$+ 2\cdot(-3)(-1)^{2+1}\cdot\det\begin{pmatrix}3-\lambda & -2\\ 3 & 8-\lambda\end{pmatrix}$$

$$= (1-\lambda)(6-\lambda)\cdot\left[(3-\lambda)(8-\lambda)+6\right]$$

$$+ 6\cdot\left[(3-\lambda)(8-\lambda)+6\right] =$$

$$= \left[(3-\lambda)(8-\lambda)+6\right]\cdot\left[(1-\lambda)(6-\lambda)+6\right] =$$

$$= \left[\lambda^2 - 11\lambda + 24 + 6\right]\cdot\left[\lambda^2 - 7\lambda + 6 + 6\right]$$

$$= (\lambda^2 - 11\lambda + 30)(\lambda^2 - 7\lambda + 12)$$

$$= (\lambda-5)(\lambda-6)(\lambda-3)(\lambda-4)$$

$\Rightarrow$ vlastní čísla 3, 4, 5, 6

<u>Vlastní vektory</u> pro $\lambda_1 = 3$

$$B\cdot v = 3v$$

$$Bv - 3v = 0 \Leftrightarrow (B-3E)v = 0$$

$$B-3E = \begin{pmatrix}-2 & 0 & 2 & 0\\ 0 & 0 & 0 & -2\\ -3 & 0 & 3 & 0\end{pmatrix} \sim \begin{array}{cccc} x_1 & x_2 & x_3 & x_4 \\ \end{array}\begin{pmatrix}1 & 0 & -1 & 0\\ 0 & 3 & 0 & 5\\ 0 & 0 & 0 & 1\end{pmatrix}$$

$$\begin{pmatrix} 0 & 3 & 0 & 5 \end{pmatrix}$$

$$\begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = p \begin{pmatrix} 1 \\ 0 \\ 1 \\ 0 \end{pmatrix}$$

$x_3 = p$

$x_4 = 0$

$x_2 = 0$

$x_1 = p$

Závěr: $v_1 = (1, 0, 1, 0)$ je vlastní vektor příslušející vlastnímu číslu $\lambda_1 = 3$

Vlastní vektor v_2 pro vlastní číslo $\lambda_2 = 4$ ....