Pr 1.1 (i) $f(x,y) = x - y$

$z = f(x,y) = c \rightsquigarrow c = x - y$

$z = x - y$ je ravnina

(ii) $f(x,y) = x^2 + y^2$

$z = x^2 + y^2 = c \geq 0$

↳ krožnice v ravnini $z = c$

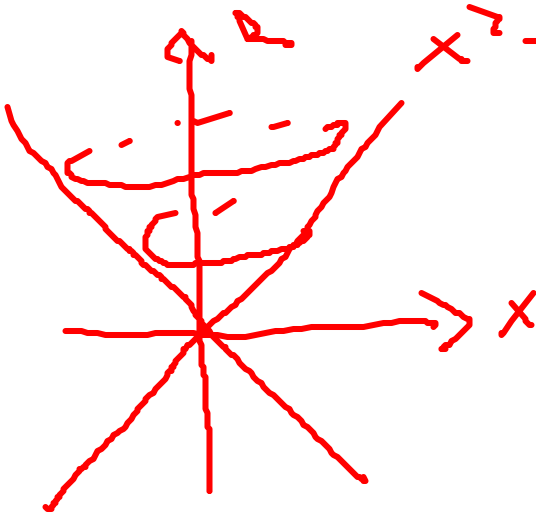
$y = 0 \rightsquigarrow z = x^2$

(iii)

$f(x,y) = \sqrt{x^2 + y^2}$

$z = \sqrt{x^2 + y^2} = c \in \mathbb{R}$

$x^2 + y^2 = c^2$



$y = 0 \rightsquigarrow z = \sqrt{x^2}$

(iv) $f(x,y) = \frac{x+y}{x-y}$ $\quad D(f) = \mathbb{R}^2 - \{(x,x) \in \mathbb{R}\}$

$\Gamma_c = \frac{x+y}{x-y} = c$ konst

$x + y = c(x-y)$ snimka

(v) $f(x,y) = \sin(x)\cos(y)$

nulina ravnina

(vi) $f(x,y) = \sin(xy)$

imak nulina ravnina

(vii) $\frac{\sin(xy)}{x^2+y^2}$

$x \to \infty$ ili $y \to \infty \Longrightarrow f(x,y) \to 0$

$(x,y) \to (0,0) \Longrightarrow ?$

Pr. 1.2: (i) $f(x,y) = \dfrac{xy}{y(x^3+x^2+x+1)}$

$y \neq 0$

$x^3+x^2+x+1 = (x+1)\underbrace{(\qquad)}_{\neq 0}$

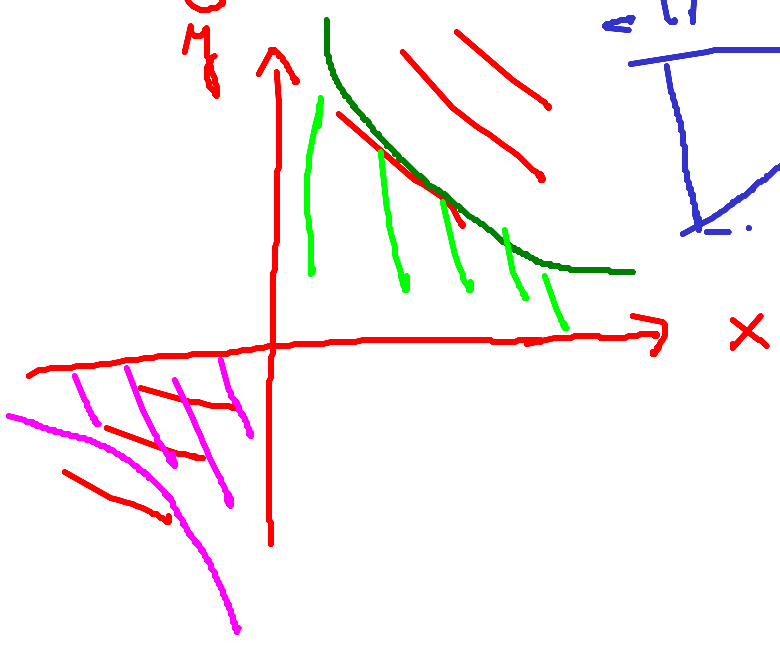
$\hookrightarrow$ korene sú $-1, \pm i$

$D(f) = \mathbb{R} - \big(\{(x,0) \mid x \in \mathbb{R}\} \cup$
$\cup \{(-1,y) \mid y \in \mathbb{R}\}\big)$

(ii) $f(x,y) = \sqrt{\arcsin(xy)}$

$\arcsin(xy) \geq 0$

$0 \leq xy \leq \pi$



$xy \leq \pi$

$y < \dfrac{\pi}{x}$ $\quad x > 0$

$y > \dfrac{\pi}{x}$ $\quad x < 0$

$$f(x,y) = \frac{\sqrt{x-y^2}}{\ln(9-x^2-y^2)}$$ spočítat sami

(iii)

1.3 Limity

(i) $f(x,y) = \dfrac{x^3+y^3}{x^2+y^2}$

$$\lim_{(x,y)\to 0} \frac{(x+y)(x^2-xy+y^2)}{x^2+y^2} =$$

$$= \lim_{(x,y)\to(0,0)} \underbrace{(x+y)}_{\to 0}\underbrace{\left(1-\frac{xy}{x^2+y^2}\right)}_{\in\langle -\frac{1}{2},\frac{1}{2}\rangle} = 0$$

$$(x\pm y)^2 \geq 0$$

$$x^2+y^2 \pm 2xy \geq 0$$

$$x^2+y^2 \geq 2xy \rightsquigarrow \frac{1}{2} \geq \frac{xy}{x^2+y^2}$$

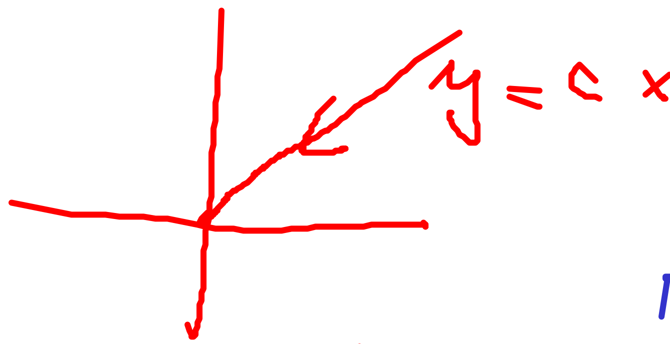
$$x^2+y^2 \geq -2xy \rightsquigarrow -\frac{1}{2} \leq \frac{xy}{x^2+y^2}$$

$$-\frac{1}{2} \leq \frac{xy}{x^2+y^2} \leq \frac{1}{2}$$

(ii) $\lim_{(x,y)\to(0,0)} \frac{\sin(xy)}{x^2+y^2} = ?$

$\lim_{x\to 0} f(x, cx) = \lim_{x\to 0} \frac{\sin(cx^2)}{x^2+c^2x^2} =$

$c \in \mathbb{R}$



$\lim_{R\to 0} \frac{\sin R}{R} = 1$

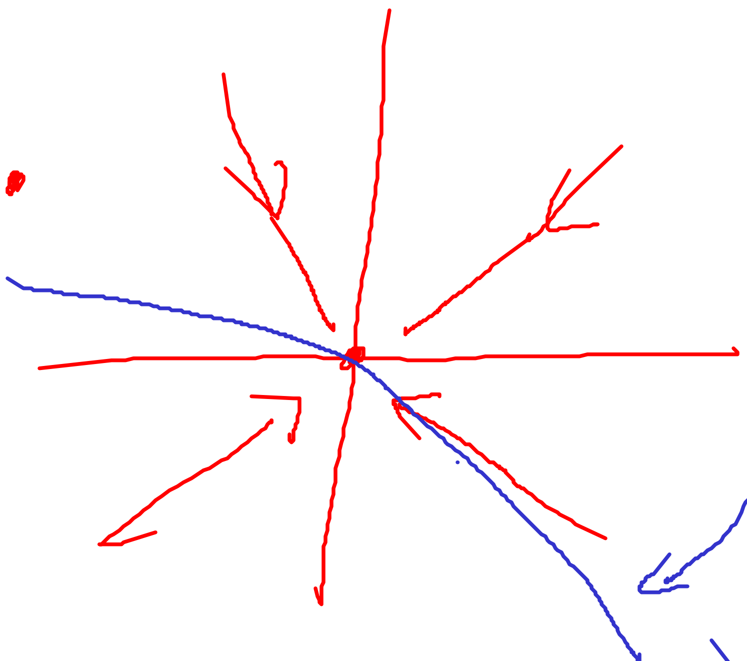
$= \lim_{x\to 0} \frac{\sin(cx^2)}{\frac{1+c^2}{c}(cx^2)} = \frac{1+c^2}{c}, \quad c \neq 0$

$\Rightarrow$ limity po přímkách jsou různé

(iii) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+y^2}{x+y} = 2$ $\qquad x \neq -y$

$\lim\limits_{x\to 0} f(x, cx) = \lim\limits_{x\to 0} \dfrac{x^2+c^2x^2}{x+cx} = 0$

Pro každé $c \in \mathbb{R},\ c \neq -1$



$y = 1 - e^x$

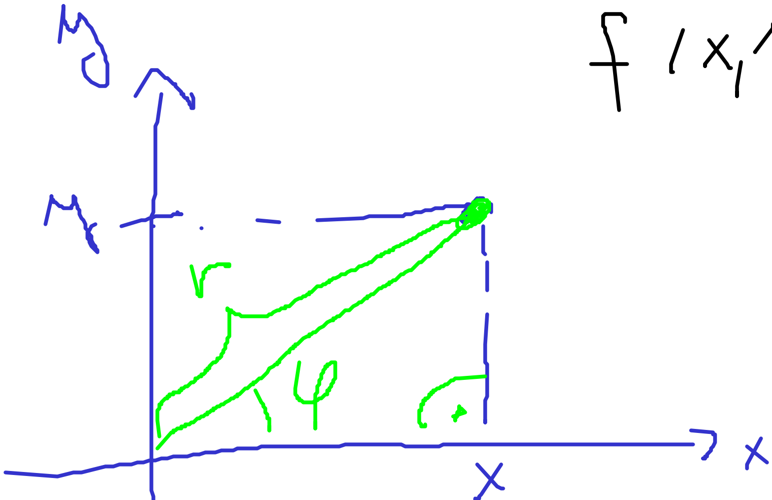
$\lim\limits_{x\to 0} f(x, 1-e^x) = \lim\limits_{x\to 0} \dfrac{x^2+(1-e^x)^2}{x+1-e^x} \overset{L'H}{=}$

$\overset{L'H}{=} \lim\limits_{x\to 0} \dfrac{2x+2(1-e^x)(-e^x)}{1-e^x} \overset{L'H}{=}$

$= \lim\limits_{x\to 0} \dfrac{2+2[(-e^x)(-e^x)+(1-e^x)(-e^x)]}{-e^x}$

$= \dfrac{2+2[1+0]}{-1} = -2$

(iv) $\lim\limits_{(x,y)\to(0,0)} \underbrace{\dfrac{x^2-y^2}{x^2+y^2}}_{f(x,y)}$



(r, φ) polární souřadnice

(x, y) kartézské

$$x = r\cos\varphi$$
$$y = r\sin\varphi$$

$$f(x,y) = \frac{r^2\cos^2\varphi - r^2\sin^2\varphi}{r^2\cos^2\varphi + r^2\sin^2\varphi}$$

$$= \cos^2\varphi - \sin^2\varphi$$

$(x,y)\to 0$ ekvivalentně znamená $r\to 0$, φ lib

$$\lim_{r\to 0}\left(\cos^2\varphi - \sin^2\varphi\right) = \cos^2\varphi - \sin^2\varphi$$

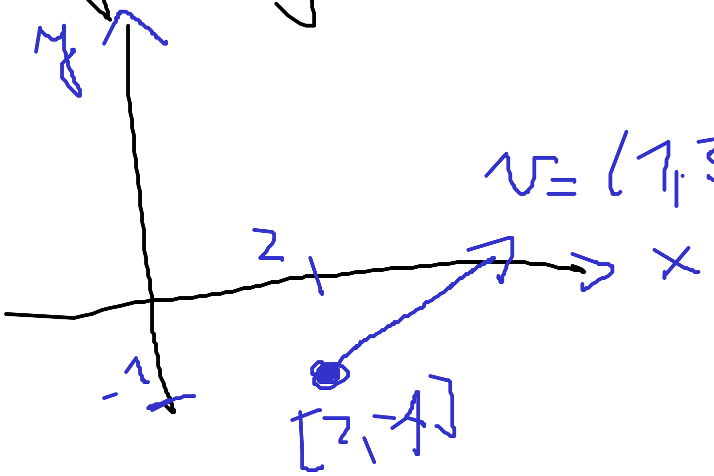
neexistuje

Pr 1.4 (i) $f(x,y) = x^3 + 4xy$

parciální derivace

$f_x(x,y) = 3x^2 + 4x$ | $f_x(2,-1) = 12-4 = 8$

$f_y(x,y) = 4x$ | $f_y(2,-1) = 8$



Směrová derivace

$$\frac{d}{dt}\Big|_{t=0} f(2+t, -1+3t) = \frac{d}{dt}\Big|_{t=0} (2+t)^3 + 4(2+t)(-1+3t)$$

$$= \underbrace{3(2+t)^2 + 4((-1+3t) + 3(2+t))}_{t=0}\Big|_{t=0}$$

$$= 12 + 4(-1+6) = 12 + 20 = 32$$

Směrné derivace pomocí parciálních derivací:

$v = (1, 3)$ bod $[2, -1]$

$$d_v f(2,-1) = 1 \cdot \underbrace{f_x(2,-1)}_{d_{(1,0)} f(2,-1)} + 3 \underbrace{f_y(2,-1)}_{d_{(0,1)} f(2,-1)}$$

$$= 1 \cdot 8 + 3 \cdot 8 = 8 + 24 = \underline{\underline{32}}$$

(iii) $f(x,y,z) = \frac{\cos(x^2 y)}{z}$ v bodě $[1,1,2]$

ve směru $(1,2,3)$

$$f_x(x,y,z) = -\frac{\sin(x^2 y)\cdot 2xy}{z}$$

$$f_y(x,y,z) = -\frac{\sin(x^2 y)\cdot x^2}{z}$$

$$f_z(x,y,z) = -\frac{\cos(x^2 y)}{z^2}$$

$$f_x(1,1,2) = -\frac{2\sin 1}{2} = -\sin 1$$

$$f_y(1,1,2) = -\frac{\sin 1}{2}$$

$$f_z(1,1,2) = -\frac{\cos 1}{4}$$

$v = (1,2,3)$

$$d_v f(1,1,2) = 1\cdot f_x(1,1,2) + 2 f_y(1,1,2) + 3 f_z(1,1,2) =$$

$$= -\sin 1 - \sin 1 + 3\left(-\frac{\cos 1}{4}\right)$$

Směrová derivace přímo z definice.