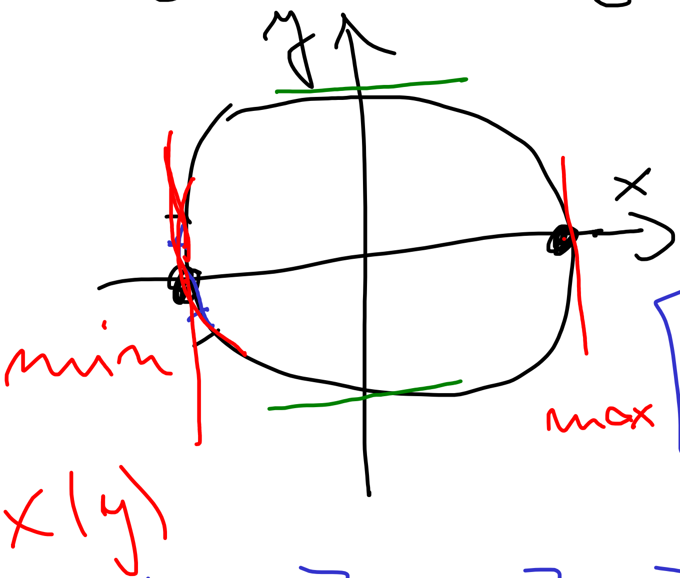
3.7 $F(x,y) = x^2+2xy-y^2-8=0$

ve kterých bodech nelze vyjádřit y jako funkci $y=y(x)$.



$$F(x,y)=0$$
$$F_y(x,y)=0$$

$$2x-2y=0$$

$$x^2+2x^2-x^2-8=0 \quad x=y$$

$$2x^2=8 \quad x=\pm 2$$
$$x^2=4 \quad [2,2],\ [-2,-2]$$

Zkusme na okolí těchto bodů psát x jako $x=x(y)$

$$x^2+2xy-y^2-8=0 \quad \Big/ \frac{d}{dy}$$

$$2xx'+2x'y+2x-2y=0$$

$$\rightarrow 2\underbrace{x'}_{=0}(x+y)+2\underbrace{(x-y)}_{=0}=0 \quad \Big/ \frac{d}{dy}$$

v bodech $x=x(y)$ má stac. bod

$$2x''(x+y) + 2x'(x'+1) + 2(x'-1) = 0$$

v bodech $[x,y] = [2,2]$

$= [-2,-2]$

kde $x' = 0$

$$2x''(x+y) - 2 = 0$$

$$x'' = \frac{1}{x+y} = \underline{{}_{[2,2]}}\ \frac{1}{4} \text{ lok min}$$

$$\overline{{}_{[-2,-2]}}\ -\frac{1}{4} \text{ lok max}$$

4.1 $h(x,y) = x - y$ najít extrémy na elipse $F(x,y) = x^2 + 2y^2 - 6 = 0$

$$L(x,y,\lambda) = h(x,y) + \lambda F(x,y)$$

$$= x - y + \lambda(x^2 + 2y^2 - 6)$$

$$\boxed{\begin{aligned} L_x(x,y,\lambda) &= 1 + \lambda \cdot 2x = 0 \\ L_y(x,y,\lambda) &= -1 + \lambda \cdot 4y = 0 \\ L_\lambda(x,y,\lambda) &= x^2 + 2y^2 - 6 = 0 \end{aligned}}$$

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \lambda \begin{pmatrix} 2x \\ 4y \end{pmatrix}$$

$F(x,y) = 0$

$$2x\lambda = -1$$

$$\lambda = -\frac{1}{2x}$$

$$-1 + 4 \cdot \left(-\frac{y}{2x}\right) = 0 \quad / \cdot x$$

$$x + 2y = 0$$

$$x = -2y$$

dosedimedo

$x^2 + 2y^2 - 6 = 0$

$4y^2 + 2y^2 - 6 = 0$

$y^2 = +1$

$y = \pm 1$

$\left.\begin{array}{l} [-2, 1] \\ [2, -1] \end{array}\right\}$ stac. body

$h(-2, 1) = -2 - 1 = -3$ min.

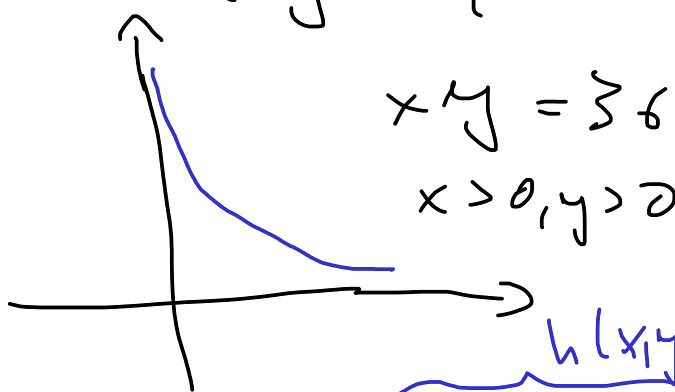
$h(2, -1) = 2 - |-1| = 3$ max

4.2 $h(x,y,z) = x + 2y + 3z$

najděte extrémy za podmínek

- $F(x,y,z) = xyz - 36 = 0$
- $x > 0,\ y > 0,\ z > 0$

nezadává kompaktní množinu



$$L(x,y,z,\lambda) = \overbrace{x + 2y + 3z}^{h(x,y,z)} + \lambda(xyz - 36)$$

$$\boxed{\begin{aligned} L_x(x,y,z,\lambda) &= 1 + \lambda yz = 0 \\ L_y(x,y,z,\lambda) &= 2 + \lambda xz = 0 \\ L_z(x,y,z,\lambda) &= 3 + \lambda xy = 0 \\ L_\lambda(x,y,z,\lambda) &= xyz - 36 = 0 \end{aligned}}$$

$$\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix} = -\lambda \begin{pmatrix} yz \\ xz \\ xy \end{pmatrix}$$

$xyz = 36$ & $\begin{cases} \lambda yz = -1 \\ \lambda xz = -2 \\ \lambda xy = -3 \end{cases}$

součin

$$\lambda^3 \cdot \underbrace{x^2 y^2 z^2}_{36^2} = -6$$

$$6^4 \lambda^3 = -6$$
$$\lambda^3 = -\frac{1}{6^3} \Rightarrow \lambda = -\frac{1}{6}$$

$$\left.\begin{aligned} yz &= 6 \\ xz &= 12 \\ xy &= 18 \end{aligned}\right\}$$ podíl $\frac{x}{y} = 2 \Rightarrow x = 2y$

$2y^2 = 18$

$y = \pm 3$

pro $x^2y^2z^2 = 36^2$ neplatí $\rightarrow xyz = 36$

$[6, 3, 2]$ $\quad$ ~~$[-6, -3, -2]$~~

stacionární body

$xyz - 36 = 0$

$x = \frac{36}{yz}$

$h(x, y, z) =$

$= h\left(\frac{36}{yz}, y, z\right) =$

$= \frac{36}{yz} + 2y + 2z$

$[y, z] = [3, 2]$

$= \cancel{[-3, -2]}$

$\rightsquigarrow$ matice druhých derivací

$g(y, z) = h\left(\frac{36}{yz}, y, z\right) = \frac{36}{yz} + 2y + 3z$

$$g_y(y,r) = -\frac{36}{y^2 r} + 2 \overset{[+3,+2]}{=} 0$$

$$g_r(y,r) = -\frac{36}{y r^2} + 3 = 0$$

v bodě $[3,2]$

$$g_{yy}(y,r) = \frac{2\cdot 36}{y^3 r} = \frac{2\cdot 36}{3^3\cdot 2} = \frac{4}{3}$$

$$g_{yr}(y,r) = \frac{36}{y^2 r^2} = \frac{36}{9\cdot 4} = 1$$

$$g_{rr}(y,r) = \frac{2\cdot 36}{y\cdot r^3} = \frac{2\cdot 36}{3\cdot 8} = \frac{2\cdot 12}{8} = 3$$

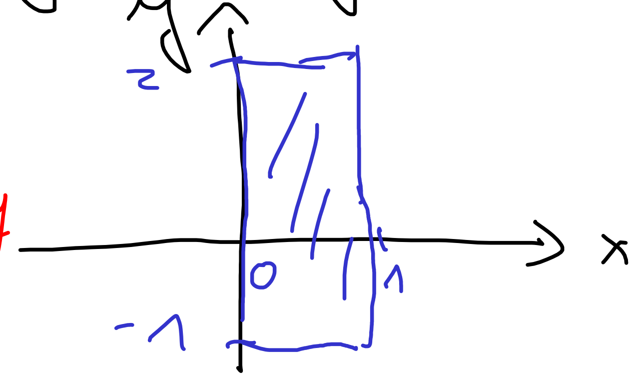
Hessián: $\begin{pmatrix} 4/3 & 1 \\ 1 & 3 \end{pmatrix}$ $\quad \det(\ldots) = 4-1 = 3 > 0$

$\Rightarrow$ poz. def. $\Rightarrow$ minimum

4.3 $\iint (x^2 + 2xy)\,dx\,dy$

$\langle 0,1\rangle \times \langle -1,2\rangle$

$$= \int_{y=-1}^{y=2} \left[\frac{1}{3}x^3 + x^2 y\right]_{x=0}^{x=1} dy$$



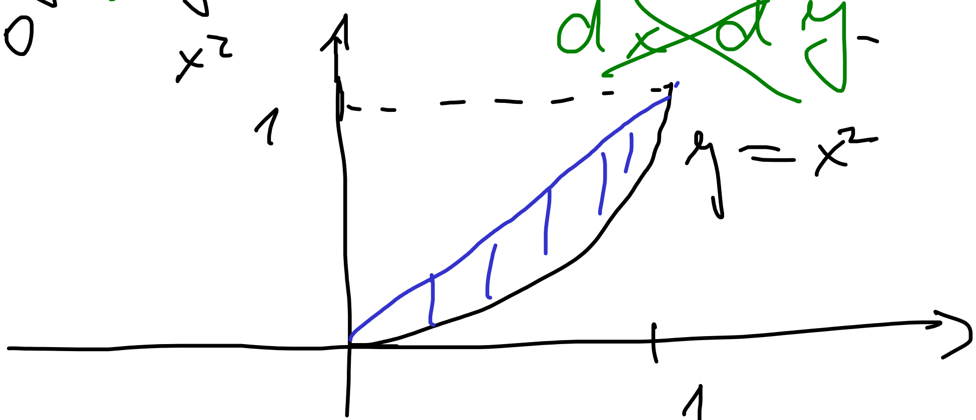
$$= \int_{-1}^{2} \left(\frac{1}{3} + y\right) dy = \left[\frac{1}{3} y + \frac{1}{2} y^2\right]_{-1}^{2} =$$

$$= \left(\frac{2}{3} + 2\right) - \left(-\frac{1}{3} + \frac{1}{2}\right) =$$

$$= 1 + 2 - \frac{1}{2} = \frac{5}{2}$$

4.4 $$\int_0^1 \left(\int_{x^2}^{x} (2 - xy)\, dy \right) dx$$

$\cancel{dx\, dy}$



$$= \int_{x=0}^{x=1} \left[2y - \frac{1}{2} x y^2\right]_{y=x^2}^{y=x} dx$$

$$= \int_0^1 \left(\left(2x - \frac{1}{2} x^3\right) - \left(2x^2 - \frac{1}{2} x^5\right) \right) dx$$

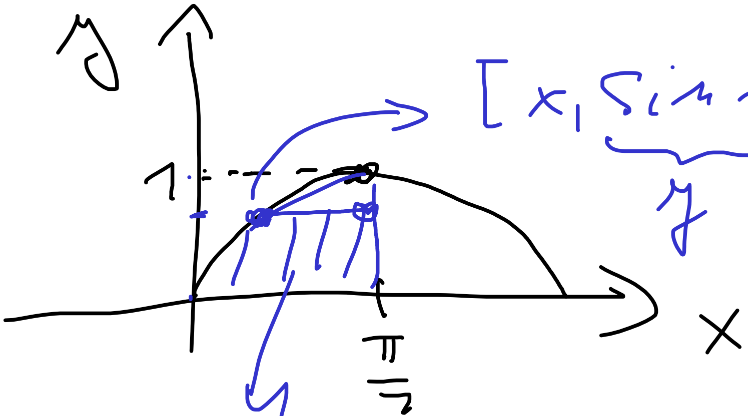
$$= \left[\frac{1}{2} \cdot \frac{1}{6} x^6 - \frac{1}{2} \cdot \frac{1}{4} x^4 - 2 \cdot \frac{1}{3} x^3 + x^2\right]_0^1$$

$$= \frac{1}{12} - \frac{1}{8} - \frac{2}{3} + 1 = \frac{2 - 3 - 16}{24} + 1 = \frac{7}{24}$$

4.6 Zaměňte pořadí integrace

$$\int_0^{\frac{\pi}{2}} \left(\int_0^{\sin x} f(x,y)\, dy \right) dx$$

$\longleftrightarrow$ nejde



$[x, \underbrace{\sin x}_{y}] \rightarrow x = \arcsin y$

$0 \le x \le \frac{\pi}{2}$

$0 \le y \le \sin x$

$0 \le y \le 1$

$\arcsin y \le x \le \frac{\pi}{2}$

$$= \int_0^1 \left(\int_{\arcsin y}^{\pi/2} f(x,y)\, dx \right) dy$$

4.7 $$\int_0^{\sqrt{\frac{\pi}{2}}} \left(\int_y^{\sqrt{\frac{\pi}{2}}} y^2 \underbrace{\sin x^2 \, dx} \right) dy$$

$\int \sin x^2 \, dx$ vyšší funkce

$$0 \le y \le \sqrt{\tfrac{\pi}{2}}$$
$$y \le x \le \sqrt{\tfrac{\pi}{2}}$$

$$0 \le x \le \sqrt{\tfrac{\pi}{2}}$$
$$0 \le y \le x$$

$$= \int_0^{\sqrt{\frac{\pi}{2}}} \left(\int_0^x y^2 \sin x^2 \, dy \right) dx$$

$$= \int_0^{\sqrt{\pi/2}} \left[\frac{1}{3} y^3 \sin x^2 \right]_{y=0}^{y=x} dx$$

$$= \int_0^{\sqrt{\pi/2}} \frac{1}{3} x^3 \sin x^2 \, dx = \left| \begin{array}{l} t = x^2 \\ dt = 2x\,dx \end{array} \right|$$

$$= \frac{1}{3} \int_0^{\frac{\pi}{2}} \frac{1}{2} (\sin t)\, t \, dt =$$

$$= \left| \begin{array}{ll} uv = \underline{\int u'v} + \int uv' & \\ u' = \sin t & u = -\cos t \\ v = t & v' = 1 \end{array} \right|$$

$$= \frac{1}{6}\left[\underbrace{\left[-t\cos t \right]_0^{\frac{\pi}{2}}}_{=0} - \int_0^{\frac{\pi}{2}} (-\cos t)\,dt \right] =$$

$$= \frac{1}{6}\int_0^{\frac{\pi}{2}} \cos t\,dt = \frac{1}{6}\left[\sin t\right]_0^{\frac{\pi}{2}} = \underline{\underline{\frac{1}{6}}}$$