Enemy-Victim Models

"Populační ekologie živočichů"

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Predator-prey system

Acarus

Cheyletus

Predator-prey model

- continuous model of Lotka & Volterra (1925-1928) used to explain decrease in prey fish and increase in predatory fish after World War I

- assumptions

- continuous predation (high population density)
- populations are well mixed
- closed populations (no immigration or emigration)
- no stochastic events
- predators are specialised on one prey species
- populations are unstructured
- reproduction immediately follows feeding

H .. density of prey

r .. intrinsic rate of prey population

a .. predation rate

P .. density of predators

m .. predator mortality rate

b .. reproduction rate of predators

- in the absence of predator, prey grows exponentially → $$\frac{\mathrm{d}H}{\mathrm{d}t} = rH$$

- in the absence of prey, predator dies exponentially → $$\frac{\mathrm{d}P}{\mathrm{d}t} = -mP$$

- predation rate is linear function of the number of prey .. aHP

- each prey contributes identically to the growth of predator .. bHP

$$\frac{\mathrm{d}H}{\mathrm{d}t} = rH - aHP$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = bHP - mP$$

Analysis of the model

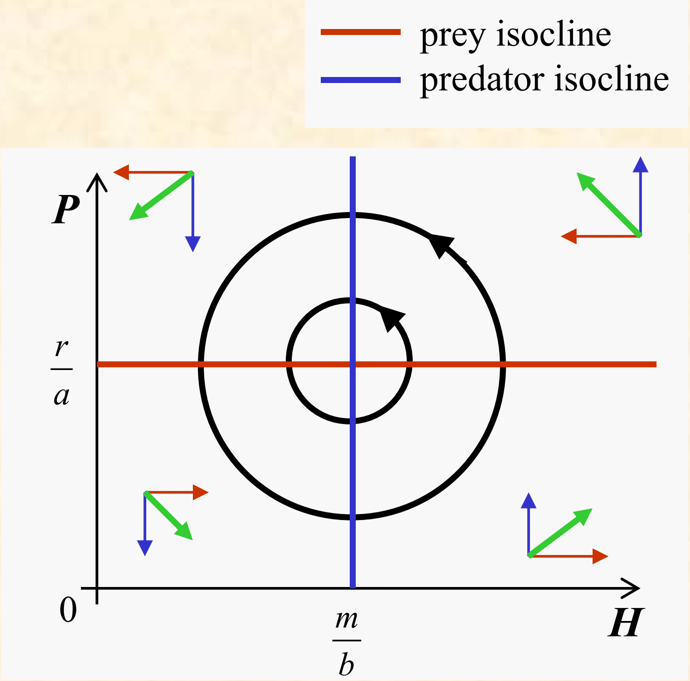
Zero isoclines:

- for prey population:

$$\frac{\mathrm{d}H}{\mathrm{d}t} = 0 \qquad 0 = rH - aHP \qquad \boxed{P = \frac{r}{a}}$$

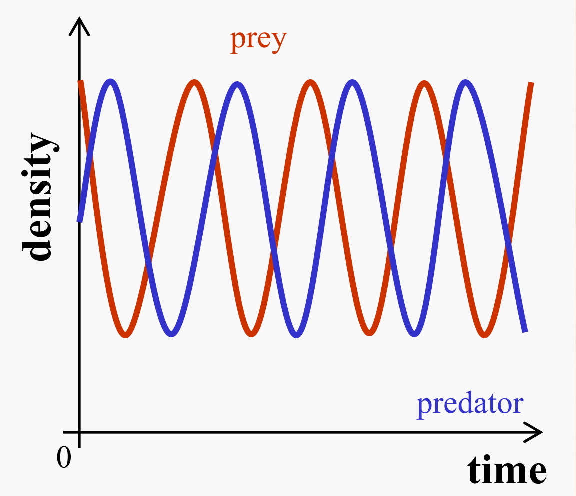
- for predator population:

$$\frac{\mathrm{d}P}{\mathrm{d}t} = 0 \qquad 0 = bHP - mP \qquad \boxed{H = \frac{m}{b}}$$



- do not converge, has no asymptotic stability (trajectories are closed lines) → **neutral stability**

- unstable system, amplitude of the cycles is determined by initial numbers



Addition of density-dependence

- in the absence of the predator prey population reaches carrying capacity K

$$\frac{\mathrm{d}H}{\mathrm{d}t} = rH\left(1 - \frac{H}{K}\right) - aHP$$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = bHP - mP$$

- for given parameter values: $r = 3$, $m = 2$, $a = 0.1$, $b = 0.3$, $K = 10$

$$\frac{\mathrm{d}H}{\mathrm{d}t} = 3H\left(1 - \frac{H}{10}\right) - 0.1HP \qquad \frac{\mathrm{d}P}{\mathrm{d}t} = 0.3HP - 2P$$

Zero isoclines:

- for prey population: $\frac{\mathrm{d}H}{\mathrm{d}t} = 0$ $\quad 0 = 3H\left(1 - \frac{H}{10}\right) - 0.1HP$

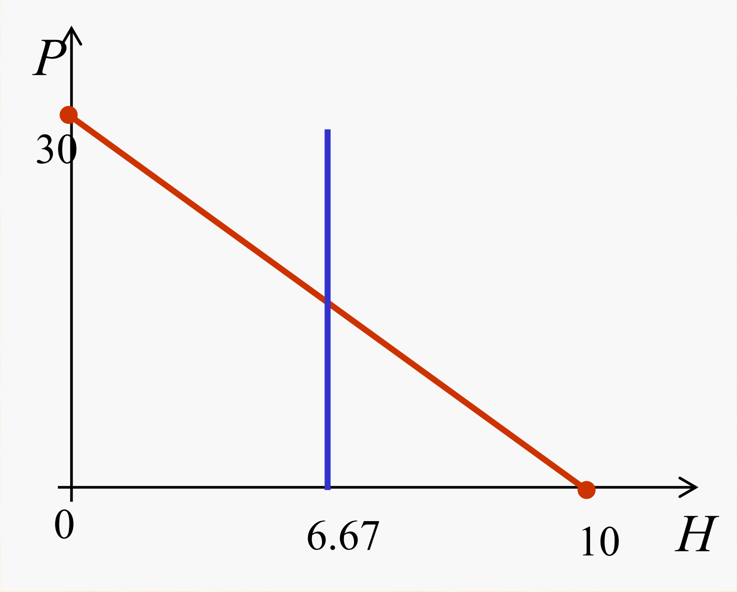
if $H = 0$ (trivial solution) or if $\quad 0 = 3\left(1 - \frac{H}{10}\right) - 0.1P$

$P = 30 - 3H$

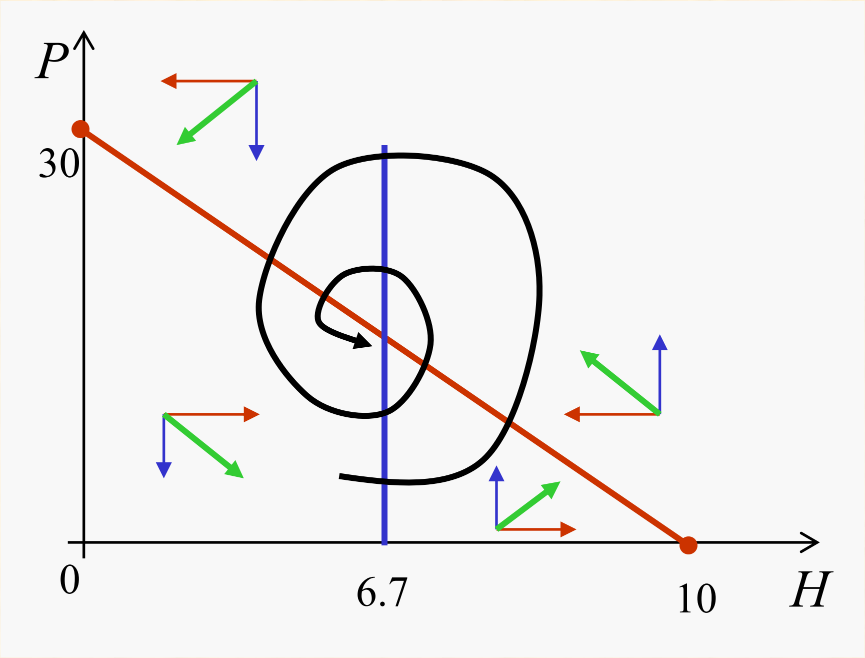
- for predator population: $\frac{\mathrm{d}P}{\mathrm{d}t} = 0$ $\quad 0.3HP - 2P = 0$

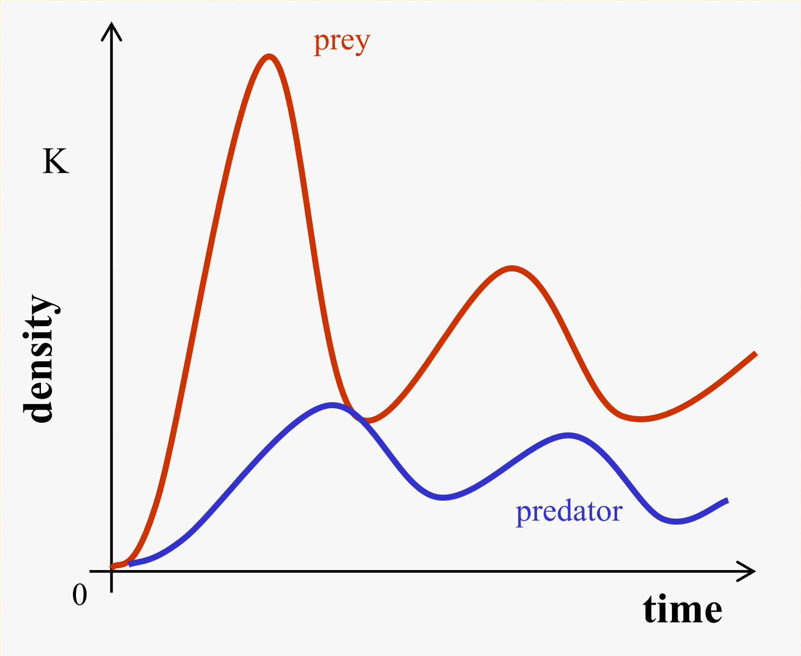
if $P = 0$ (trivial solution)
or if $0.3H - 2 = 0$

$H = 6.667$



- gradient of prey isocline is negative





- has single positive asymptotically stable equilibrium defined by crossing of isoclines
- converges to the stable equilibrium

Addition of functional response of Type II

- functional response Type II: $$H_a = \frac{aHT}{1 + aHT_h}$$

- rate of consumption by all predators: $$\frac{H_a P}{T} = \frac{aHP}{1 + aHT_h}$$

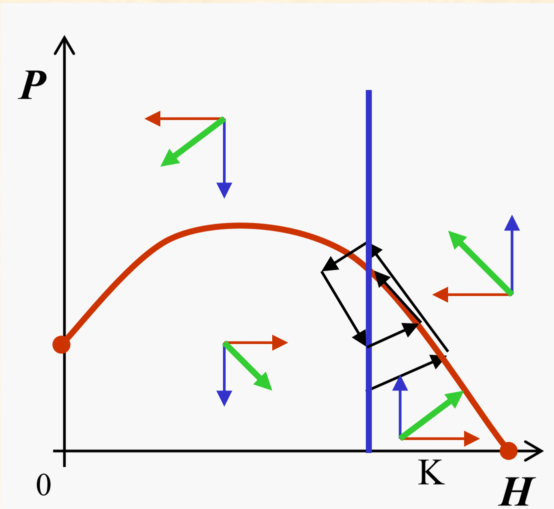
$$\frac{\mathrm{d}H}{\mathrm{d}t} = r_H H\left(1 - \frac{H}{K}\right) - \frac{aHP}{1 + aHT_h} \qquad \frac{dP}{\mathrm{d}t} = bHP - mP$$

- for parameters: $r_H = 3$, $a = 0.1$, $T_h = 2$, $K = 10$

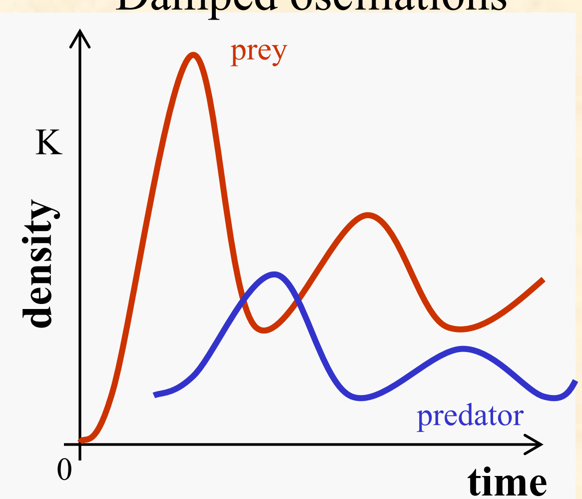
$$\frac{\mathrm{d}H}{\mathrm{d}t} = 0 \qquad 0 = 3H\left(1 - \frac{H}{10}\right) - \frac{0.1HP}{1 + 0.1H2} \qquad H = \frac{m}{b}$$

prey isocline: $P = 30 + 6H - 0.6H^2$ predator isocline: $H = constant$

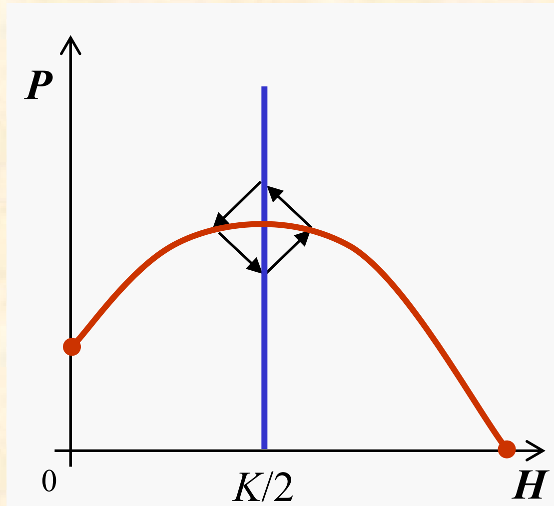
- predator exploits prey close to *K*
  - isocline: *H* = 9



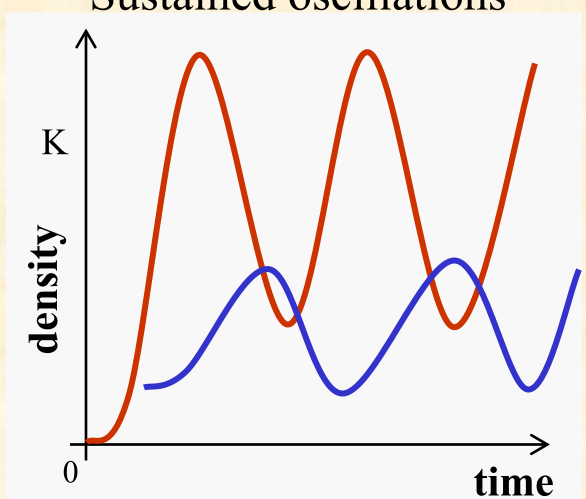
Damped oscillations



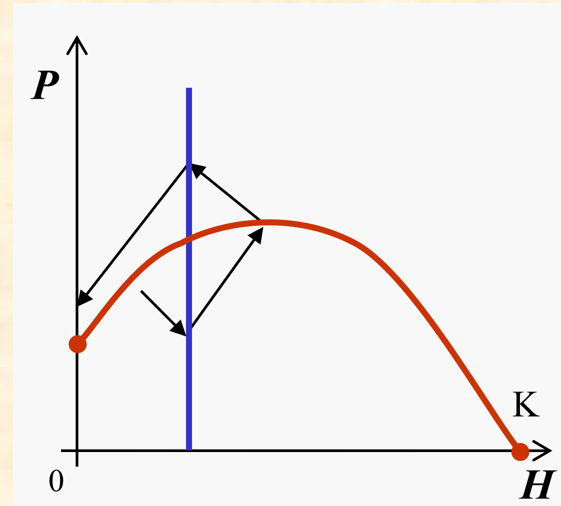
- predator exploits prey close to *K*/2
  - isocline: *H* = 5



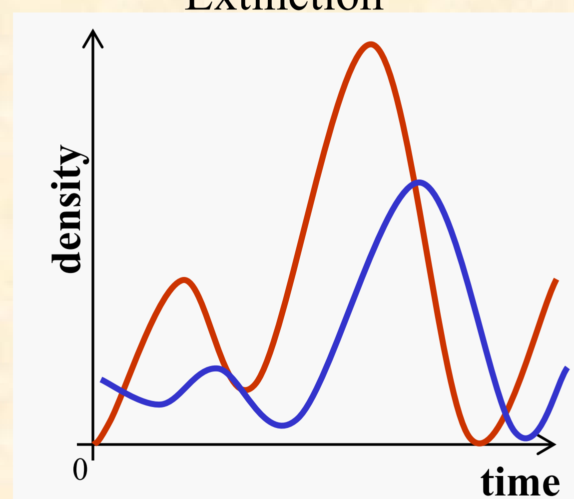
Sustained oscillations



- predator exploits prey at low density
  - isocline: *H* = 2



Extinction



Rosenzweig & MacArthur (1963)

Addition of predator's carrying capacity

- logistic model with carrying capacity proportional to *H*
- *k* .. parameter of carrying capacity of the predator
- $r_P = bH - m$

$$\frac{\mathrm{d}P}{\mathrm{d}t} = bHP - mP$$

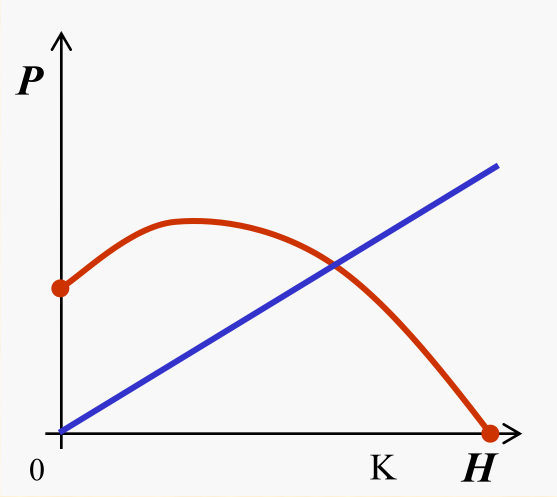
$$\frac{\mathrm{d}P}{\mathrm{d}t} = r_P P\left(1 - \frac{P}{kH}\right) \qquad \frac{\mathrm{d}H}{\mathrm{d}t} = r_H H\left(1 - \frac{H}{K}\right) - \frac{aHP}{1 + aHT_h}$$

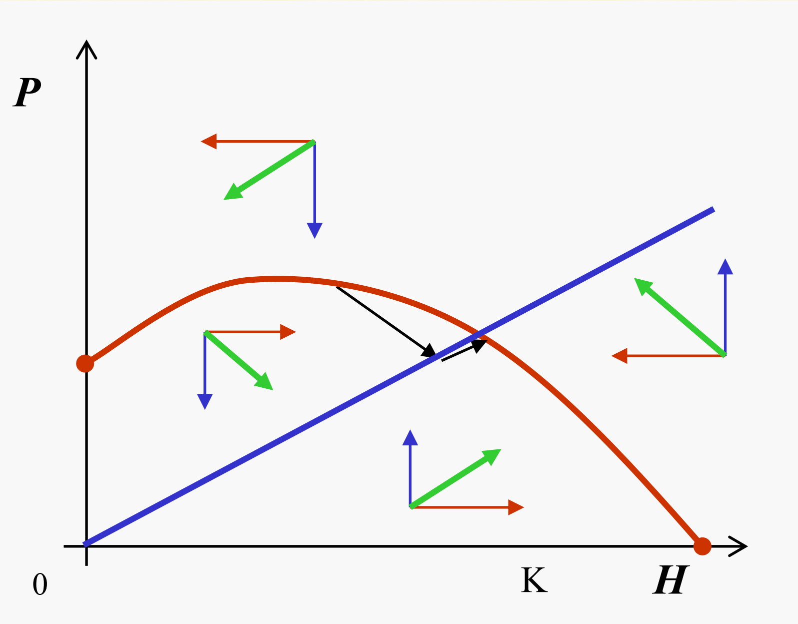
- for parameters: $r_P = 2, k = 0.2$

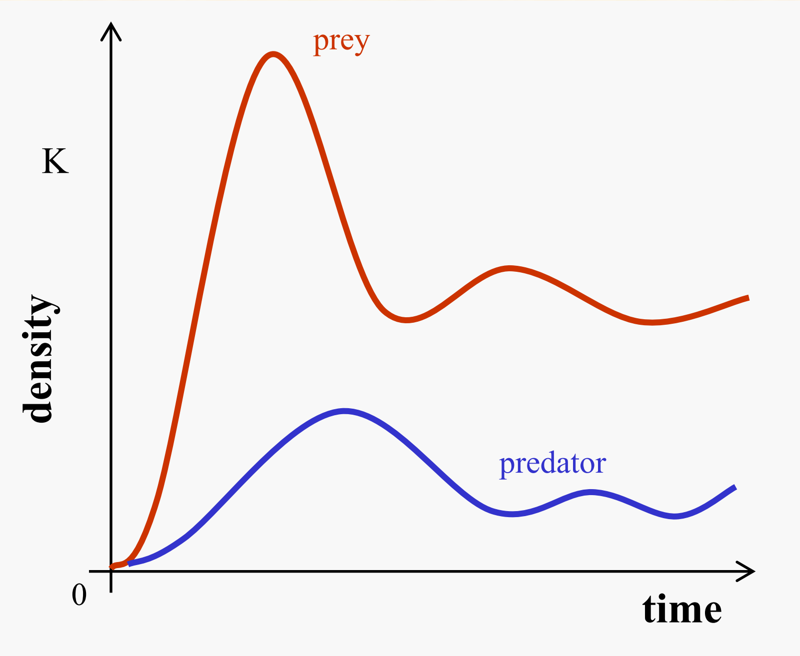
$$\frac{\mathrm{d}P}{\mathrm{d}t} = 0 \qquad 0 = 2P\left(1 - \frac{P}{0.2H}\right)$$

predator isocline: $H = 5P$

prey isocline: $P = 30 + 6H - 0.6H^2$







- quick approach to stable equilibrium

Host-parasitoid system

Zatypota

Theridion

Host-parasitoid model

- discrete model of Nicholson & Bailey (1935)

- discrete generations
- attack happens at reproduction
- 1, .., several, or less than 1 host
- random host search and functional response Type III
- lay eggs in aggregation

H_t = number of hosts in time t
H_a = number of attacked hosts
λ = finite rate of increase of the host

$$H_{t+1} = \lambda(H_t - H_a)$$
$$P_{t+1} = cH_a = H_a$$

P_t = number of parasitoids
c = conversion rate, no. of parasitoids for 1 host

Incorporation of random search

- parasitoid searches randomly
- encounters (x) are random (Poisson distribution)

$$p_x = \frac{\mu^x e^{-\mu}}{x!} \qquad x = 0, 1, 2, ... \qquad p_0 = e^{-\mu}$$

p_0 = proportion of not encountered, μ .. mean number of encounters

E_t = total number of encounters
a = searching efficiency

$$E_t = a\, H_t\, P_t \longrightarrow \frac{E_t}{H_t} = aP_t = \mu \longrightarrow p_0 = e^{-aP_t}$$

- proportion of encounters (1 or more times): $p = (1 - p_0)$

$$p = (1 - e^{-aP_t})$$

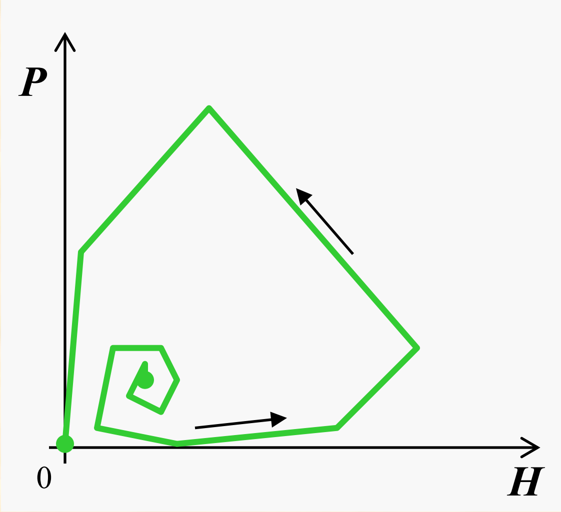
$$H_a = H_t\left(1 - e^{-aP_t}\right)$$

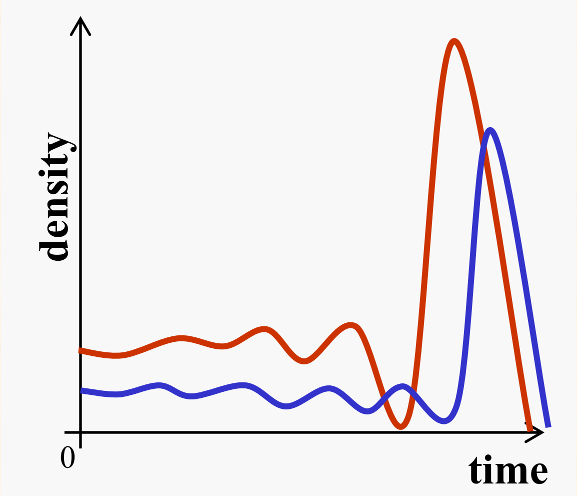
$$H_{t+1} = \lambda(H_t - H_a)$$
$$P_{t+1} = H_a$$

$$H_{t+1} = \lambda H_t e^{-aP_t}$$
$$P_{t+1} = H_t\left(1 - e^{-aP_t}\right)$$

- highly unstable model for all parameter values:

- equilibrium is possible but the slightest disturbance leads to divergent oscillations (extinction of parasitoid)





Addition of density-dependence

- exponential growth of hosts is replaced by logistic equation

$$H_{t+1} = \lambda H_t e^{\left(1-\frac{H_t}{K}\right)-aP_t}$$

$$P_{t+1} = H_t\left(1-e^{-aP_t}\right)$$

$$q = \frac{H^*}{K}$$

*H**.. new host carrying capacity

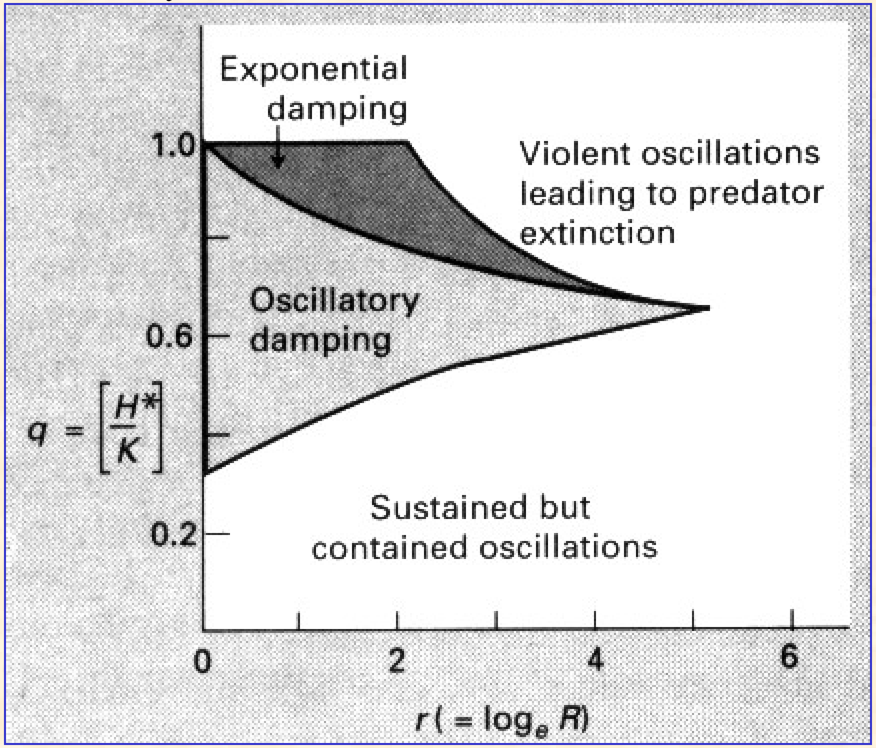
- depends on parasitoids' efficiency

- when a is low then $q \to 1$

- when a is high then $q \to 0$

- density-dependence have stabilising effect for moderate r and q

Stability boundaries



Beddington et al. (1975)

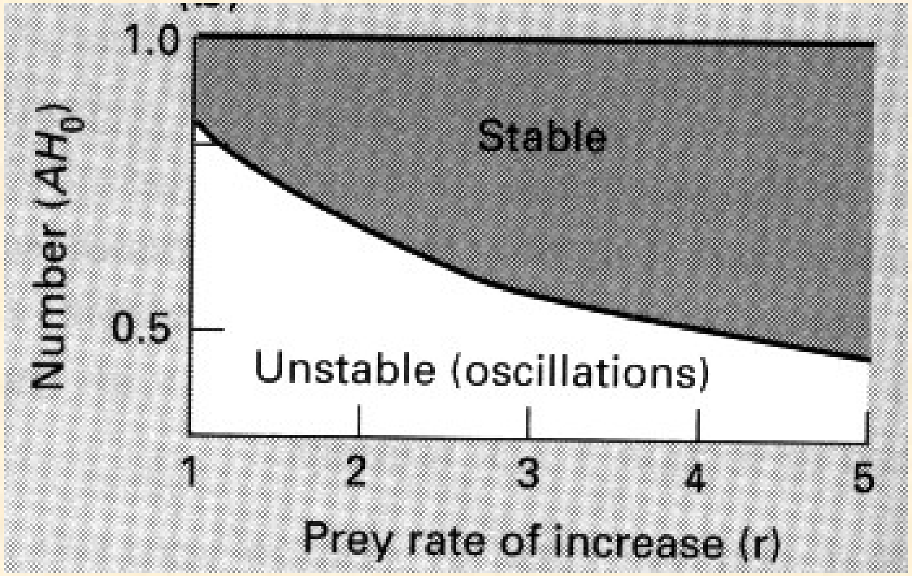
Addition of the refuge

- if hosts are distributed non-randomly in the space

<u>Fixed number in refuge</u>: H_0 hosts are always protected

$$H_{t+1} = \lambda H_0 + \lambda(H_t - H_0)e^{-aP_t}$$
$$P_{t+1} = (H_t - H_0)\left(1 - e^{-aP_t}\right)$$

- have strong stabilising effect even for large r



Hassell & May (1973)

Addition of aggregated distribution

- distribution of encounters is not random but aggregated (negative binomial distribution)

- proportion of hosts not encountered (p_0):

$$p_0 = \left(1 + \frac{aP_t}{k}\right)^{-k}$$

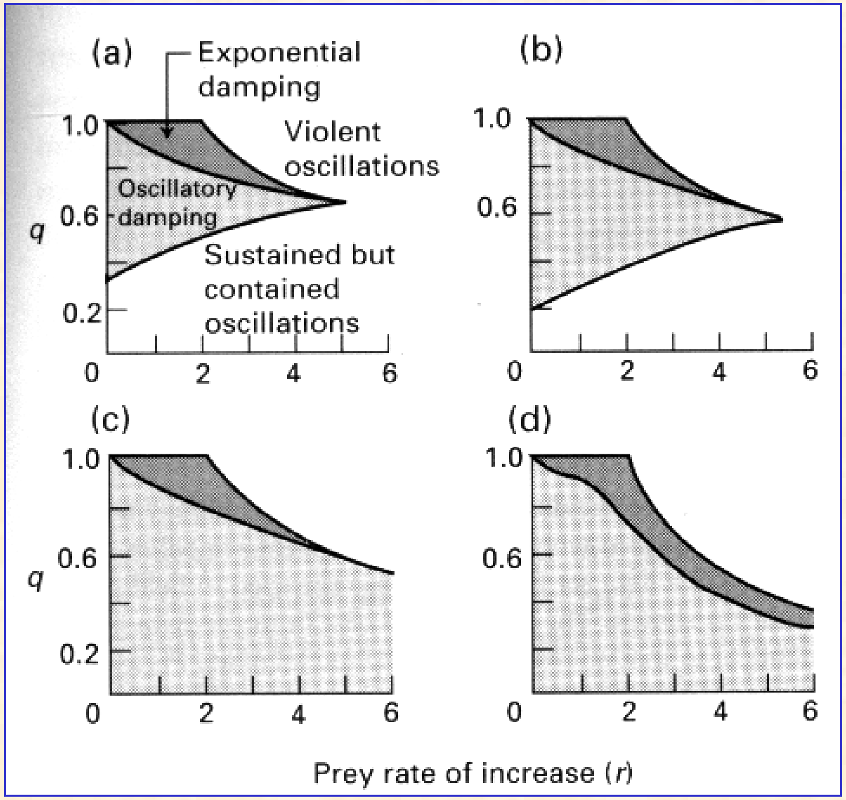
where k = degree of aggregation

$$H_{t+1} = \lambda H_t e^{\left(1-\frac{H_t}{K}\right)\left(1+\frac{aP_t}{k}\right)^{-k}}$$

$$P_{t+1} = H_t\left(1-\left(1+\frac{aP_t}{k}\right)^{-k}\right)$$

- very stable model system if $k \leq 1$

Stability boundaries:
a) $\boldsymbol{k}=\infty$, b) $\boldsymbol{k}=2$, c) $\boldsymbol{k}=1$, d) $\boldsymbol{k}=0$



Hassell (1978)