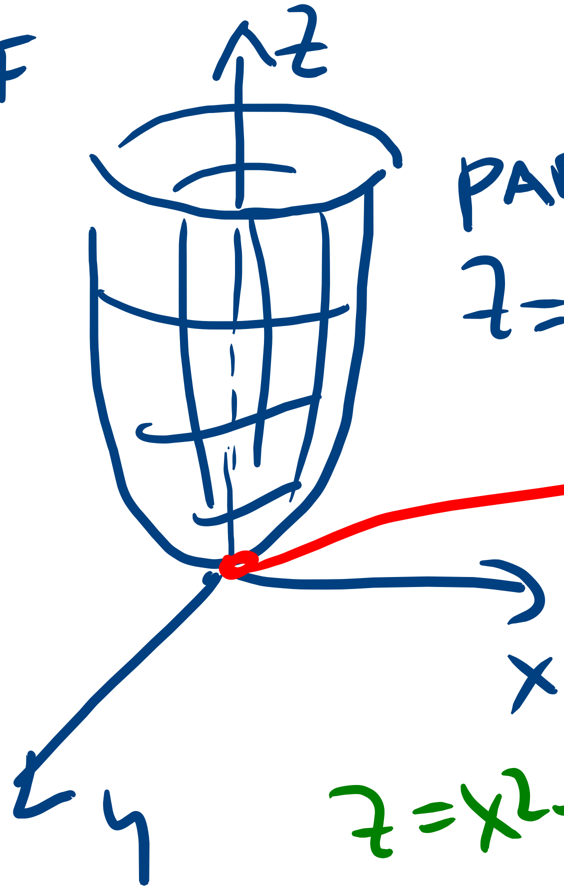
1a) SUDÁ FCE $\Leftrightarrow$ $\forall x \in D$ : $f(x) = f(-x)$

NE — SPECIÁLNĚ MUSÍ PLATIT: $-1 \in D \rightarrow f(-1) = f(1)$ ALE $1 \notin D$

$(D = \mathbb{R} \setminus \{1\})$

2a) $z = x^2 + y^2 + k$, $k \in \mathbb{R}$ ANO

A) GRAF




PARABOLOID
$z = x^2 + y^2$

1 EXTRÉM (MINIMUM)

$z = x^2 + y^2 + k$
PARABOLOID

POSUNUTÍ O k VE SMĚRU OSY z

B) VÝPOČET

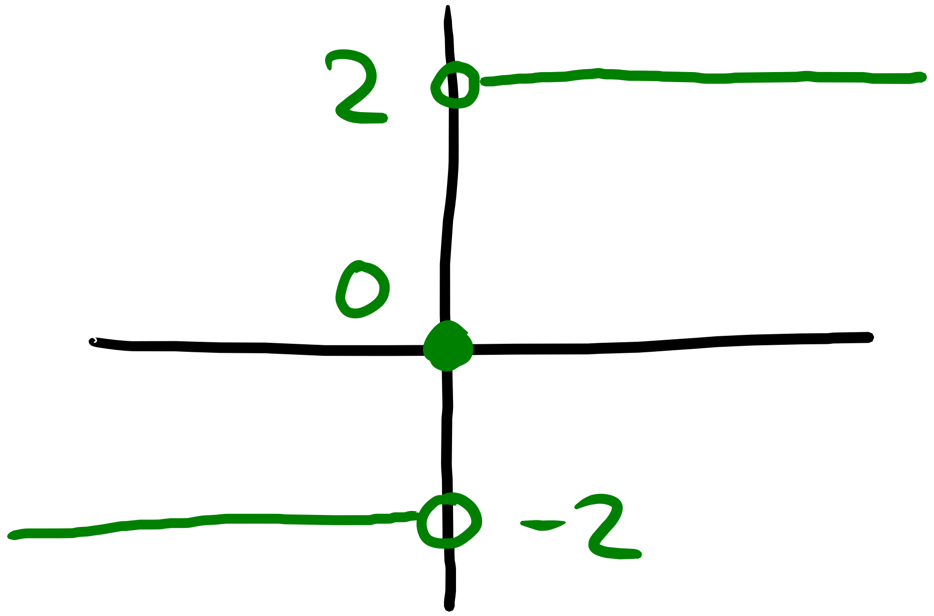
$\frac{\partial z}{\partial x} = 2x$, $\frac{\partial z}{\partial y} = 2y$

$2x = 0$
$2y = 0$ $\rightarrow A[0;0]$ POUZE 1 STACIONÁRNÍ BOD

$H = \det \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = 4$

$H(A) = 4 > 0 \rightarrow$ EXTRÉM

2)
$$f(x) = \begin{cases} 2 & \text{PRO } x > 0 \\ 0 & \text{PRO } x = 0 \\ -2 & \text{PRO } x < 0 \end{cases}$$




$$\lim_{x \to 0^+} f(x) = 2$$

$$\lim_{x \to 0^-} f(x) = -2$$

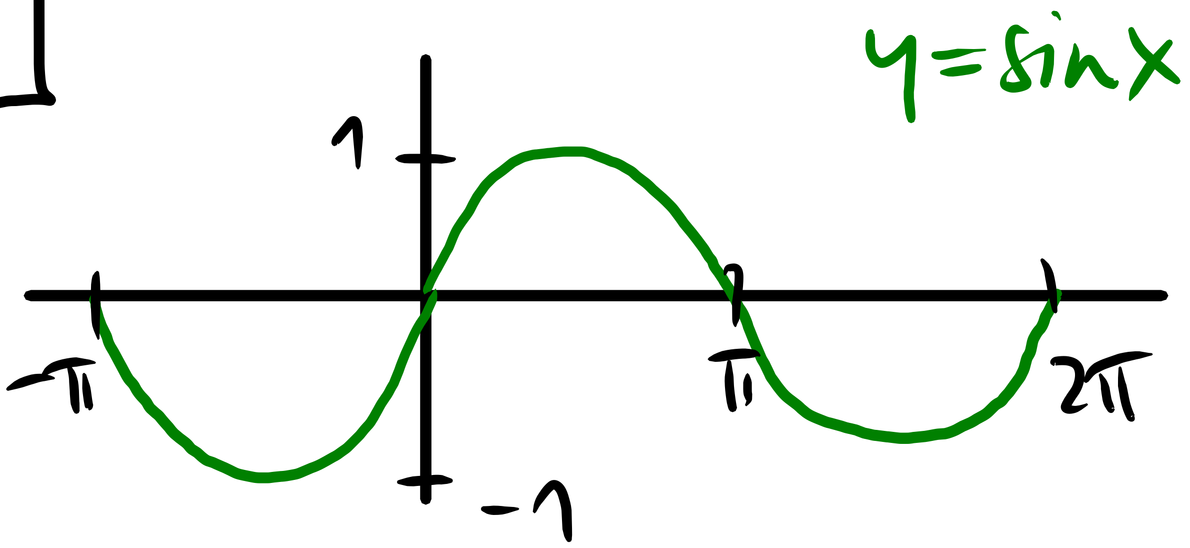
⟹ JEDNOSTRANNÉ LIMITY SE NEROVNAJÍ → $\lim_{x \to 0} f(x)$ NEEXISTUJE

3a)
$$\int_a^b f(x)\,dx = F(b) - F(a) = -\big(F(a) - F(b)\big) = -\int_b^a f(x)\,dx$$

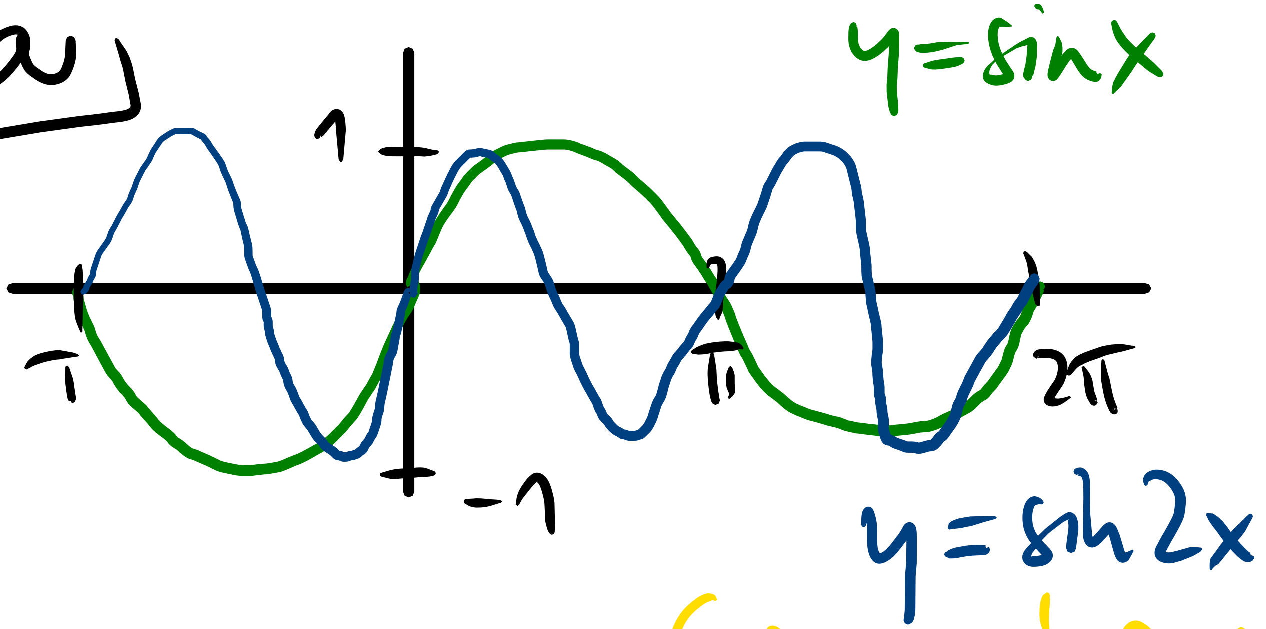
F(x) JE PRIMITIVNÍ FCE K f(x)

3b)
$$\int_a^c f(x)\,dx + \int_c^b f(x)\,dx = \cancel{F(c)} - F(a) + F(b) - \cancel{F(c)} =$$
$$= F(b) - F(a) = \int_a^b f(x)\,dx$$

4)




4a)



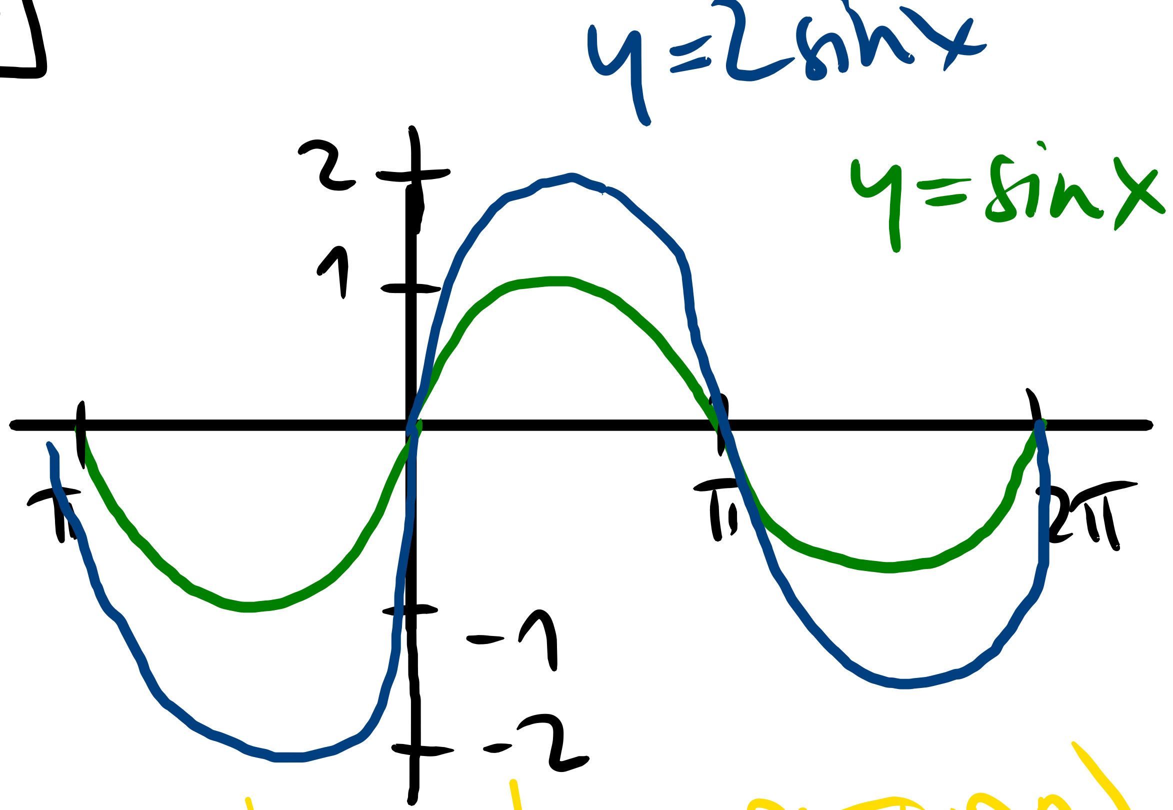

(DVOJNÁSOBNÁ FREKVENCE)

PŘ: $x = \pi/2$

$\sin \frac{\pi}{2} = 1$

$\sin(2 \cdot \frac{\pi}{2}) = \sin \pi = 0$

4b)



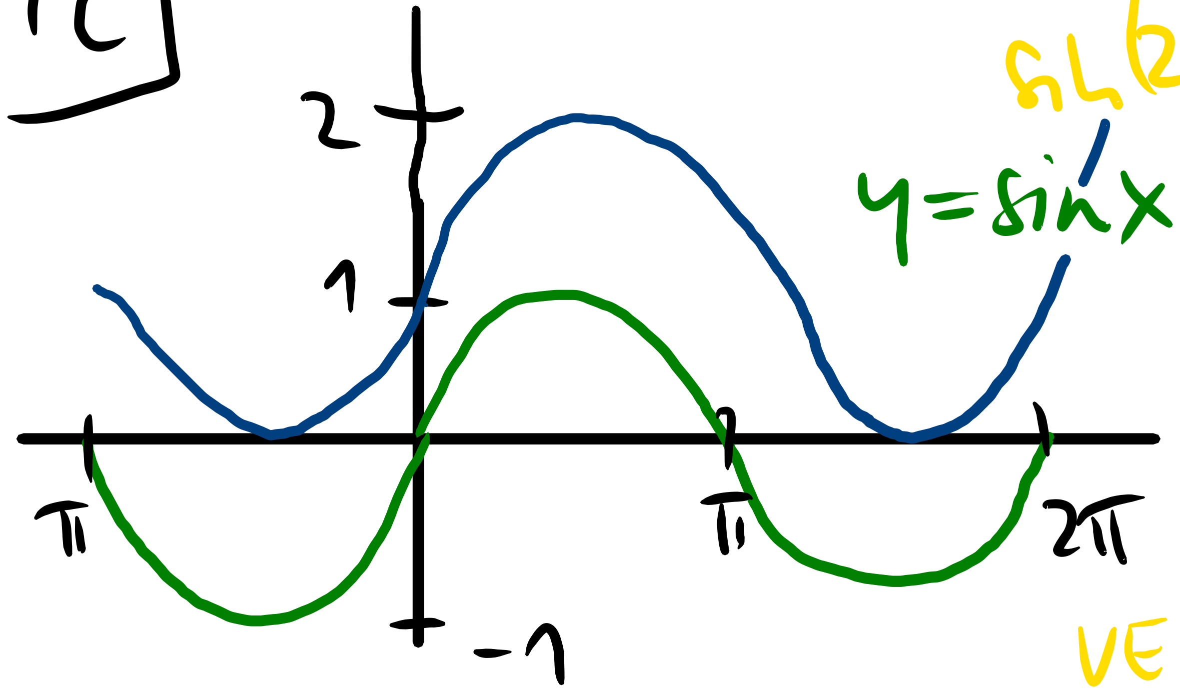

(DVOJNÁSOBNÁ AMPLITUDA)

PŘ: $x = \pi/2$

$\sin \pi/2 = 1$

$2 \cdot \sin \pi/2 = 2$

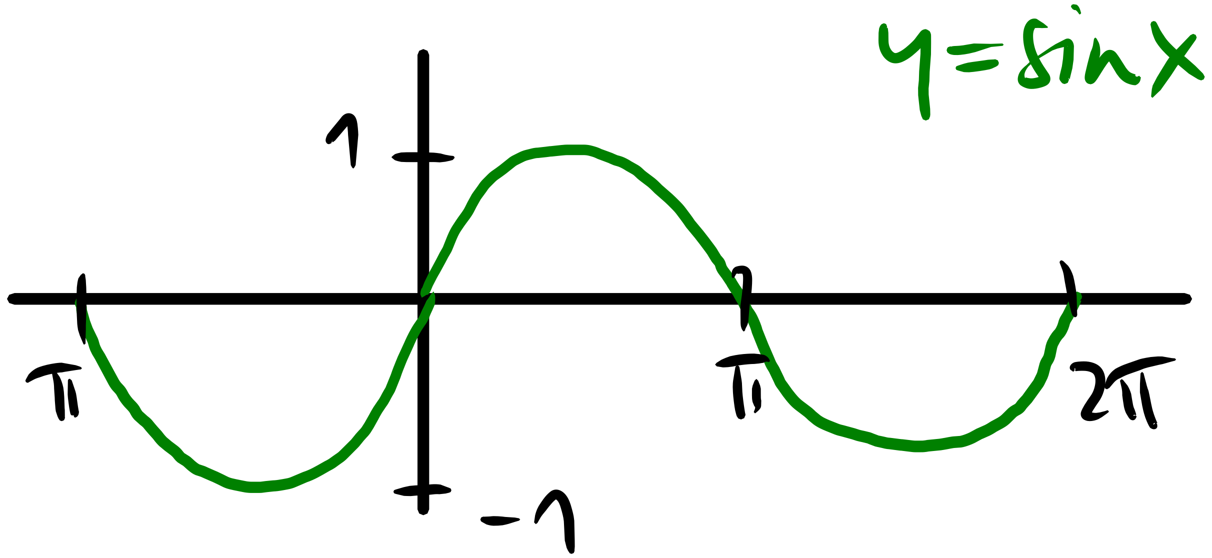
4c)




$y = 1 + \sin x$

POSUN O 1 VE SMĚRU OSY y

4D)

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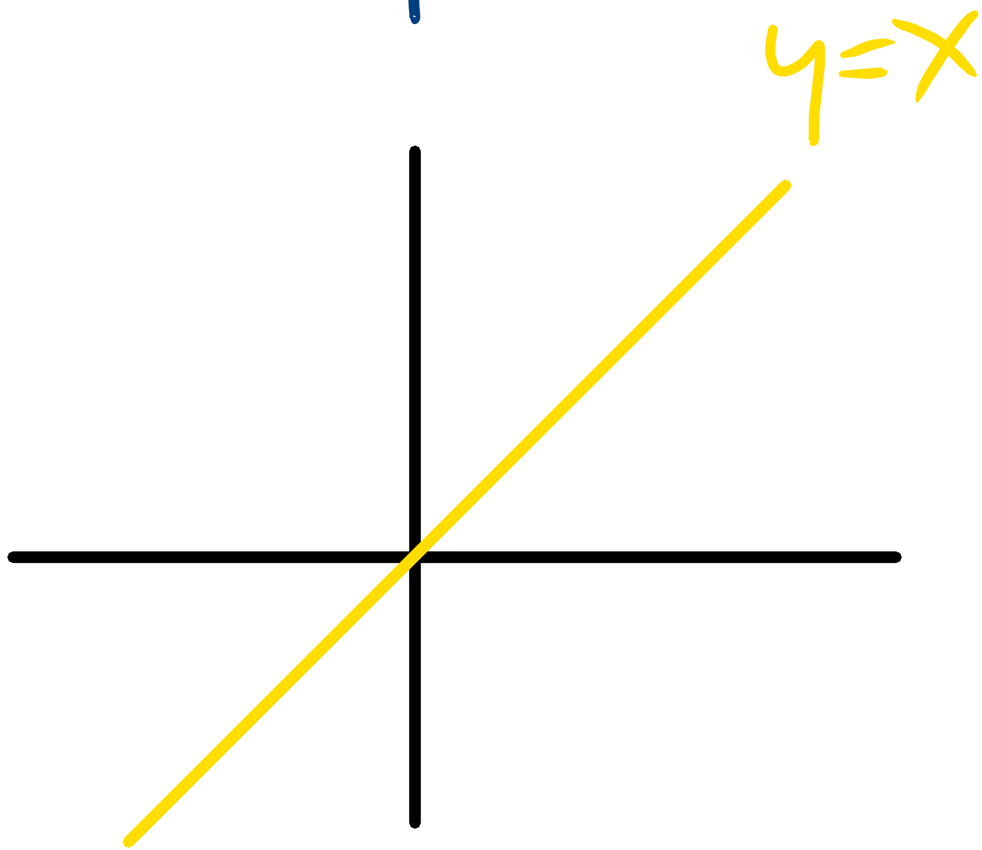


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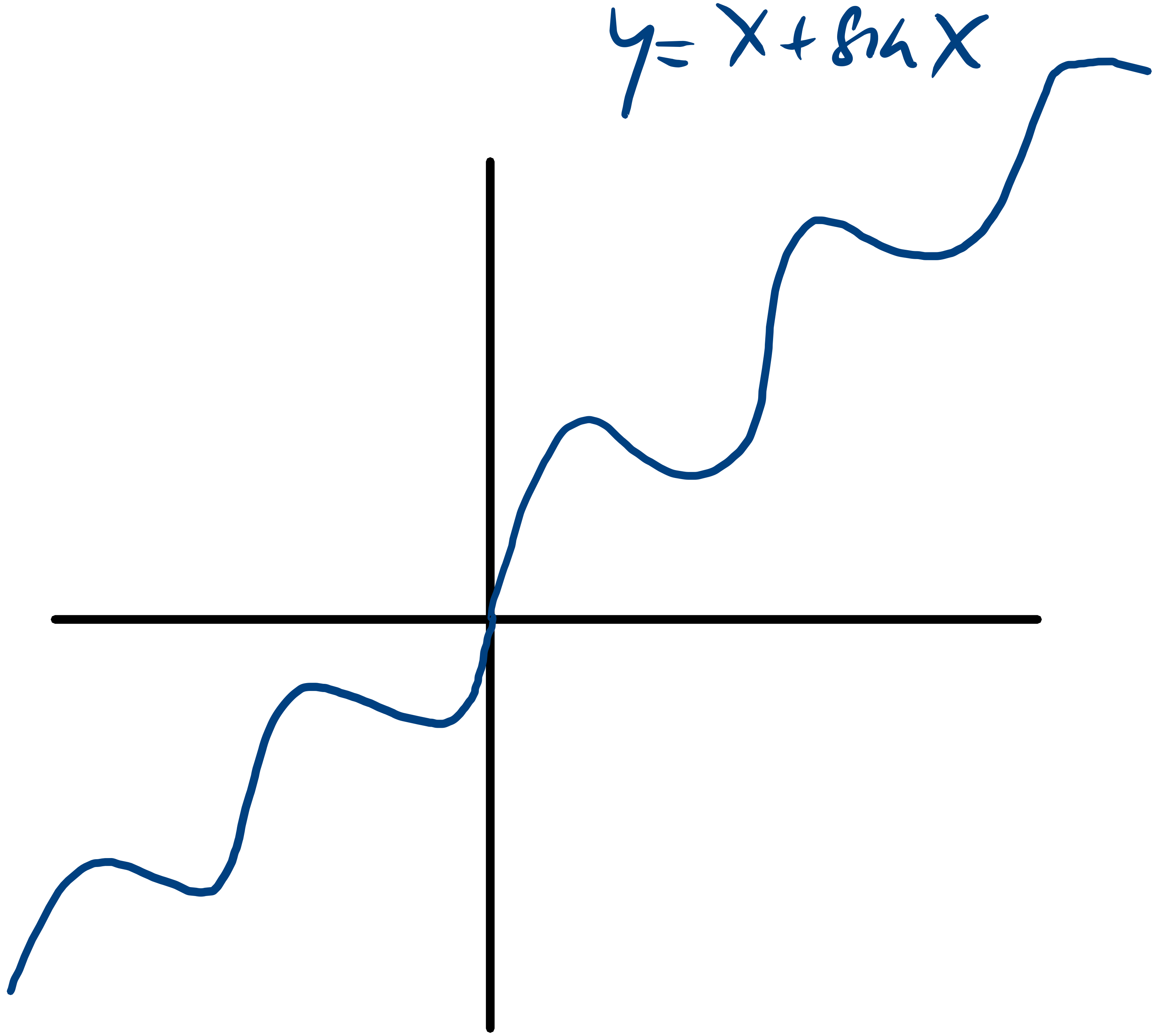
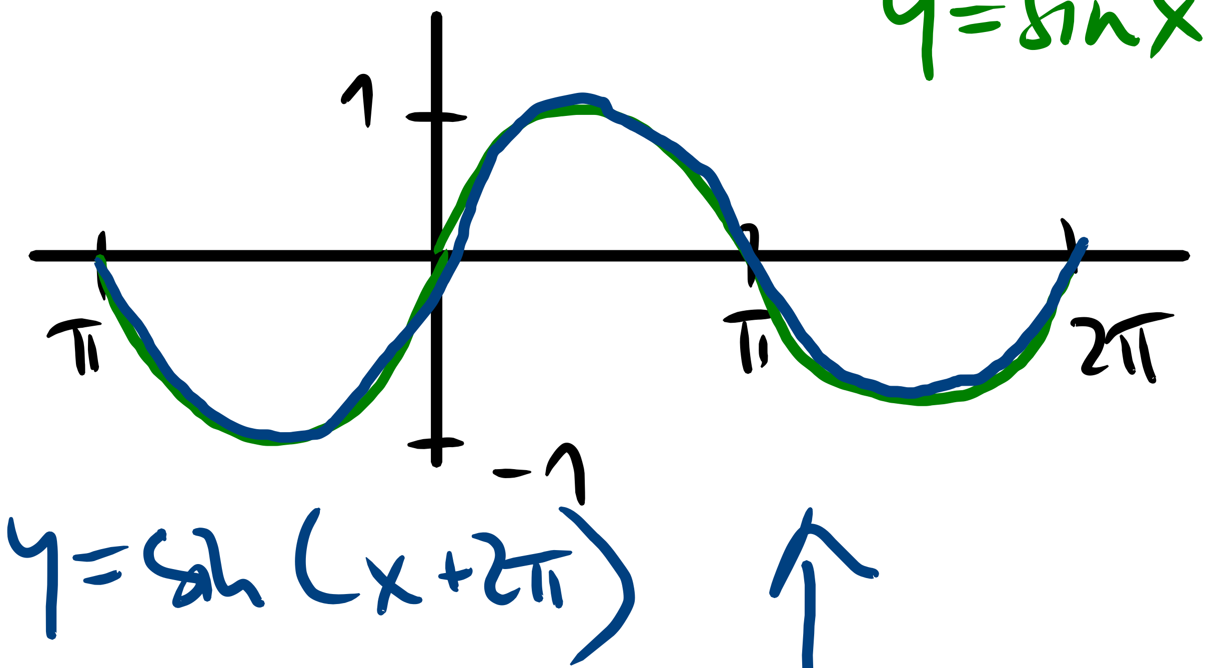


4E)




FCE $\sin x$ JE $2\pi$-PERIODICKÁ,

TJ: $\forall x \in D: \sin x = \sin(x + k2\pi)$

SPECIÁLNĚ PRO $k = 1$:

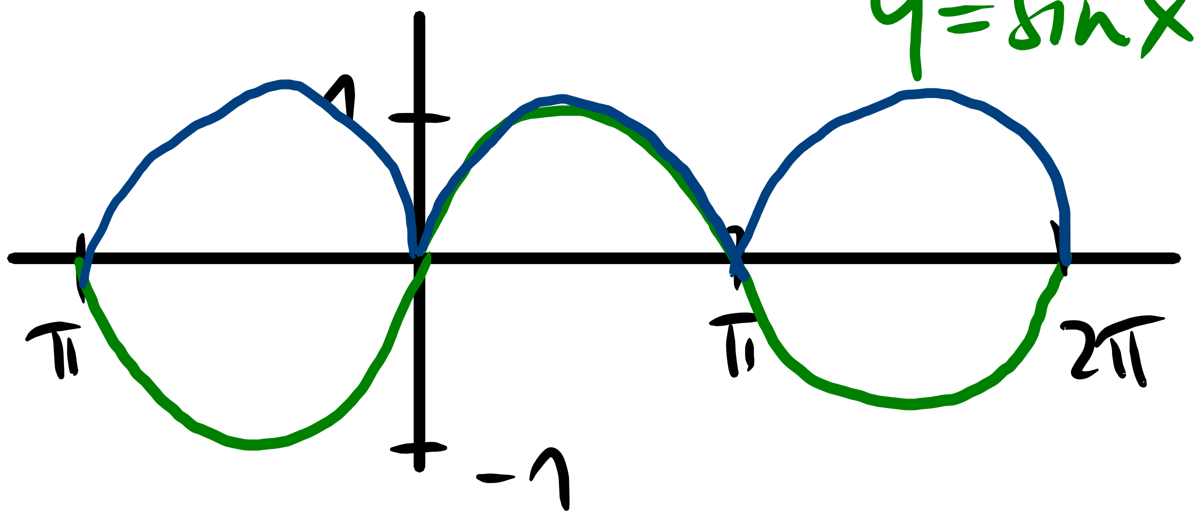
$\sin x = \sin(x + 2\pi)$

TOTOŽNÁ S $\sin x$

NEBO:

$\sin(x+2\pi)$ JE SINUSOIDA POSUNUTÁ O $2\pi$ DOLEVA, TEDY TOTOŽNÁ S $\sin x$

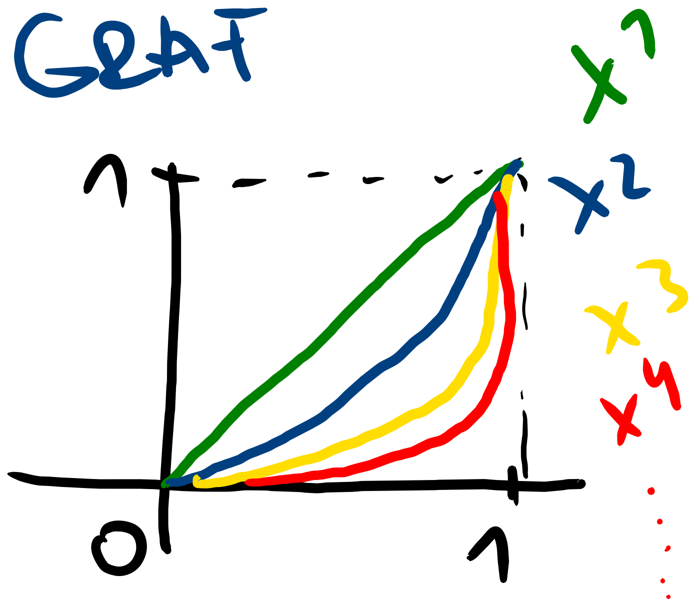
4F) $y = |\sin x|$




5) URČETE NEJMENŠÍ MOŽNÉ K :

$\int_0^1 x^n \, dx \le K$ pro libovolné $n \in \mathbb{N}, n \ge 1$

A) GRAF




$\int_0^1 f(x)\,dx =$ OBSAH PLOCHY MEZI KŘIVKOU $f(x)$ A OSOU X NA INTERVALU $[0;1]$

NEJVĚTŠÍ OBSAH PRO $n=1$

$\Rightarrow K = 1/2$ [POLOVINA OBSAHU ČTVERCE]

B) VÝPOČET

$$\int_0^1 x^n\,dx = \left[\frac{x^{n+1}}{n+1}\right]_0^1 = \frac{1^{n+1}}{n+1} - \cancel{\frac{0^{n+1}}{n+1}}^{\,0} = \frac{1^{n+1}}{n+1} = \frac{1}{n+1}$$

$\frac{1}{n+1}$ NABÝVÁ NEJVĚTŠÍ HODNOTY PRO $n=1$, A FCE

$$\frac{1}{1+1} = \frac{1}{2} = K$$

C) $f'(x) \geq 0$ pro $\forall x \in D$

→ FCE JE NEKLESAJÍCÍ

PŘÍKLADY ŘEŠENÍ:

$f(x) = e^x$

$f(x) = 3x$

$f(x) = 2$

$f(x) = x^3$

6b) PRŮSEČÍKY S OSOU X PRO $x \in \{-1; 2, 4\}$

$\Leftrightarrow \quad f(-1)=0, \; f(2)=0, \; f(4)=0$

ŘEŠENÍ: $f(x) = (x+1)(x-2)(x-4)$

[KOŘENY POLYNOMU = PRŮSEČÍKY S OSOU X]

6c) $f(x,y)$, BEZ EXTRÉMŮ

$\downarrow$

STAČÍ, ABY NEEXISTOVAL ŽÁDNÝ STACIONÁRNÍ BOD

$\downarrow$

PRVNÍ PARC. DERIVACE MUSÍ BÝT NENULOVÉ

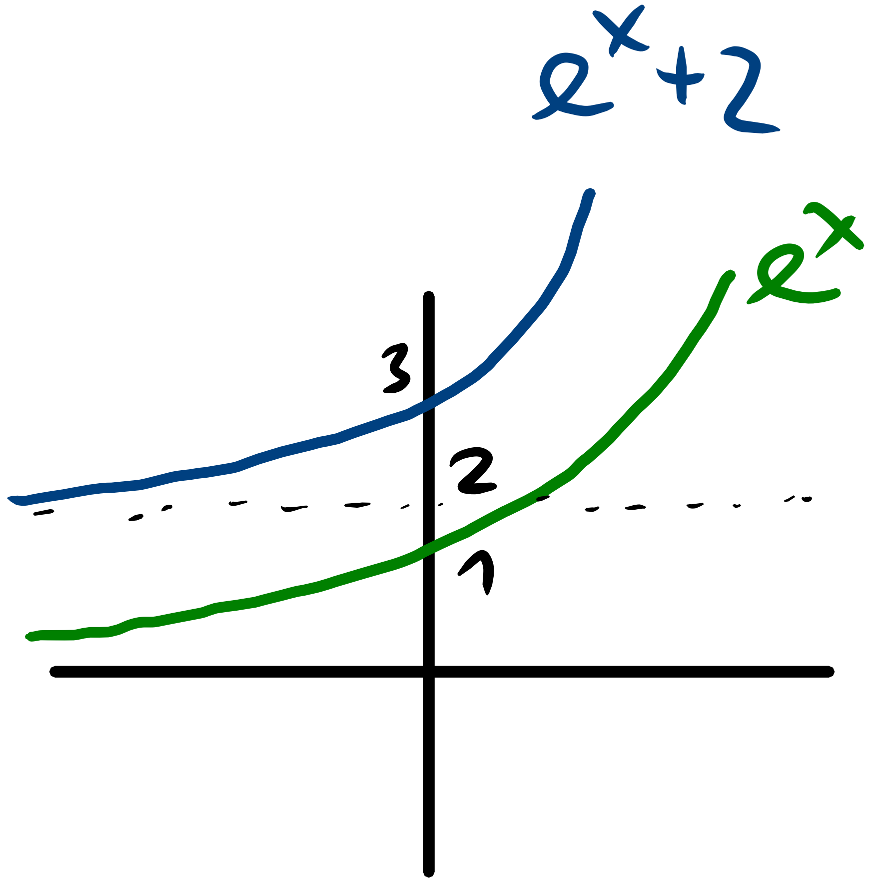
$\downarrow$

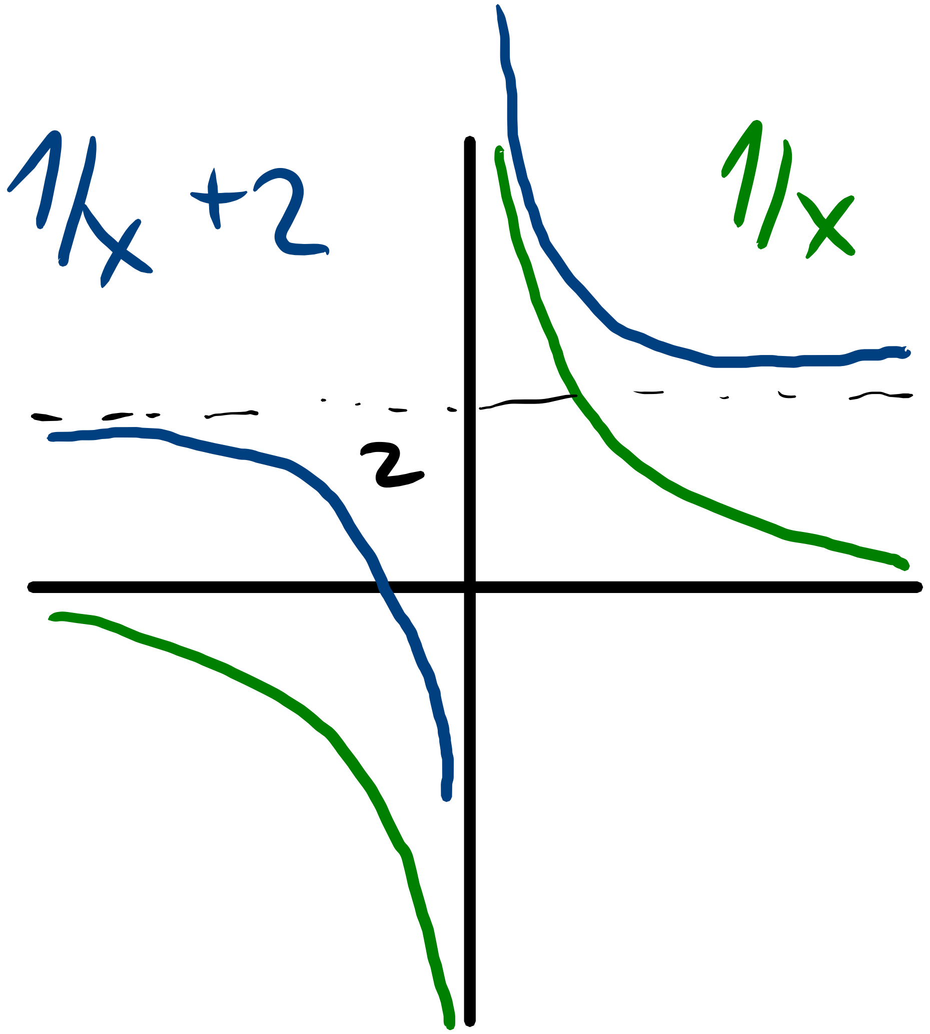
NAPŘ. $\dfrac{\partial f}{\partial x} = 1, \; \dfrac{\partial f}{\partial y} = 1$ PRO $f(x,y) = x+y$

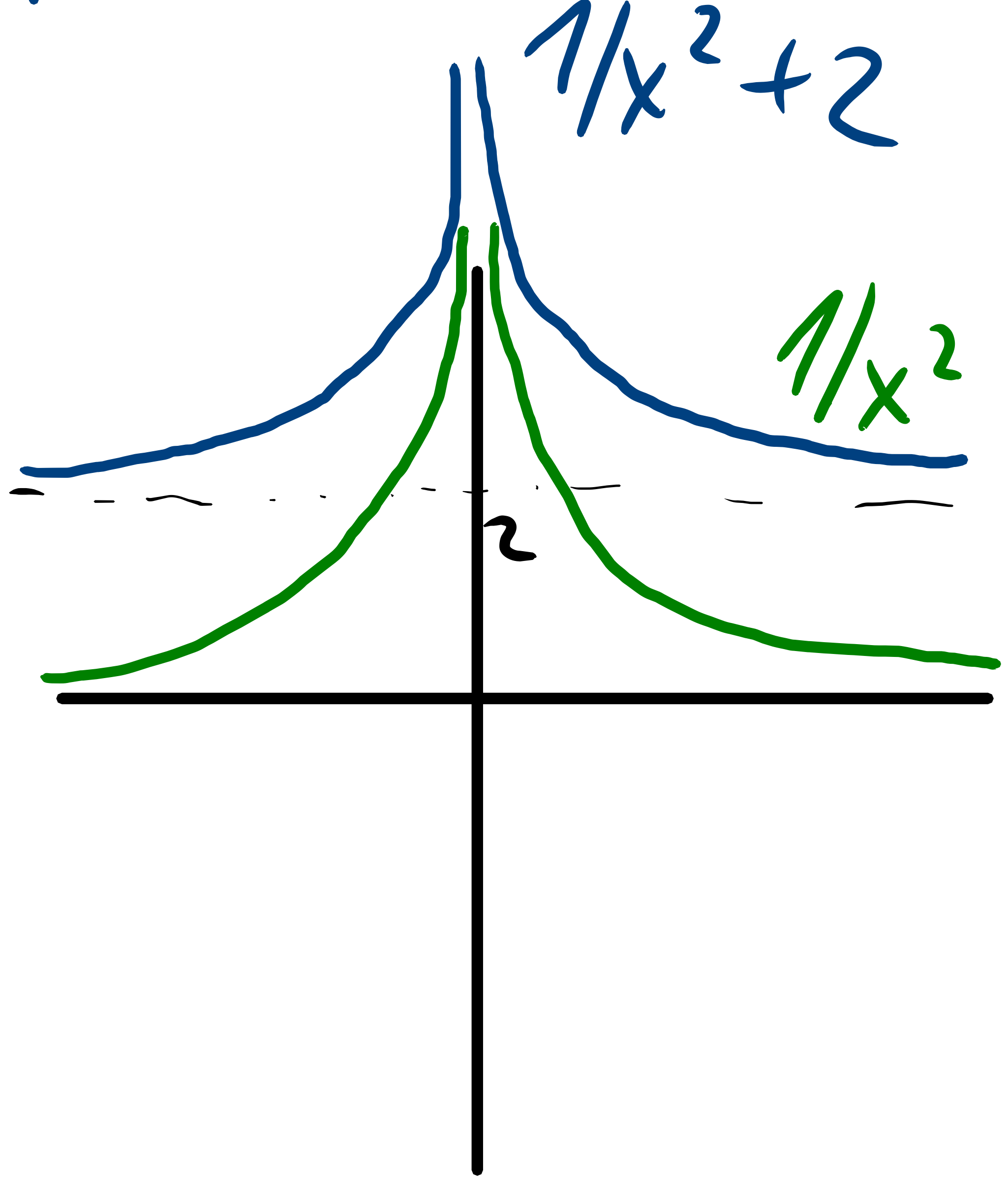
GDJ ASYMPTOTA $y=2$

↳ STAČÍ VZÍT FCI S ASYMPTOTOU $y=0$ A POSUNOUT JI O 2 NAHORU

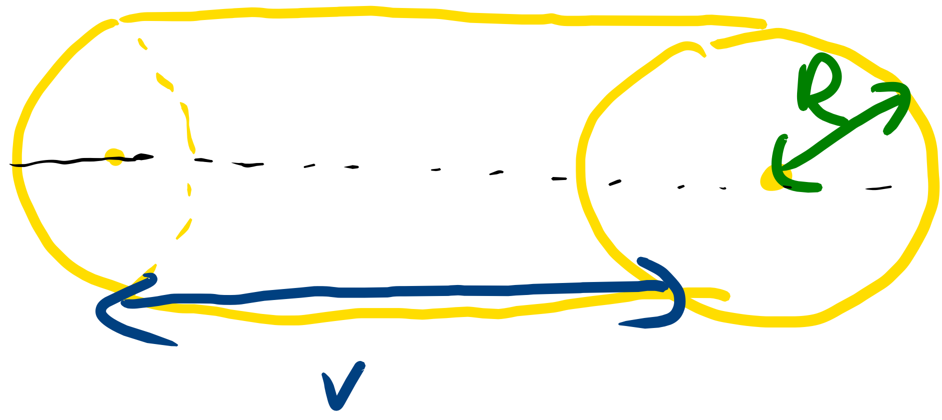
Př.



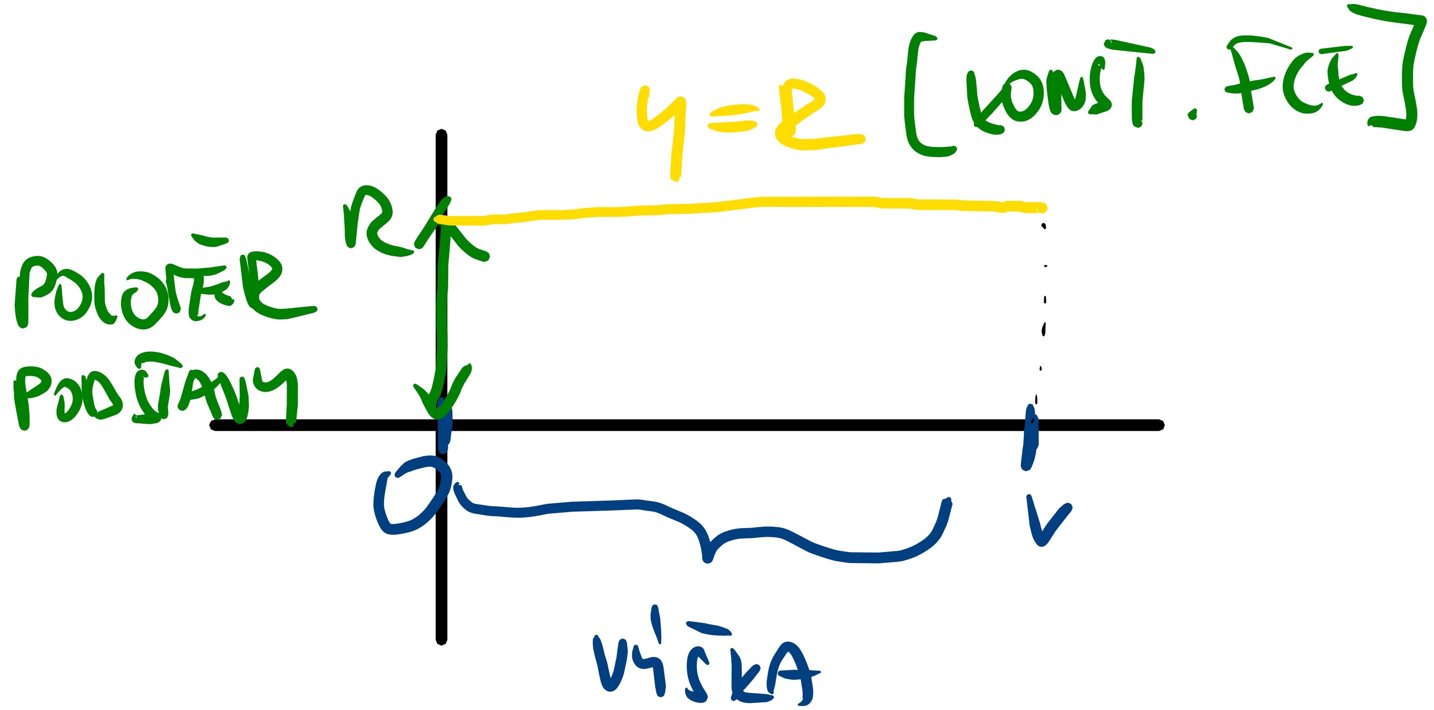



7)




$\Rightarrow$




OBECNĚ:

$$V = \pi \int_a^b f^2(x)\,dx$$

VÁLEC:

$$V = \pi \int_0^v (R)^2\,dx = \pi \left[R^2 x\right]_0^v = \pi\left(R^2 \cdot v - R^2 \cdot 0\right) = \pi R^2 \cdot v$$

INTEGRACE DLE X, $R^2$ JE KONSTANTA

b) $z = e^{y^2x}$

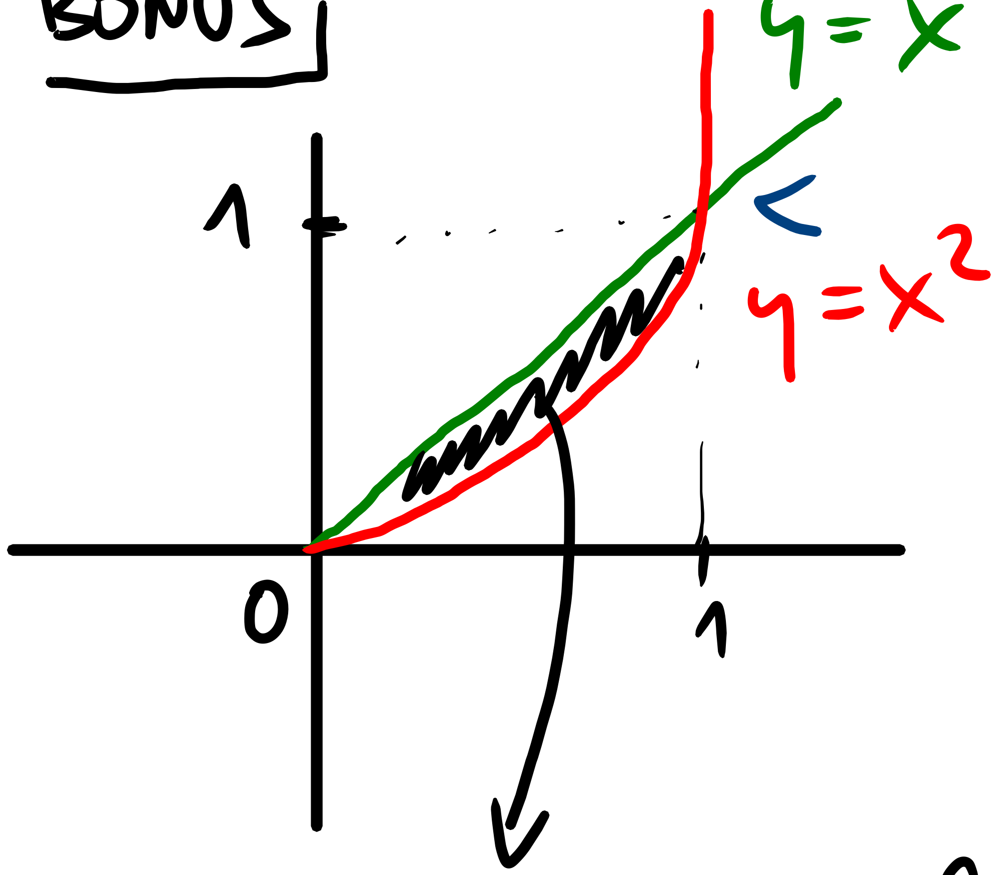
$$\frac{\partial z}{\partial x} = e^{y^2x} \cdot y^2 \qquad \frac{\partial z}{\partial y} = e^{y^2x} \cdot 2xy$$

$$\frac{\partial^2 z}{\partial x^2} = e^{y^2x} \cdot y^4$$

$$\frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = e^{y^2x} \cdot 2xy \cdot y^2 + e^{y^2x} \cdot 2y$$

$$\frac{\partial^2 z}{\partial y^2} = e^{y^2x} \cdot 2xy \cdot 2xy + e^{y^2x} \cdot 2x$$

## BONUS




**SOUŘADNICE PRŮSEČÍKŮ?**

$$x = x^2 \rightarrow x^2 - x = 0$$

$$\rightarrow x(x-1) = 0$$

$$\hookrightarrow x = 0 \rightarrow y = 0$$

$$x = 1 \rightarrow y = 1$$

$$\text{OBSAH} = \int_0^1 x\,dx - \int_0^1 x^2\,dx = \int_0^1 x - x^2\,dx =$$

$$= \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_0^1 = \left(\frac{1}{2} - \frac{1}{3}\right) - \left(\frac{0}{2} - \cancelto{0}{\frac{0}{3}}\right) =$$

$$= \frac{1}{2} - \frac{1}{3} = \frac{1}{6}$$