Analysis of variance
ANOVA

E0420

Week 5

What for?

- To compare more than two independent means = more than two groups

- ANOVA = regression
- More precisely, both are linear models
  - More about that later on in the course

Logic

- To see whether there is a “signal” in the noise
- To see whether the variance due to group membership is systematic
- To compare whether group means differ from grand mean

ANOVA model

1. Categorical independent variable
2. Continuous dependent variable

- All of the assumptions we know from *t* tests apply here
  - Normality of distribution of DV within each group
  - Homogeneity of variance
  - Independence of observations

ANOVA types

- One-way ANOVA
- Two-way ANOVA
- Repeated measures ANOVA
- ANCOVA
- MANOVA
- MANCOVA

One-way ANOVA

- Comparing whether means of groups differ from the baseline
- H0: the group means are not significantly different from the grand mean (i.e. the sample average)

- Terms:
  - Sums of squares (SS)
  - degrees of freedom (df)
  - Mean square (MS)
  - F-test

Sums of squares (SS)

- SS/(n-1) = variance

$$\sum_{i=0}^{n} (X_i - \overline{X})^2$$

- SS between = variance between groups = IV
- SS within = variance within groups = error

- SS Total = SS between + SS within

- **Mean square**: SS/df

Degrees of freedom

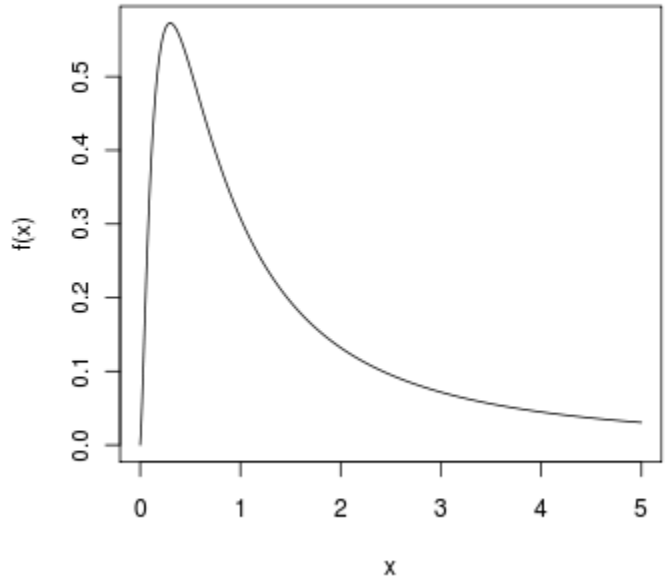
- The number of independent values that can vary in an analysis
- df = n – # parameters
- for 2 groups = 1df, for 3 groups = 2df…
- Ex: average is 10 out of 3 numbers. What are the 3 numbers?
  - X1, X2, X3
  - Once we know X1 and X2, X3 has only one solution = 2 degrees of freedom

F-test

- Test of explained/unexplained variance

$$MS_{between} / MS_{within}$$

- Follows an F-distribution
- Critical F-value for statistical significance



http://www.r-tutor.com/elementary-statistics/probability-distributions/f-distribution

Post-hoc tests

- F-test is an omnibus test
  - Tells you whether the variance explained is significantly greater than random variance
  - Does not tell you which group means are different from grand mean
- Need to run post-hoc tests – pairwise comparisons
- Running multiple t tests inflates Type I error rate

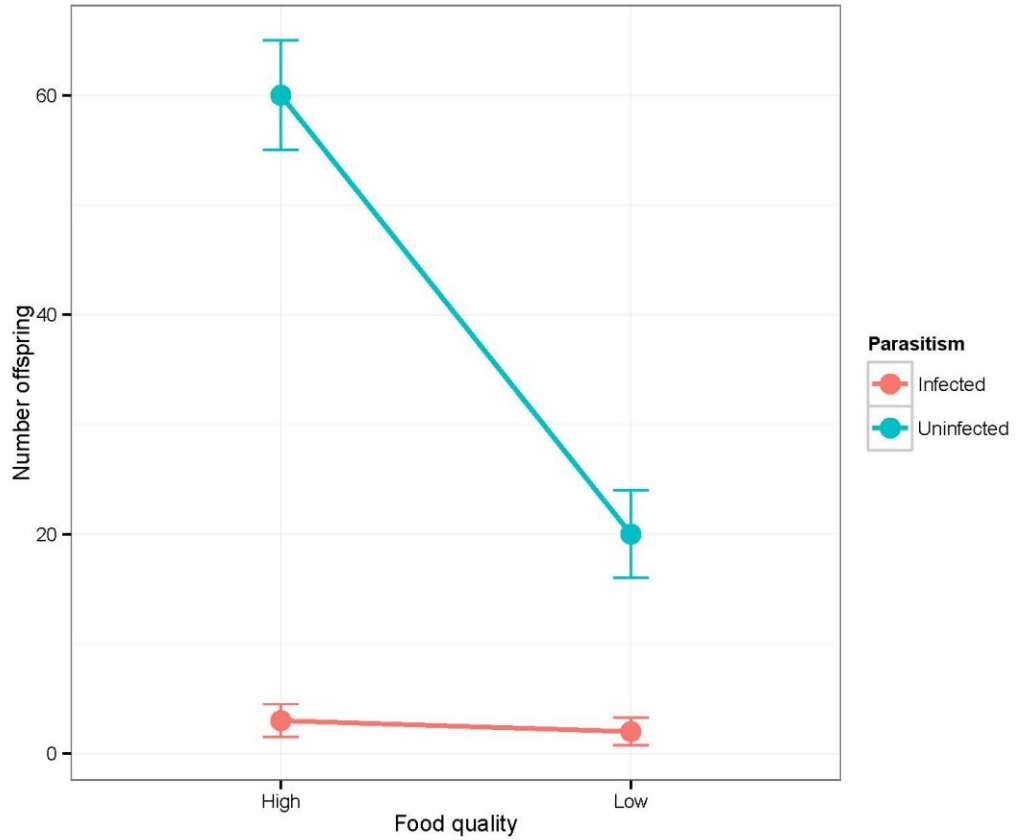
n of groups	comparisons	Type I error rate
2	1	5%
3	3	14%
4	6	26%
5	10	40%
6	15	54%
7	21	66%
8	28	76%
9	36	84%
10	45	90%

Different types

- Bonferroni
  - Easy to compute
  - Conservative – decreases statistical power
- Tukey – for all pairwise comparisons
- Dunnett – for comparing group means to control, not to each other
- LSD
- HSD
- Scheffé

Two-way ANOVA

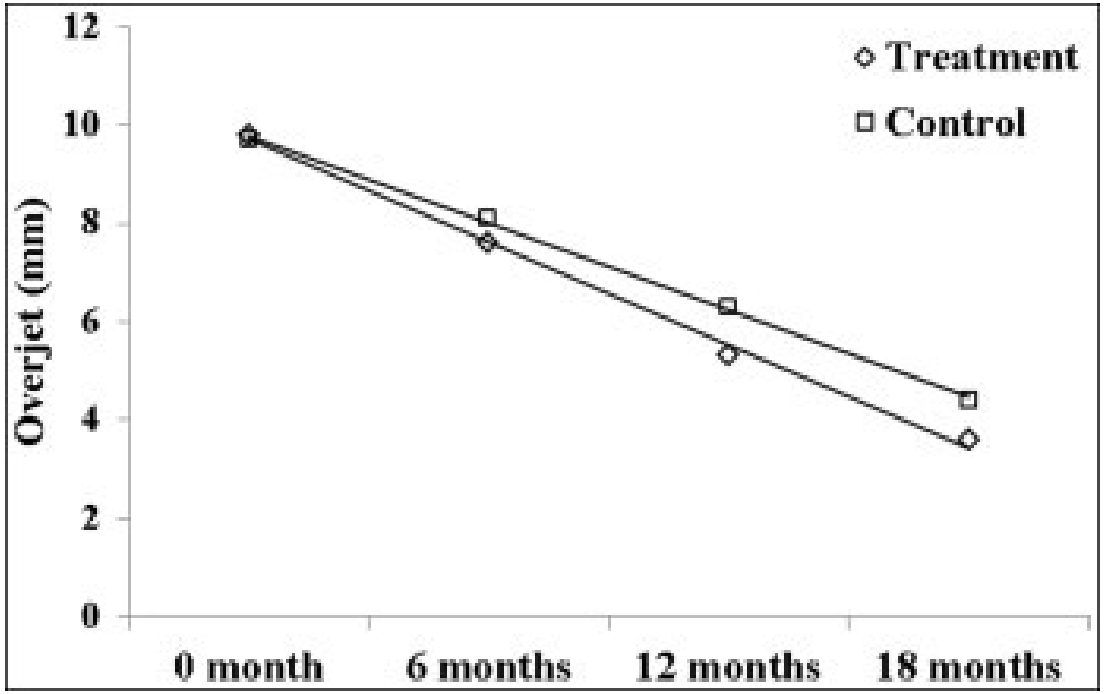
- Two categorical variables
- Can estimate interactions (IV1 conditional on levels of IV2 and vice versa)



https://dynamicecology.wordpress.com/2014/10/02/interpreting-anova-interactions-and-model-selection/

Repeated measures ANOVA

- For asessing the effect of categorical IV with more timepoints of the DV
- E.g., pretest-posttest



ANCOVA

- ANalysis Of COVAriance
- ANOVA + controlling for other variables

MANOVA

- Multiple Analysis Of VAriance
- Estimates effects on multiple DVs
  - Using a combination of DVs
- multivariate F value （Wilks‘ λ）

Statistical write-up

There was a statistically significant difference between groups as determined by one-way ANOVA, $F(2,27) = 4.467$, $p = .021$.

A post hoc Tukey test showed that the Group A and Group B differed significantly at $p < .05$; the Group C was not significantly different from the other two groups.