

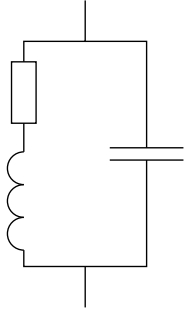
1 Capacitively coupled discharges – basic characterisation

Basic literature: [9, 1]

$$\omega_{pi} < \omega \ll \omega_{pe} \quad (1)$$

$$l \ll \lambda \quad (2)$$

2 RF plasma conductivity



$$\sigma = \frac{ne^2}{m(\nu + i\omega)} + i\omega\epsilon_0 \quad (3)$$

$$\frac{1}{\sigma} = \frac{m(\nu + i\omega)}{ne^2 + i\omega\epsilon_0 m(\nu + i\omega)} = \frac{1}{\epsilon_0} \frac{\nu + i\omega}{\omega_{pe}^2 - \omega^2 + i\nu\omega} \quad (4)$$

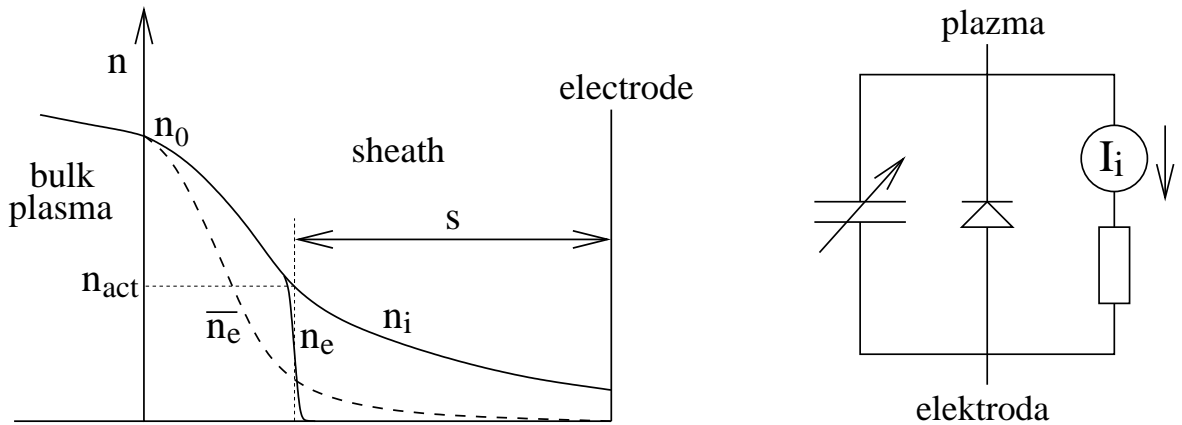
$$\omega_{pe} = e \sqrt{\frac{n}{m\epsilon_0}} \quad (5)$$

Ohmic heating:

$$\langle p \rangle = \frac{1}{2} j_1 E_1 \cos \alpha = \frac{1}{2} E_1^2 \text{Re}(\sigma) = \frac{E_1^2}{2} \frac{ne^2\nu}{m(\nu^2 + \omega^2)} \xrightarrow{\nu \rightarrow 0} \frac{E_1^2}{2} \frac{ne^2}{m\omega^2} \nu \quad (6)$$

3 RF. sheath

Literature: [1, 8]



Sheath current density:

$$j = \epsilon_0 \frac{dE}{dt} - \frac{en_0}{4} \sqrt{\frac{8kT_e}{\pi m}} e^{-eU_{sh}(t)/(kT_e)} + en_0 \sqrt{\frac{kT_e}{m_i}} \quad (7)$$

$$\epsilon_0 \frac{dE}{dt} = n_{act} e \frac{ds}{dt}$$

DC sheath voltage:

$$0 = -\frac{en_0}{4} \sqrt{\frac{8kT_e}{\pi m}} \left\langle e^{-eU_{sh}(t)/(kT_e)} \right\rangle + en_0 \sqrt{\frac{kT_e}{m_i}}$$

Let us assume $U_{sh} = U_0 + U_1 \sin \omega t$:

$$e^{-\frac{eU_0}{kT_e}} \frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} e^{-\frac{eU_1}{kT_e} \sin \omega t} dt = \sqrt{\frac{2\pi m}{m_i}}$$

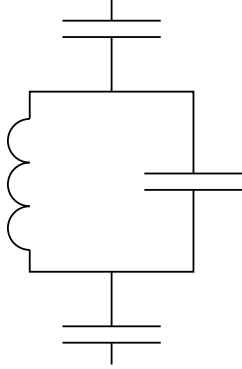
We will use

$$\frac{\omega}{2\pi} \int_0^{\frac{2\pi}{\omega}} e^{a \sin \omega t} dt = I_0(a),$$

where $I_0(a)$ is the modified Bessel function of the order zero. We will obtain

$$\frac{eU_0}{kT_e} = \frac{1}{2} \ln \left(\frac{m_i}{2\pi m} \right) + \ln \left[I_0 \left(\frac{eU_1}{kT_e} \right) \right] \quad (8)$$

4 Series plasma-sheath resonance



$$\begin{aligned} Z_b &\approx \frac{l_b}{\sigma S} \approx \frac{l_b}{i\epsilon_0 S \omega} \frac{1}{-\frac{\omega_{pe}^2}{\omega^2} + 1} \\ Z_{sh} &\approx \frac{1}{i\omega C_{sh}} \approx -i \frac{s_{tot}}{\omega \epsilon_0 S} \\ Z &\approx \frac{i}{\epsilon_0 \omega S} \left(-s_{tot} + \frac{l_b}{\frac{\omega_{pe}^2}{\omega^2} - 1} \right) \end{aligned}$$

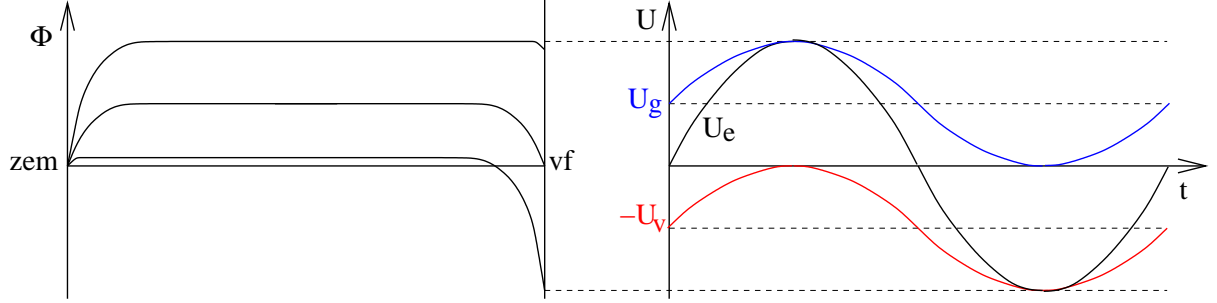
The resonance ($Z \approx 0$) occurs for the frequency

$$\omega_{sr} = \omega_{pe} \sqrt{\frac{s_{tot}}{l}}, \quad (9)$$

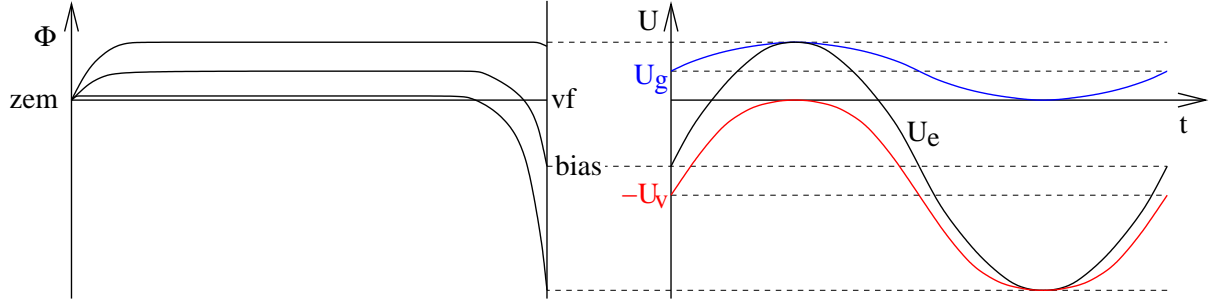
where l_b is the length of the bulk plasma, s_{tot} is the total thickness of the both sheaths and $l = l_b + s_{tot}$ is the distance of electrodes.

5 Discharge asymmetry

Symmetrical discharge:



Asymmetrical discharge:



First approach:

We will use $U_{sh} \propto s^\kappa$ and $U_{sh} \propto \frac{I}{C_{sh}} \propto \frac{I}{S} s$, so that $U_{sh} \propto \frac{U_{sh}^{1/\kappa}}{S}$ and we will obtain

$$U_{sh} \propto \frac{1}{S^{\frac{\kappa}{\kappa-1}}}$$

The index g marks the sheath at the grounded electrode, the index v sheath at the powered electrode and the index e will mark the voltage at the powered electrode. We can write

$$\begin{aligned} \frac{U_v}{U_g} &= \left(\frac{S_g}{S_v} \right)^\alpha \\ \alpha &= \frac{\kappa}{\kappa-1} \in \langle 1; 4 \rangle, \end{aligned} \quad (10)$$

frequently $\alpha \approx 2$ (valid for matrix sheath, $\kappa = 2$). The DC component of the voltage at the powered electrode can be calculated by

$$U_{e0} = U_{g0} - U_{v0} = -U_{e1} \frac{1 - \left(\frac{S_v}{S_g} \right)^\alpha}{1 + \left(\frac{S_v}{S_g} \right)^\alpha}, \quad (11)$$

where the notation $U = U_0 + U_1 \sin \omega t$ was used.

Second approach:

$$E(x) = \frac{e}{\varepsilon_0} \int_0^x n_i(y) dy \quad (12)$$

$$U_{sh} = \frac{e}{\varepsilon_0} \int_0^s dx \int_0^x n_i(y) dy \quad (13)$$

For $n_i = konst.$ we would get

$$U_{sh} = \frac{en_i s^2}{2\varepsilon_0} = \frac{Q^2}{2en_i \varepsilon_0 S^2},$$

where $Q = en_i S s$. We can write the eq. (13) by means of the total sheath charge (Q) even in a more general case. We will define $\xi = x/s$ and we will label the actual averaged ion concentration in a sheath \bar{n}_i :

$$U_{sh} = \frac{Q^2}{2e \bar{n}_i \varepsilon_0 S^2} \mathcal{I} \quad (14)$$

$$\mathcal{I} = 2 \int_0^1 d\xi \int_0^\xi \frac{n_i(\xi')}{\bar{n}_i} d\xi'$$

Because the total charge in both sheaths $Q_M = Q_g(t) + Q_v(t)$ is constant and because each sheath must almost collapse during each RF period, the maximum charge of each sheath is approximately equal to Q_M . we can write

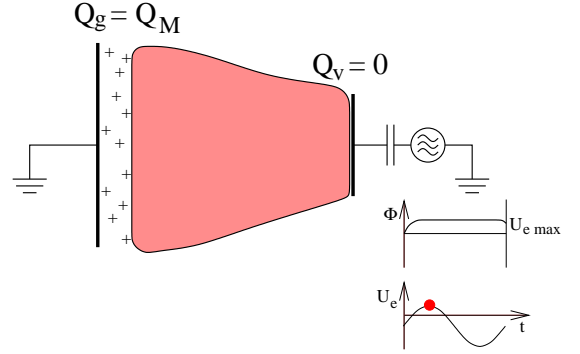
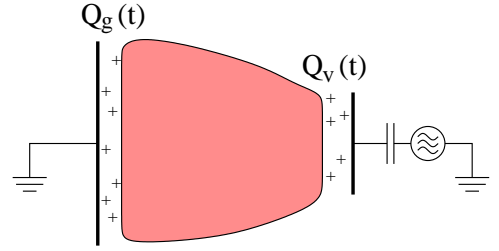
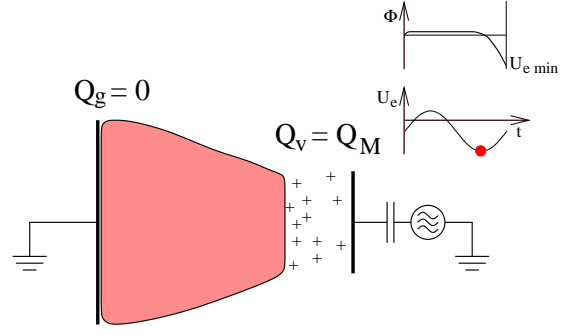
$$U_{e max} = U_{g max} = \frac{Q_M^2}{2e \bar{n}_{ig} \varepsilon_0 S_g^2} \mathcal{I}_g \quad (15)$$

$$U_{e min} = -U_{v max} = -\frac{Q_M^2}{2e \bar{n}_{iv} \varepsilon_0 S_v^2} \mathcal{I}_v \quad (16)$$

and we can define the asymmetry parameter \mathcal{E}

$$\mathcal{E} = \frac{U_{g max}}{U_{v max}} = \left(\frac{S_v}{S_g} \right)^2 \frac{\bar{n}_{iv}}{\bar{n}_{ig}} \frac{\mathcal{I}_g}{\mathcal{I}_v} \quad (17)$$

For the bias calculation we will mark $U_e = U_{e0} + U_{eRF}(t)$, where U_{e0} is the spontaneously generated DC component of the voltage at the powered electrode (bias) and U_{eRF} is the known RF voltage supplied from the RF generator. When we neglect the bulk-plasma voltage, we can write $U_e = U_g - U_v$. We can take advantage of the situation in the two extremes of the supplied



voltage

$$\begin{aligned} U_{gmax} &= U_{e0} + U_{eRFmax} \\ -U_{vmax} &= U_{e0} + U_{eRFmin} \end{aligned}$$

and we can get

$$U_{e0} = -\frac{U_{eRFmax} + \mathcal{E}U_{eRFmin}}{1 + \mathcal{E}} \quad (18)$$

For symmetrical supplied voltage ($U_{eRFmax} = -U_{eRFmin}$, e.g. $U_{eRF} = U_{eRFmax} \sin \omega t$) we will get

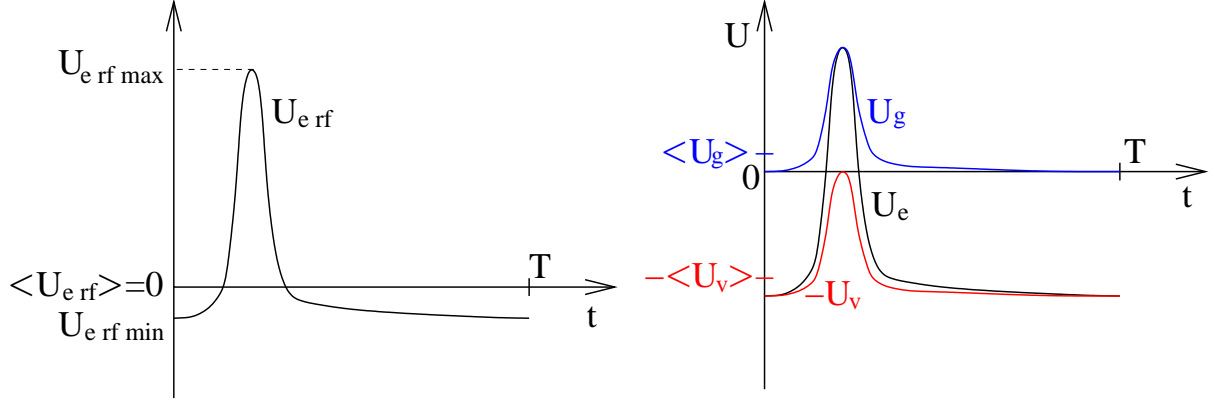
$$U_{e0} = -U_{eRFmax} \frac{1 - \mathcal{E}}{1 + \mathcal{E}} \quad (19)$$

6 Electric asymmetric effect

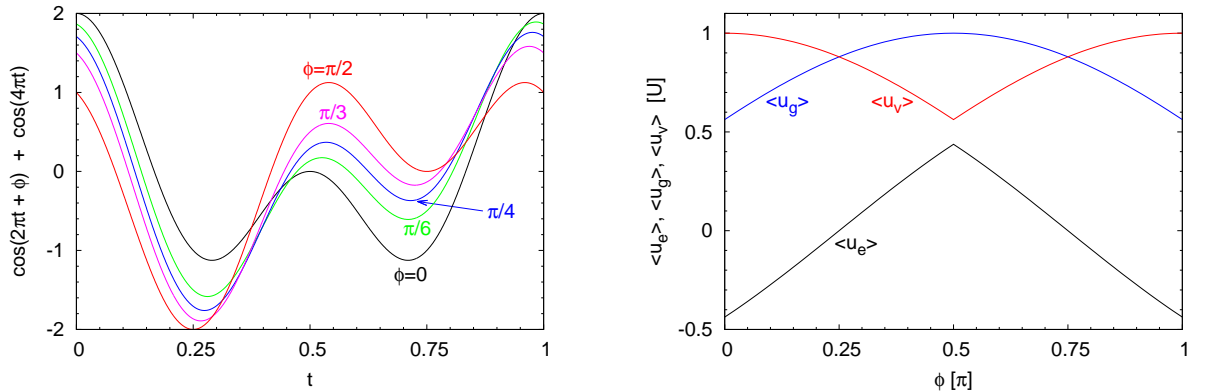
For symmetric discharges the eq. (18) has the following form:

$$U_{e0} = -\frac{U_{eRFmax} + U_{eRFmin}}{2}.$$

Consequently, if we use asymmetric waveform of the supplied voltage, we can generate an electric asymmetry (DC bias) in spite of geometric symmetry of the discharge [2].



Electric asymmetrical effect for a strongly asymmetric supplied voltage.



An example of the electric asymmetric effect for $U_{eRF} = U \cos(\omega t + \Phi) + U \cos(2\omega t)$.

7 Nonlinear nature of sheaths

Literature: [3, 5]

For $n_i = konst.$ we can approximate

$$\begin{aligned} E &= \frac{nes}{\varepsilon_0} \\ U_{sh} &= \frac{nes^2}{2\varepsilon_0} \\ j &= \varepsilon_0 \frac{dE}{dt} = ne \frac{ds}{dt} \end{aligned}$$

The two simplest cases are:

1) **Monofrequency current** $I = I_1 \cos \omega t$:

One sheath:

$$\begin{aligned} s &= \frac{I_1}{Sne\omega} (\sin \omega t + 1) \\ U_{sh} &= \frac{1}{\varepsilon_0 en} \left(\frac{I_1}{S\omega} \right)^2 \left(\frac{3}{4} + \sin \omega t - \frac{1}{4} \cos 2\omega t \right) \end{aligned} \quad (20)$$

Two sheaths:

$$U_e(\omega t) = U_g(\omega t) - U_v(\omega t + \pi) \quad (21)$$

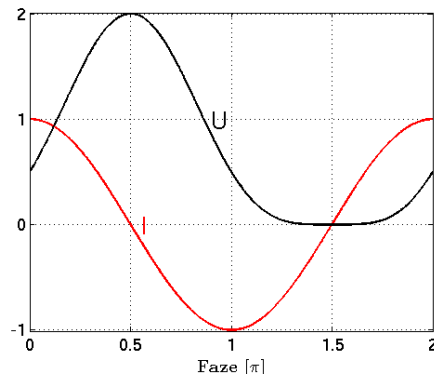
$$U_e = \frac{1}{\varepsilon_0 en} \left(\frac{I_1}{\omega} \right)^2 \left[\frac{3}{4} \left(\frac{1}{S_g^2} - \frac{1}{S_v^2} \right) + \left(\frac{1}{S_g^2} + \frac{1}{S_v^2} \right) \sin \omega t - \frac{1}{4} \left(\frac{1}{S_g^2} - \frac{1}{S_v^2} \right) \cos 2\omega t \right] \quad (22)$$

$$U_{e0} = -\frac{3}{4} U_{e1} \frac{1 - \left(\frac{S_v}{S_g} \right)^2}{1 + \left(\frac{S_v}{S_g} \right)^2} \quad (23)$$

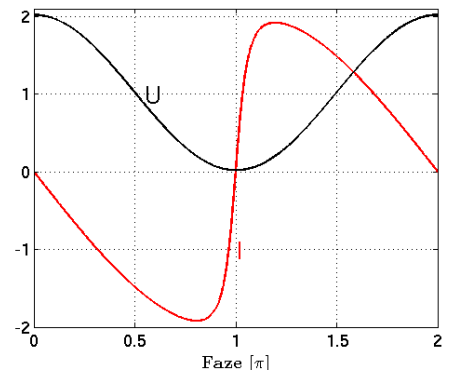
2) **Monofrequency voltage** $U_{sh} = U_0 + U_1 \cos \omega t$:

$$I = Sne \frac{ds}{dt} = S \sqrt{\frac{\varepsilon_0 ne}{2}} \frac{1}{\sqrt{U_{sh}}} \frac{dU_{sh}}{dt} \quad (24)$$

$$I = -S \sqrt{\frac{\varepsilon_0 ne}{2}} \frac{U_1}{\sqrt{U_0}} \frac{\omega \sin \omega t}{\sqrt{1 + \frac{U_1}{U_0} \cos \omega t}} \quad (25)$$



Monofrequency sheath current



Monofrequency sheath voltage

8 Ion energy distribution function (IEDF)

Basic parameters:

- Ratio between the mean transit time of an ion through the sheath and the RF period:

$$\frac{T_i}{T} \approx \frac{3\bar{s}\omega}{2\pi} \sqrt{\frac{m_i}{2eU_0}} \quad (26)$$

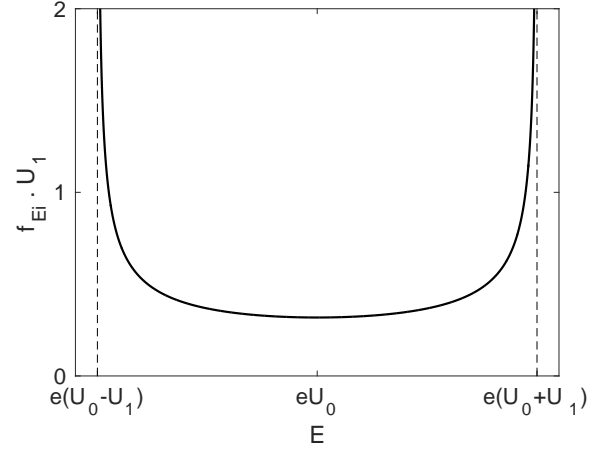
- Number of ion collisions inside the sheath $\approx 1/(\nu_i T_i)$

Literature: [7]

8.1 Collisionless low-frequency regime

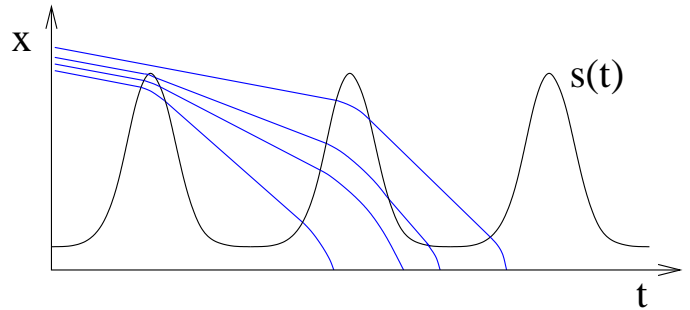
$T_i \ll T$, ion energy corresponds to the actual sheath voltage in the moment of ion impact. Let us assume $U_{sh} = U_0 + U_1 \cos \omega t$.

$$\begin{aligned} E &= e(U_0 + U_1 \cos \omega t) \\ |dt| &= \frac{dE}{\omega e U_1 \sqrt{1 - \left(\frac{E - eU_0}{eU_1}\right)^2}} \\ f_{Ei} &= \frac{1}{\pi e U_1 \sqrt{1 - \left(\frac{E - eU_0}{eU_1}\right)^2}} \quad (27) \\ E &\in \langle e(U_0 - U_1); e(U_0 + U_1) \rangle \end{aligned}$$



8.2 Collisionless middle-frequency regime

The saddle structure narrows with the increase of the T_i/T ratio.



8.3 Collisionless high-frequency regime

$T_i \gg T$, for $\frac{T_i}{T} \rightarrow \infty$ the saddle structure f_E transforms itself to a single peak at the energy $E = eU_0$.

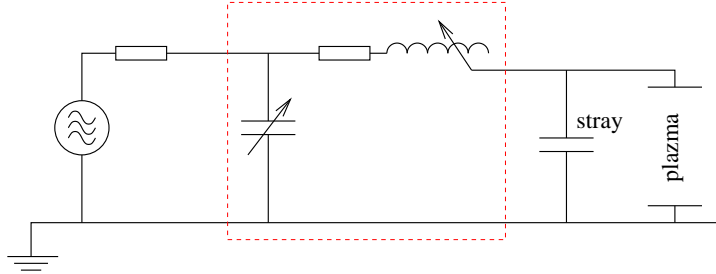
$$\begin{aligned}
 f_{Ei} &= \frac{1}{\pi \Delta E \sqrt{1 - \left(\frac{E - eU_0}{\Delta E}\right)^2}} \\
 \Delta E &= \frac{2eU_1}{\pi} \left(\frac{T}{T_i}\right) \\
 E &\in \langle eU_0 - \Delta E; eU_0 + \Delta E \rangle
 \end{aligned} \tag{28}$$

8.4 Collisions

- elastic collisions – continuous decrease of ion energy
- charge transfer – generation of new peaks in the EEDF (f_{Ei})
- broadening of the ion angle distribution

9 Matching box

For example [1]:



10 Local/non-local plasma character

Local regime:

$$\begin{aligned}
 \vec{j}(\vec{r}, t) &= \sigma(\vec{r}, t) \vec{E}(\vec{r}, t) \\
 f_E(\vec{r}, t) &= f_E[\vec{E}(\vec{r}, t)]
 \end{aligned}$$

Nonlocal regime:

$$\begin{aligned}
 \vec{j}(\vec{r}, t) &= \iiint_{\vec{r}'} d\vec{r}' \int_{t' \leq t} dt' \sigma(\vec{r} - \vec{r}', t - t') \vec{E}(\vec{r}', t') \\
 f_E(\vec{r}, t) &= f_E[\vec{E}(\vec{r}', t')], \quad \vec{r}' \in V, t' \leq t
 \end{aligned}$$

11 Plasma heating

- collisional (Ohmic) heating [9, 1, 6]
- stochastic heating [9, 1, 16]
 - bounce resonance [12]
- field reversal [15]
- γ -heating, α and γ regimes [14, 18, 11, 13]

11.1 γ regime

Potential emission – electron emission from electrode caused by an ion impact, $\sim 0,01$.

Electron avalanche in the sheath:

$$\begin{aligned}
 \frac{dj_e}{dx} &= \alpha [E(x, t)] j_e \\
 j_e(x) &= j_e(0) e^{\int_0^x \alpha[E(x', t)] dx'} \\
 j_e(0) &= \gamma j_i \\
 \langle j_i \rangle &= \left\langle \gamma j_i \left\{ e^{\int_0^{s(t)} \alpha[E(x, t)] dx} - 1 \right\} \right\rangle + env_B
 \end{aligned} \tag{29}$$

For negligible nv_B we can use the last equation for calculation of the ignition voltage for the transition of the sheath from the α to the γ regime. The current density for this transition can be estimated by means of

$$\begin{aligned}
 j &= ne \frac{ds}{dt} \approx \frac{\varepsilon_0}{s} \frac{dU}{dt} \\
 j_1 &\approx \varepsilon_0 \frac{\omega U_1}{s}
 \end{aligned} \tag{30}$$

$\alpha \rightarrow \gamma$ transition:

- increase of electron concentration, plasma conductivity and current density
- decrease of sheath thickness, generation of a discharge structure that is analogous to the structure of the DC glow discharge
- discharge contraction to a smaller area, VA characteristics with a constant discharge voltage
- increase of supplied power
- in some cases an abrupt transition with hysteresis
- EEDF variations (shift to the Maxwell EEDF, increase of electron concentration, decrease of electron temperature)

12 Global models

Input parameters: pressure, electrode distance (l), angular frequency of the electric field (ω), RF current amplitude (I_1), gas composition (K_i , E_i , ν , K_{exc} , E_{exc} , K_{el} , m_n)

Output parameters: electron concentration (n), electron temperature (T_e), mean sheath thickness (s) [1].

- Balance of the number of electrons:

$$n_n \bar{n} K_i (l - s) = 2h_l n_c u_B \quad (31)$$

$$K_i = K_{i0} e^{-\frac{E_i}{kT_e}} \quad (32)$$

$$u_B = \sqrt{\frac{kT_e}{m_i}} \quad (33)$$

(n_n is the concentration of neutrals, \bar{n} is the mean electron concentration in the bulk plasma, K_i is the rate constant for ionization, n_c is the electron concentration in the discharge center, h_l is the ratio between the electron concentration at the bulk-sheath border and in the plasma center, u_B is the Bohm velocity, E_i is the ionization energy of neutrals.)

- Balance of the mean electron energy:

$$\frac{1}{2} (R_{ohm} + 2R_{stoch} + 2R_{ohm,sh}) I_1^2 = 2h_l n_c u_B E_T (T_e) S \quad (34)$$

$$E_T = E_i + \frac{K_{exc}}{K_i} E_{exc} + \frac{3m}{m_n} \frac{K_{el}}{K_i} kT_e + 2kT_e + e\Delta\Phi \quad (35)$$

$$R_{stoch} = 0.72 (mkT_e)^{1/2} \frac{\omega s}{eI_1} \quad (36)$$

$$R_{ohm} = 1.55 hm\nu (l - 2s) \left(\frac{\omega}{eI_1} \right)^{3/2} (S\varepsilon_0 skT_e)^{1/2} \quad (37)$$

$$R_{ohm,sh} = 0.33 m\nu s \frac{\omega s}{eI_1} \quad (38)$$

(R_{ohm} , R_{stoch} and $R_{ohm,sh}$ are resistivities caused by the collisional heating in the bulk plasma, stochastic heating and collisional heating in the sheath, E_T is the mean energy supplied to one electron, K_{exc} is the rate constant for excitation of neutrals, E_{exc} is the excitation energy, K_{el} is the rate constant for elastic collisions between electrons and neutrals, m_n is the mass of neutrals, $\Delta\Phi$ is the average voltage that must be overcome by an electron that leaves plasma.)

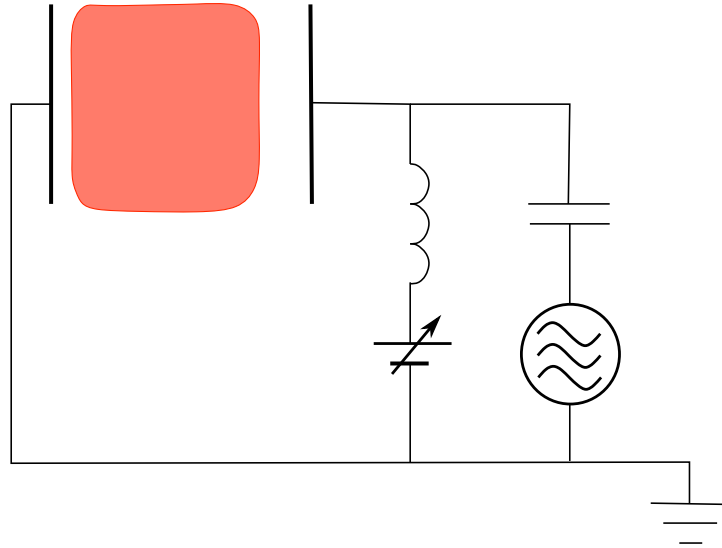
- Sheath thickness:

$$s = \frac{5}{12eh_l^2 n_c^2 \varepsilon_0 kT_e} \left(\frac{I_1}{S\omega} \right)^3 \quad (39)$$

The equations listed above are valid for low-pressure plasma with no negative ions and with no collisions in the sheaths.

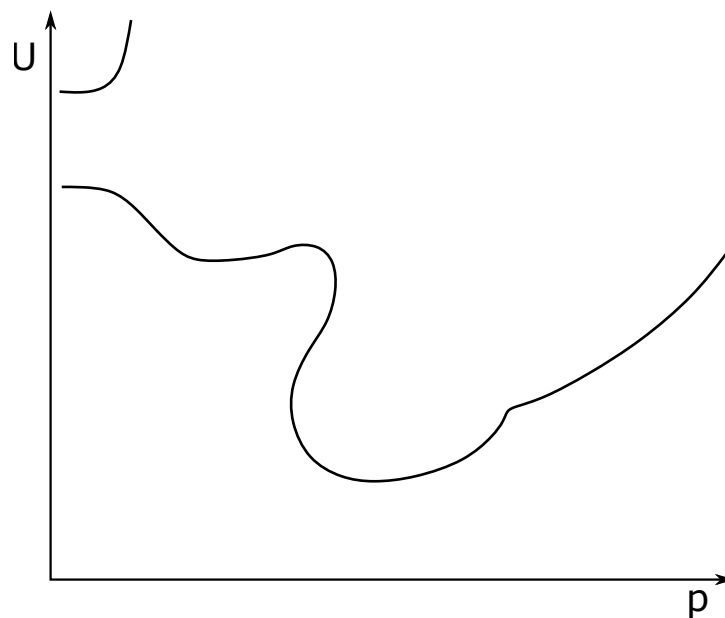
13 Independent control of the reactive species concentration and ion energy

- DC + RF [19, 10]



- Capacitive biasing of an electrode in a different discharge (ICP, MW)
- Double-frequency CCP [1, 4]
- Electric asymmetric effect [2, 4, 17]

14 CCP ignition



Breakdown voltage in the high-pressure branch:

$$\begin{aligned} \frac{dn}{dt} &= \nu_i n + D \frac{d^2 n}{dx^2} \\ \frac{dn}{dt} &\geq 0 \\ n &\approx n_{\text{centr}} \sin \left(\sqrt{\frac{\nu_i}{D}} x \right) \\ \nu_i &= D \left(\frac{\pi}{l} \right)^2 \\ \nu_i &= K_1 p e^{-K_2 \frac{lp}{U}} \\ U &= \frac{K_2 pl}{\ln \left(\frac{pl}{K_1 \pi^2} \frac{l}{D} \right)} \end{aligned}$$

Used marking of quantities

ω_{pi}	plasma frequency of ions
ω	(angular) frequency of el. field
ω_{pe}	plasma frequency of electrons
l	electrode separation
λ	wavelength
σ	plasma conductivity
n	electron concentration
e	elementary charge
m	electron mass
ν	mean collisional frequency of transition of momentum of the electron (to neutrals)
ε_0	vacuum permittivity
p	power density
j	current density
j_1	current density amplitude
E	electric field intensity energy
E_1	amplitude of electric field intensity
n_0	electron concentration at the plasma-sheath border
n_i	ion concentration
k	Boltzmann constant
T_e	electron temperature
U_{sh}	sheath voltage
m_i	ion mass
s	sheath thickness
U_0	DC voltage component
U_1	amplitude of the fundamental voltage component
I_0	modified Bessel function of the 1. kind, order 0
Z_b	bulk plasma impedance
l_b	bulk plasma length
S	electrode area
Z_{sh}	sheath impedance
C_{sh}	sheath(s) capacity
s_{tot}	total thickness of both sheaths
Z	discharge impedance
ω_{sr}	(angular) frequency of the plasma-sheath resonance
U_g	sheath voltage (at the grounded electrode)
U_v	sheath voltage (at the powered electrode)
U_e	powered electrode voltage
κ	power in the dependence of the sheath voltage on the sheath thickness
α	phase difference between current and voltage power in the dependence of the electric and geometric discharge asymmetry

	designation of the CCP regime with negligible role of γ -electrons
	1. Townsend coefficient
S_g	grounded electrode area
S_v	powered electrode area
Q	el. charge
Q_g	sheath charge at the grounded electrode
Q_v	sheath charge at the powered electrode
Q_M	total charge in both sheaths
Φ	el. potential
$U_{x\max}$	maximum value of the voltage U_x
$U_{x\min}$	minimum value of the voltage U_x
U_{eRF}	RF component of the powered electrode voltage (i.e. $U_e - U_{e0}$)
\mathcal{I}	quantity that describes profile of ion concentration inside a sheath
\mathcal{E}	discharge asymmetry parameter
I	electric current
T_i	mean transit time of an ion through a sheath
T	discharge period
ν_i	mean collisional frequency of an ion
f_{Ei}	IEDF
f_E	EEDF (electron energy distribution function)
ν_i	ionization frequency
D	coefficient of diffusion

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