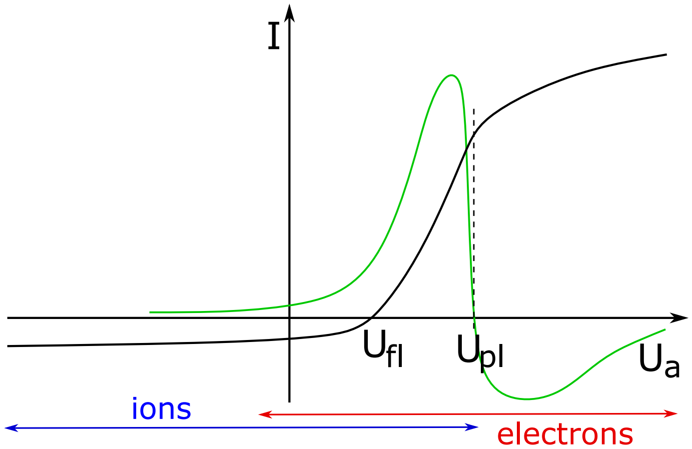

Figure 1: *VA characteristics of a cylindrical Langmuir probe.*

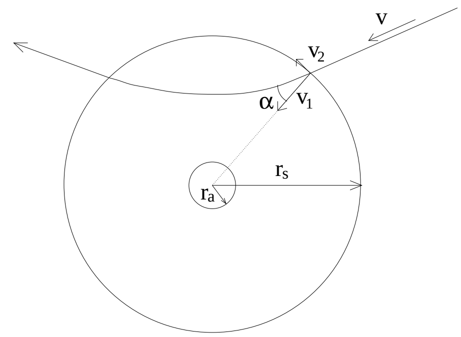

Figure 2: *Trajectory of a charged particle in a sheath around a probe.*

1 Langmuir probe

Langmuir probe is a small conductor inserted into plasma, whose VA characteristics can be used for determination of some plasma parameters, e.g. concentration and temperature of electrons, plasma potential and electron energy distribution function. We will summarize here the results of the probe theory, which calculates probe current by study of trajectories of charged particles in a collisionless sheath around the probe, determination of the part of the velocity-space that contains particles that will reach the probe and calculation of the probe current by integration of radial velocity component and velocity distribution function over this part of the velocity-space, i.e.

$$I = qS_s \iiint\limits_C v_1 \, g(\vec{v}) \, \mathrm{d}^3\vec{v}, \tag{1}$$

where q is the charge of a particle, S_s is the area of the plasma-sheath boundary, g is the velocity distribution function. The following text will show formulas obtained for a cylindrical probe.

1.1 Particles repelled from the probe

(i.e. electrons for $U_a < U_{pl}$ and positive ions for $U_a > U_{pl}$) Particles can reach the probe surface only if their kinetic energy is sufficiently high

$$\frac{1}{2}mv^2 \geq q(U_a - U_{pl})$$

and if they enter the sheath with sufficiently small angle α

$$\sin^2\alpha \leq \frac{r_a^2}{r_s^2}\left[1 - \frac{2q\,(U_a - U_{pl})}{mv^2}\right].$$

By integration of the flux of repelled particles over this part of the velocity space we obtain the VA characteristics

$$I = qS\frac{1}{2\sqrt{2m}}\int\limits_{-qU}^{\infty}\frac{E+qU}{\sqrt{E}}\,f(E)\,\mathrm{d}E, \tag{2}$$

where $U = U_{pl} - U_a$, E is the kinetic energy, $f(E)$ is the energy distribution function and S is the surface area of the probe. This formula is valid also for planar and spherical probes. If the studied species have the Maxwell energy distribution function, the formula (2) can be simplified to

$$\begin{aligned} I &= qS\,\frac{1}{4}n\bar{v}\,\exp\left(\frac{qU}{kT}\right) \\ \bar{v} &= \sqrt{\frac{8kT}{\pi m}} \end{aligned} \tag{3}$$

The distribution function can be determined from (2) by means of the so-called Druyvesteyn formula

$$f(qU) = \frac{2\sqrt{2m|U|}}{q^{5/2}S}\,\frac{\mathrm{d}^2 I}{\mathrm{d}|U|^2} \tag{4}$$

1.2 Particles attracted to the probe

The calculation of electric current carried by species attracted to the probe is more complicated and in general depends on the dimensions of the sheath around the probe. If the species have Maxwell EDF, the current flowing to the cylindrical probe can be calculated as

$$I = \frac{1}{4}Sqn\bar{v}\left\{\frac{r_s}{r_a}\sqrt{\mathrm{erf}\left(\frac{r_a^2}{r_s^2 - r_a^2}\frac{qU}{kT}\right)} + \left[1 - \sqrt{\mathrm{erf}\left(\frac{r_s^2}{r_s^2 - r_a^2}\frac{qU}{kT}\right)}\right]\exp\left(\frac{qU}{kT}\right)\right\}, \tag{5}$$

where

$$\mathrm{erf}\,x = \frac{2}{\sqrt{\pi}}\int\limits_0^x e^{-t^2}\mathrm{d}t,$$

which can be for $r_s \gg r_a$, (so-called OML, orbital motion limited theory) simplified to

$$I \approx \frac{1}{4}Sqn\bar{v}\sqrt{1 + \frac{qU}{kT}}. \tag{6}$$

In practice, thanks to the OML theory the electron current for $U_a \gg U_{pl}$ can be described by

$$I_e = \frac{1}{4}Sqn\bar{v}\sqrt{1 + \frac{e\left(U_a - U_{pl}\right)}{kT_e}} \tag{7}$$

and the positive ion current for $U_a \ll U_{fl}$ by

$$I_i = I_{i0}\left[1 + \frac{e\left(U_{pl} - U_a\right)}{kT_e}\right]^{\kappa}. \tag{8}$$

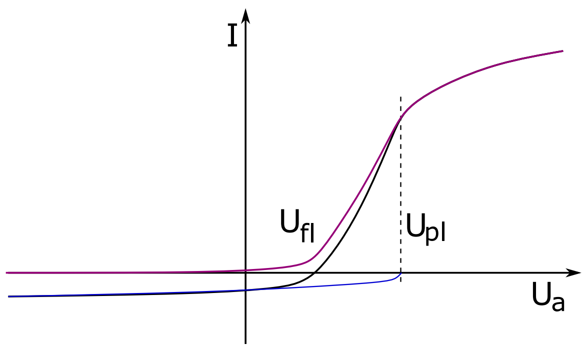

Figure 3: *VA characteristics of an (unheated) Langmuir probe.*

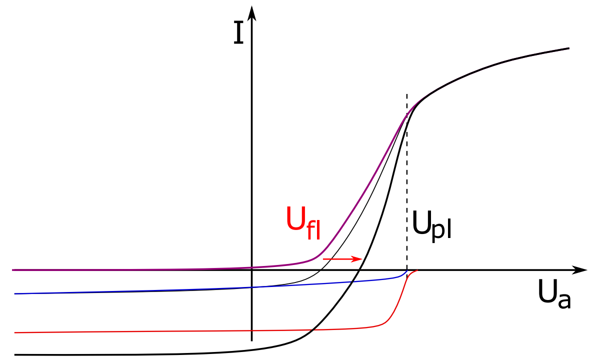

Figure 4: *VA characteristics of an emissive probe.*

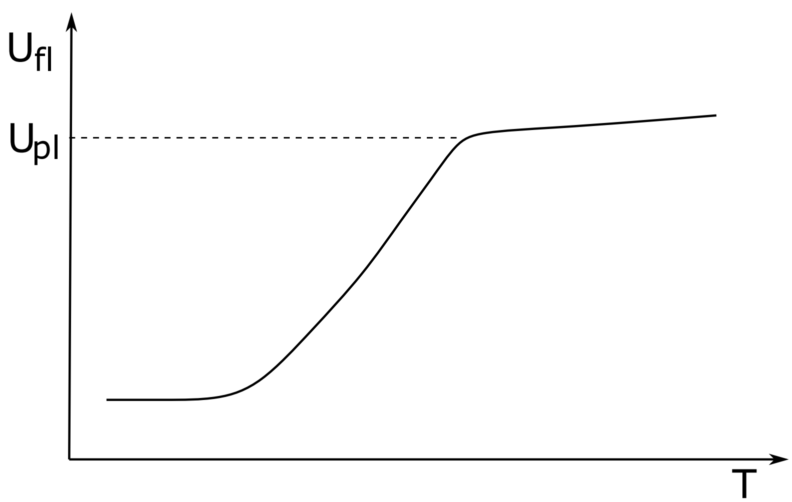

Figure 5: *Dependence of the floating potential of an emissive probe on the probe temperature.*

2 Emissive probe

Heating of probe results in thermoemission of electrons, which shifts the floating potential to more positive values. The floating potential will grow when probe temperature will be increased. But, because emitted electrons can not escape from probe to plasma for $U_a > U_{pl}$, the floating potential increase will stop at the value of plasma potential.

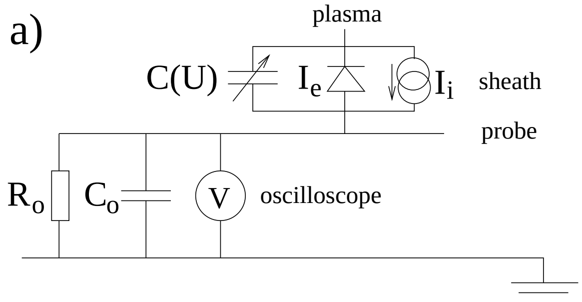

Figure 6: *Electric scheme of the uncompensated probe circuit: The real situation (a) and the approximation of sheath by a capacitor (b).*

3 Measurement of RF components

Langmuir probes usually contain RF compensation, which blocks RF probe currents in order to prevent the RF components from distortion of the probe VA characteristics. Unfortunately, the compensation usually disables measurement of RF components of plasma potential. Consequently, the RF components can be measured by means of probes without any RF compensation, which can be made as a simple wire connected to a high-impedance probe of an oscilloscope.

However, the measured probe potential waveform differs from the plasma potential waveform, because the probe is separated from the plasma by a sheath. Therefore, it is necessary to determine the waveform of the voltage on the sheath around the probe (U) from the measured waveform of the probe current (I). The electric current flowing to the probe can be described as a sum of the displacement (I_d), electron (I_e) and ion (I_i) current

$$I = I_\mathrm{d} + I_\mathrm{e} + I_\mathrm{i} \tag{9}$$

$$I_d = S\varepsilon_0 \frac{\mathrm{d}E}{\mathrm{d}t} \tag{10}$$

$$I_\mathrm{e} = -\frac{enS}{4}\sqrt{\frac{8kT_\mathrm{e}}{\pi m_\mathrm{e}}}\,\exp\left\{-\frac{eU}{kT_\mathrm{e}}\right\}, \tag{11}$$

where S denotes the surface area of the probe, ε_0 the vacuum permitivity, E the electric field intensity at the probe surface, e the elementary charge, n the electron concentration in the bulk plasma, k the Boltzmann constant, T_e the electron temperature and m_e the electron mass. The ion current can be assumed to be constant during the whole discharge period, since the ion plasma frequency is usually significantly smaller than the frequency of the applied electric field.

The total probe current (I) can be easily obtained from the measured probe voltage waveform and from the known input impedance of the oscilloscope. When the values of ion current (I_i), sheath voltage in one moment of the RF period ($U(t = 0)$), electron concentration (n) and electron temperature (T_e) are known and when the relation between electric field intensity (E) and sheath voltage (U) is defined (see fig. 7), it is possible to use the equations (9) – (11) for numerical calculation of the temporal evolution of the sheath voltage: From known values of I, I_i and I_e in the initial ($t = 0$) moment, the actual displacement current and the actual temporal

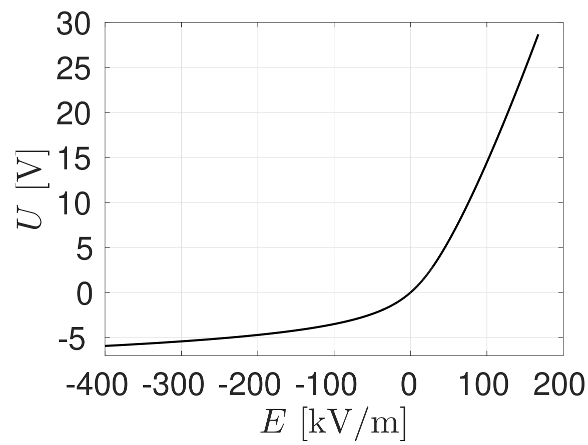

Figure 7: *An example of the relation between electric field intensity at the probe surface and sheath voltage.*

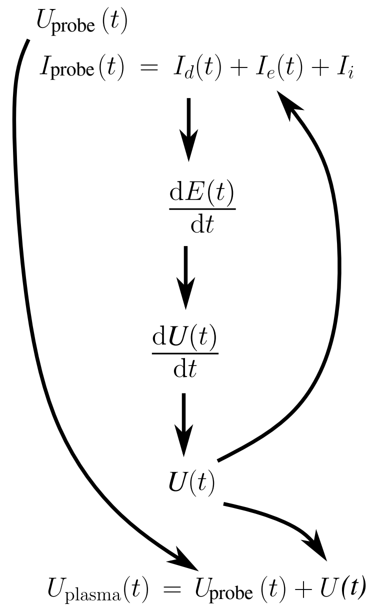

Figure 8: *Scheme of the numerical calculation of the plasma potential.*

derivative of electric field intensity are calculated. The temporal derivative of electric field intensity is converted to the temporal derivative of the sheath voltage. This derivative is used for the determination of the sheath voltage in the forthcoming moment. Now it is possible to calculate the value of electron current in this second moment and the described procedure can be repeated until the whole period of the sheath voltage is acquired, see scheme 8. After that, it is possible to find the plasma potential waveform as the sum of the probe voltage measured by the oscilloscope and the sheath voltage

$$U_{\mathrm{pl}} = U_a + U. \tag{12}$$

The values of ion current and initial sheath voltage can be found by means of two requirements: First is the periodicity of the sheath voltage. The second claims that the mean value of plasma potential waveform calculated by equations (9) − (11) must be equal to the value measured by compensated Langmuir probe.

In some cases, the sheath around the probe can be modeled as a capacitor. In this approximation, the electric circuit drawn in the fig. 6b is solved and each frequency component of plasma potential (u_{pl}) is calculated according to

$$u_{\mathrm{pl}} = u_a \left(1 + \frac{C_{\mathrm{o}}}{C} - \frac{\mathrm{i}}{\omega C R_{\mathrm{o}}}\right), \tag{13}$$

where u_a are the frequency components of the probe voltage measured by the oscilloscope, C_{o} and R_{o} are the capacity and resistance of the probe to ground, respectively, and C is the

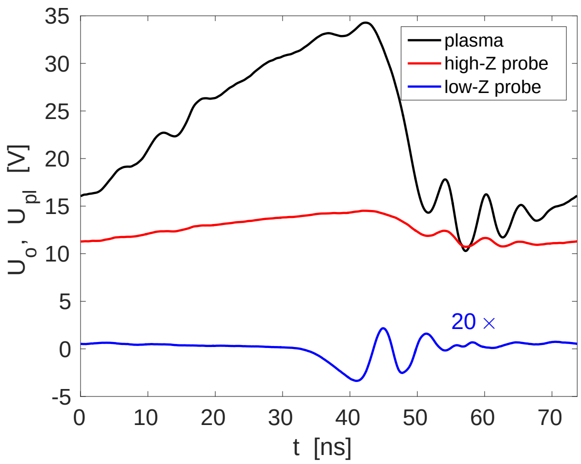

Figure 9: *Waveforms of plasma (black), high-impedance probe (red) and low-impedance probe (blue) potentials measured in a capacitively coupled discharge ignited in argon at 6 Pa.*

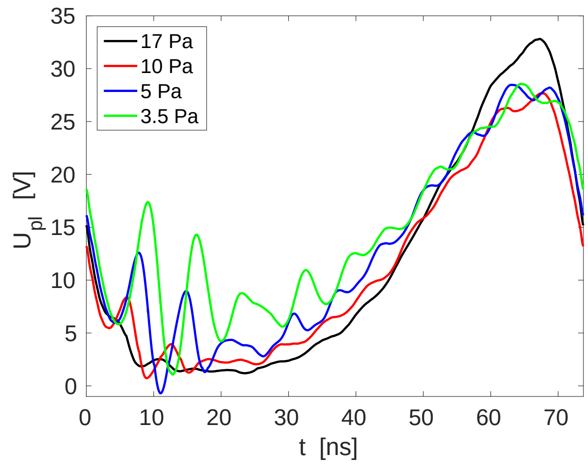

Figure 10: *Plasma potential waveforms measured in a nitrogen capacitively coupled discharge at various pressures.*

capacity of the sheath around the probe. In this laboratory work, the values $C_o = 3.9\,\mathrm{pF}$, $R = 10^8\,\Omega$ and $C = 0.65\,\mathrm{pF}$ can be used.