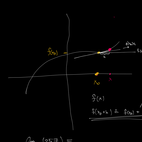
0) diferenciál



$\Delta y = f(x_0+\Delta x) - f(x_0) \approx f'(x_0)\cdot\Delta x$

$dy = f'(x_0)\cdot dx$

$dy = f'(x)\,dx$

$\Delta x \to 0$

$\Delta y \approx dy$

$\Delta y = dy + \varepsilon$

$\varepsilon \to 0$

$(\log_a(x+1))' = \left(\frac{\ln(x+1)}{\ln a}\right)' = \frac{1}{\ln a}\cdot\frac{1}{x+1}\cdot 1 = \frac{1}{(x+1)\ln a}$

$\log_a x = \frac{\ln x}{\ln a}$

$\log_a x = y$

$a^y = x$

$\ln a^y = \ln x$

$y\ln a = \ln x$

$y = \frac{\ln x}{\ln a}$

$(\ln|x|)' = \left(\frac{x}{|x|}\cdot\ln x\right)' = \frac{1}{x}$

$\frac{x}{|x|} = \pm 1$

1 Primitív függvény

A) $F(x) \xrightarrow{\text{deriválás}} f(x)$ deriváltfüggvény

$f(x) \xrightarrow{\text{integrálás}} F(x)$ primitívfüggvény

$F(x)$ az $f(x)$ egy primitívfüggvénye $\Leftrightarrow F'(x) = f(x)$

+ határ: vagy a feladatból

$S(x) = x^2$

$(x^2)' = 2x$

$\left(\frac{x^2}{2}\right)' = x$

$\left(\frac{x^2}{2}+17\right)' = x$

$\int x^n\,dx = \frac{x^{n+1}}{n+1} + C$

$(\sin x)' = \cos x$

$\int f(x)\,dx$

határozatlan integrál vagy függvény

$\int \cos x\,dx = \sin x + C \quad C \in \mathbb{R}$

$(\sin x + c)' = \cos x$

$\left(\frac{x^{n+1}}{n+1}\right)' = x^n$

ha $n \neq -1$

$\int \frac{1}{x}\,dx = \ln|x| + C$

$\ln|x|$

$\ln x$

$\ln x$ $\Rightarrow$ $x > 0$

$\ln|x|$ $\Rightarrow$ $x \neq 0$

B) $\int \frac{1}{5x^2}\,dx =$

$\frac{1}{5}\int \frac{1}{x^2}\,dx = \frac{1}{5}\int x^{-2}\,dx = \frac{1}{5}\cdot\frac{x^{-1}}{-1} + C$

$= -\frac{1}{5x} + C$

C) $\int \frac{1}{\ln 3}\,dx = \frac{1}{\ln 3}\ln|x| + C$

$\left(\frac{1}{\ln 3}\ln|x| + c\right)' = \frac{1}{\ln 3}\cdot\frac{1}{x} \checkmark$

D) $\int \frac{1}{2x-5}\,dx = \int \frac{1}{2}\left(\frac{1}{x-\frac{5}{2}}\right)dx = \frac{1}{2}\int \frac{1}{x-\frac{5}{2}}$

$= \frac{1}{2}\ln\left|x-\frac{5}{2}\right| + C$

$\left(\ln|2x-5|\right)' = \frac{1}{2x-5}\cdot 2$

$\left(\frac{1}{2}\ln|2x-5|\right)' = \frac{2}{2(2x-5)}$

E) $\int \tan^2 x\,dx = \int\left(\frac{1}{\cos^2 x} - 1\right)dx = \int \frac{1}{\cos^2 x}\,dx - \int 1\,dx = \tan x + C_1 - (x + C_2)$

$= \tan x - x + C$

$\tan x = \left(\frac{\sin x}{\cos x}\right)^2$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

$\tan^2 x = \frac{1-\cos^2 x}{\cos^2 x}$

$= \frac{1}{\cos^2 x} - 1$

F) $\int \frac{1}{5x^2+1}\,dx = \int \frac{1}{(\sqrt{5}x)^2+1}\,dx = \operatorname{arctg}\sqrt{5}x \cdot \frac{1}{\sqrt{5}} + C$

$y^2 = 5x^2$

$\frac{1}{y^2+1}$

$(\operatorname{arctg}\sqrt{5}x)' = \left(\frac{1}{5x^2+1}\right)\cdot\sqrt{5}$

$\left(\frac{1}{\sqrt{5}}\operatorname{arctg}\sqrt{5}x\right)' = \frac{1}{\sqrt{5}}\left(\frac{1}{5x^2+1}\right)\cdot\sqrt{5} = \frac{1}{5x^2+1}$

2) Per partes

$\int (u(x)\cdot v(x))' = \int u(x)\,v(x) + \int u(x)\,v'(x)$

$u(x)\cdot v(x)$

$\int u'(x)\,v(x)\,dx = u(x)\,v(x) - \int u(x)\,v'(x)\,dx$

A) $\int x\cdot\sin x\,dx \quad \left|\begin{matrix} u' = \sin x & u = -\cos x \\ v = x & v' = 1 \end{matrix}\right| = -\cos x\cdot x - \int -\cos x\cdot 1\,dx$

$(-\cos x)' = -(-\sin x) = \sin x$

$= -\cos x\cdot x + \int \cos x\,dx = -\cos x\cdot x + \sin x + C$

C) $\int x\cdot\operatorname{arctan} x\,dx \quad \left|\begin{matrix} u' = x & u = \frac{x^2}{2} \\ v = \operatorname{arctan} x & v' = \frac{1}{1+x^2} \end{matrix}\right|$

$= \frac{x^2}{2}\operatorname{arctan} x - \int \frac{x^2}{2}\cdot\frac{1}{1+x^2}\,dx = \frac{x^2}{2}\operatorname{arctan} x - \frac{1}{2}\int 1 + \frac{-1}{1+x^2}\,dx = \frac{x^2}{2}\operatorname{arctan} x - \frac{1}{2}\int 1\,dx + \frac{1}{2}\int \frac{1}{1+x^2}\,dx$

$\frac{x^2+1-1}{1+x^2} = 1 + \frac{-1}{1+x^2}$

$= \frac{x^2}{2}\operatorname{arctan} x - \frac{x}{2} + \frac{1}{2}\operatorname{arctan} x + C$

$x^2 : (1+x^2) = 1 - \frac{1}{1+x^2}$

$-1+x^2$

-1

B) $\int \operatorname{arccos} x\,dx = \int 1\cdot\operatorname{arccos} x\,dx \quad \left|\begin{matrix} u' = 1 & u = x \\ v = \operatorname{arccos} x & v' = \frac{1}{\sqrt{1-x^2}} \end{matrix}\right| =$

$= x\operatorname{arccos} x - \int \frac{x}{\sqrt{1-x^2}}\,dx = x\operatorname{arccos} x - \left(-\sqrt{1-x^2}\right) + C$

$= x\operatorname{arccos} x + \sqrt{1-x^2} + C$

3) SUBSTITUCIÓ

A) $\int \frac{x}{\sqrt{1-x^2}}\,dx = \int \frac{x}{\sqrt{y}}\cdot\frac{dy}{-2x} = -\frac{1}{2}\int \frac{1}{\sqrt{y}}\,dy$

① $y = 1-x^2$

② $\frac{dy}{dx} = -2x$

③ $dx = \frac{dy}{-2x}$

$= -\frac{1}{2}\cdot\frac{y^{\frac{1}{2}}}{\frac{1}{2}} + C$

$= -y^{\frac{1}{2}} + C$

$= -\sqrt{1-x^2} + C$

Substitució gyorsabban

$\int \frac{1}{x+3}\,dx = \int \frac{1}{y}\,dy = \ln|y|$

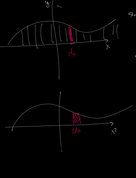
$\ln|x+3|$

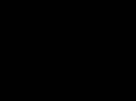
$y = x+3$

$\int 1\,dy = \int 1\,dx$

$1\,dy = 1\,dx$

$\int \frac{1}{y+3}\,dy$





② $\int \frac{1}{5x^2+1}\,dx = \int \frac{1}{y+1}\cdot\frac{dy}{5x} = \frac{1}{5}\int \frac{1}{y+1} = \frac{1}{5}\cdot\operatorname{arctan}(y) + C$

$y = 5x^2$

$y = \sqrt{5}x^2 = \sqrt{5}\cdot x$

$\int 1\,dy = \int \sqrt{5}\,dx$

$1\,dy = \sqrt{5}\cdot 1\,dx$

$dx = \frac{dy}{\sqrt{5}}$

$= \frac{1}{\sqrt{5}}\operatorname{arctan}(\sqrt{5}\cdot x) + C$

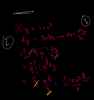
B) $\int \frac{x^2}{(8x^3+27)^{\frac{3}{2}}}\,dx = \int \frac{x^2}{(y)^{\frac{3}{2}}}\cdot\frac{dy}{24x^2} = \frac{1}{24}\int y^{-\frac{3}{2}}\,dy$

① $y = 8x^3+27$

② $dy = 24x^2\,dx$

③ $dx = \frac{dy}{24x^2}$

$= \frac{1}{24}\cdot\frac{y^{-\frac{1}{2}}}{-\frac{1}{2}} + C$

$= -\frac{1}{12}(8x^3+27)^{-\frac{1}{2}} + C$

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