$x^\alpha \checkmark \quad \frac{1}{x} \checkmark \quad e^x \checkmark$

$\cos x \quad \sin x$

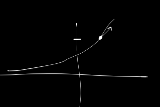
$\int f(x)\,dx = \{F(x) \mid F'(x) = f(x)\}$

$\int x^7\,dx = \frac{x^8}{8} + C \qquad \int x^\alpha\,dx \overset{\alpha \neq -1}{=} \frac{x^{\alpha+1}}{\alpha+1} + C$

$\int \frac{1}{x}\,dx = \ln|x| + C$

x^{-1}

$\int e^x\,dx = e^x + C$



$(e^x)' = e^x$

$\int \cos x\,dx = \sin x + C$

$\int \sin x\,dx = -\cos x + C$

①

$\int x^n e^x\,dx \qquad \int x^4 e^x\,dx = \dots$

$\hookrightarrow \int u'(x)\,v(x)\,dx = u(x)\,v(x) - \int u(x)\,v'(x)\,dx$

$\underset{C^n_{(x)}}{\int x^n e^x\,dx} \left| \begin{matrix} u' = e^x & u = e^x \\ v = x^n & v' = n \cdot x^{n-1} \end{matrix} \right| =$

$= e^x x^n - n \cdot \int \underline{e^x x^{n-1}\,dx}$

$C^n(x) = e^x x^n - n \cdot C^{n-1}_{(x)}$

$C^4(x) = e^x x^4 - 4 \cdot C^3(x)$

$C^3(x) = e^x x^3 - 3C^2(x) \qquad C^0(x) = \int x^0 e^x\,dx = e^x$

$C^2(x) = e^x x^2 - 2C^1(x)$

$C^1(x) = e^x \cdot x - 1C^0(x)$

$C^4(x) = e^x x^4 - 4(e^x x^3 - 3(e^x x^2 - 2(e^x \cdot x - e^x)))$

$\int \underset{u}{e^x} \underset{v'}{\cos x}\,dx \left| \begin{matrix} u = e^x & u' = e^x \\ v = \sin x & v' = \cos x \end{matrix} \right| = e^x \cdot \sin x - \int e^x \cdot \sin x\,dx$

$\left| \begin{matrix} u = e^x & u' = e^x \\ v = \cos x & v' = \sin x \end{matrix} \right.$

$= e^x \cdot \sin x - \left(-e^x \cos x - \int -e^x \cos x\right)$

$\int e^x \sin x\,dx = e^x \cdot \cancel{\sin x}(-\cos x) - \int e^x \cdot (-\cos x)\,dx =$

$\left| \begin{matrix} u = e^x & u' = e^x \\ v = \cos x & v' = \sin x \end{matrix} \right.$

$$\boxed{\underbrace{\int e^x \cos x\,dx}_{C(x)} = e^x \sin x + e^x \cos x - \underset{C(x)}{\int e^x \cdot (\cos x)\,dx}}$$

$2\int e^x \cos x\,dx = e^x \sin x + e^x \cos x$

$\int e^x \cos x\,dx = \frac{e^x \sin x + e^x \cos x}{2}$

$\int \sqrt{x} \ln^2 x\,dx = \left| \begin{matrix} u = \ln^2 x & u' = 2 \cdot \ln(x) \cdot \frac{1}{x} \\ v' = \sqrt{x} = x^{\frac{1}{2}} & v = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \end{matrix} \right| \ln^2 \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \int 2 \cdot \ln(x) \cdot \frac{1}{x} \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} =$

$= \ln^2 x \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - 2 \cdot \frac{2}{3} \int \ln(x)\, x^{\frac{1}{2}} \left| \begin{matrix} u = \ln(x) & u' = \frac{1}{x} \\ v' = x^{\frac{1}{2}} & v = \frac{x^{\frac{3}{2}}}{\frac{3}{2}} \end{matrix} \right| \qquad \to \frac{3}{2} = \frac{1}{2}$

$= \ln^2 x \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{4}{3}\left(\ln x \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \int \frac{1}{x} \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) = \underline{\ln^2 x \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{4}{3}\left(\ln x \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}} - \frac{2}{3} \cdot \frac{x^{\frac{3}{2}}}{\frac{3}{2}}\right) + C}$

$\int \frac{x}{\sqrt{\underset{t}{1 - x^2}}}\,dx = \int \frac{\cancel{x}}{\sqrt{t}} \cdot \frac{dt}{-2\cancel{x}} =$

① $t = 1 - x^2$

② $dt = -2x\,dx$

③ $dx = -\frac{dt}{2x}$

$= \int \frac{1}{\sqrt{t}} \cdot -\frac{dt}{2} =$

$= -\frac{1}{2} \int \frac{1}{\sqrt{t}}\,dt =$

$= -\frac{1}{\cancel{2}} \cdot \frac{t^{\frac{1}{2}}}{\cancel{\frac{1}{2}}} = -t^{\frac{1}{2}} = \underline{\underline{-\sqrt{1 - x^2} + C}}$

b) $\int \frac{dx}{(1+x)\sqrt{x}}$

$t = \sqrt{x}$

c) $\int \frac{dx}{e^x + e^{-x}}$

d) $\int \frac{\ln^2 x}{x}\,dx \qquad \underline{t = \ln x}$

e) $\int \sin^5 x \cos x\,dx$

$t = \sin x$

$\int \frac{dx}{(1+x)\sqrt{x}} = \int \frac{2\cancel{t}\,dt}{(1+t^2)\,\cancel{t}} = 2 \cdot \int \frac{1}{1+t^2}\,dt = 2 \cdot \operatorname{arctg}(t) + C$

$= 2 \operatorname{arctg}(\sqrt{x}) + C$

$t = \sqrt{x} \qquad t^2 = x$

$2t\,dt = dx$

c) $\int \frac{dx}{e^x + e^{-x}} = \int \frac{dx}{e^x + \frac{1}{e^x}} = \int \frac{1}{t + \frac{1}{t}} \frac{dt}{t} = \int \frac{1}{t^2 + 1}\,dt$

1) $t = e^x$

2) $1 \cdot dt = e^x \cdot dx$

3) $dx = \frac{dt}{e^x} = \frac{dt}{t}$

$= \operatorname{arctg}(t)\,dt + C$

$= \operatorname{arctg}(e^x) + C \quad \checkmark$

$\int \frac{P(x)}{Q(x)} \quad \leftarrow$ polynomy

a) $\int \frac{2x+3}{(x-2)(x+5)}\,dx$

① je mocnina nahoře < mocnina dole? ✓

② parciální zlomky

$\frac{2x+3}{(x-2)(x+5)} = \frac{\overset{1}{A}}{(x-2)} + \frac{\overset{1}{B}}{(x+5)} \quad \Big| \, (x-2)(x-5)$

$2x + 3 = A(x+5) + B(x-2)$

L:

$x \quad 2 = A + B$

$1 \quad 3 = 5A - 2B$

$(2) + 2 \cdot (2) \qquad 3 + 2 \cdot 2 = 5A - 2B + 2A + 2B$

$7 = 7A$

$A = 1$

$z\,(1): \quad B = 1$

$2x + 3 = A(x+5) + B(x-2)$

dosaď $x = 2$

$4 + 3 = A(2+5) + 0$

$7 = 7A$

$A = 1$

dosaď $x = -5$

$-10 + 3 = 0 + B(-5-2)$

$-7 = -7B$

$B = 1$

$\int \frac{2x+3}{(x-2)(x+5)}\,dx = \int \left(\frac{1}{x-2} + \frac{1}{x+5}\right) dx$

$= \int \frac{1}{x-2}\,dx + \int \frac{1}{x+5}\,dx = \ln|x-2| + \ln|x+5| + C$

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c) $\int \frac{x}{x^3 - 3x + 2}\,dx$

Čern

Hrubý

① mocnina nahoře < moc dole ✓

② $\frac{x}{x^3 - 3x + 2} = \frac{A}{(x-1)} + \frac{B}{(x-1)^2} + \frac{C}{(x+2)}$

$x^3 - 3x + 2 = (x+2)(x^2 - 2x + 1) = (x+2)(x-1)^2$

$$\begin{array}{r|rrrr} & 1 & 0 & -3 & 2 \\ \hline -2 & 1 & -2 & 1 & 0 \\ 1 & 1 & -1 & 0 & \end{array}$$

$x^2 - 2x + 1 =$

$x = A(x-1)(x+2) + B(x+2) + C(x-1)^2$

dosadíme $x = 1 \qquad 1 = 0 + B(1+2) + 0$

$1 = 3B$

$B = \frac{1}{3}$

dosadíme $x = -2 \qquad -2 = 0 + 0 + C(-2-1)^2$

$-2 = 9C$

$C = -\frac{2}{9}$

$x^2: \quad 0 = A + C$

$\rightsquigarrow A = \frac{2}{9}$

③ $\int \left(\frac{2}{9} \cdot \frac{1}{x-1} + \frac{1}{3} \frac{1}{(x-1)^2} - \frac{2}{9} \frac{1}{x+2}\right) dx$

$\frac{2}{9} \int \frac{1}{x-1}\,dx + \frac{1}{3} \int \frac{1}{(x-1)^2}\,dx - \frac{2}{9} \int \frac{1}{x+2} =$

$= \frac{2}{9} \ln|x-1| + \frac{1}{3} \frac{(x-1)^{-1}}{-1} - \frac{2}{9} \ln|x+2| + C$

$= \frac{2}{9} \ln|x-1| - \frac{1}{3} \frac{1}{x-1} - \frac{2}{9} \ln|x+2| + C$

4b) $\int \frac{x^4}{x^4 + 5x^2 + 4}\,dx = \int \frac{x^4 + 5x^2 + 4 - 5x^2 - 4}{x^4 + 5x^2 + 4}\,dx = \int \left(1 + \frac{-5x^2 - 4}{x^4 + 5x^2 + 4}\right) dx$

①

$(x^4) : (x^4 + 5x^2 + 4) = 1 + \frac{-5x^2 - 4}{x^4 + 5x^2 + 4}$

$\underline{-x^4 - 5x^2 - 4}$

$-5x^2 - 4$

②

$\frac{-5x^2 - 4}{\underbrace{x^4 + 5x^2 + 4}_{(x^2+1)(x^2+4)}} = \frac{Ax + B}{x^2 + 1} + \frac{Cx + D}{x^2 + 4}$