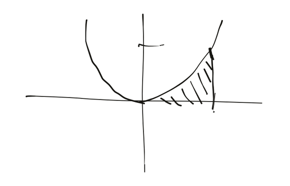
množina funkcí



$$\int x^2\,dx = \underbrace{\frac{x^3}{3}+C}$$

$$\int_0^1 x^2\,dx = \left[\frac{x^3}{3}\right]_0^1 = \left(\frac{1^3}{3}\right)-\left(\frac{0^3}{3}\right) = \frac{1}{3}$$

číslo

$\lim\limits_{x\to 0} \sin x$

Chceme:

$\sin^2 x \overset{?}{=} \frac{1}{2}-\frac{1}{2}\cos 2x$

Začneme: $1-\cos 2x = 1-(\cos^2 x-\sin^2 x)$

$= \sin^2 x+\cos^2 x-\cos^2 x+\sin^2 x$

$= 2\sin^2 x$ ✓

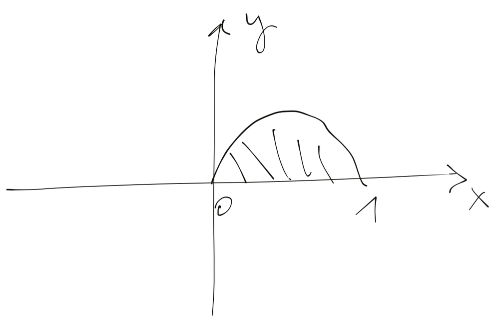
$\cos^2 x = 1-\sin^2 x$

$\cos 2x = \cos^2 x-\sin^2 x$

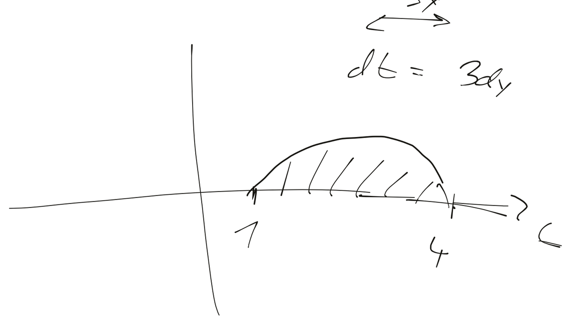
dostaneme: $\downarrow \cdot\frac{1}{2}$

$\frac{1}{2}-\frac{1}{2}\cos 2x = \sin^2 x$

$(\sin 2x)' = \cos 2x\cdot 2$



Substituce



$3x$

$dt = 3dy$

$t = 1+3x$

1.

$$\int_0^\pi \sin^2 x\,dx = \int_0^\pi \frac{1}{2}-\frac{1}{2}\cos 2x\,dx$$

$$\left(\frac{\sin 2x}{2}\right)' = \cos 2x$$

$$= \frac{1}{2}\int_0^\pi 1\,dx - \frac{1}{2}\int_0^\pi \cos 2x\,dx = \frac{1}{2}\left[x\right]_0^\pi - \frac{1}{2}\left[\frac{\sin 2x}{2}\right]_0^\pi$$

$$= \frac{\pi}{2}-0-\frac{1}{2}\left(\frac{0}{2}-0\right) = \frac{\pi}{2}$$

2.

$$\frac{y^1}{y^{\frac{1}{2}}} = y^1\cdot y^{-\frac{1}{2}} = y^{\frac{1}{2}}$$

$$\int_0^5 \frac{x}{\sqrt{1+3x}}\,dx = \int_1^{16}\frac{\frac{y}{3}-\frac{1}{3}}{\sqrt{y}}\,\frac{dy}{3} = \frac{1}{9}\left(\int_1^{16}\frac{y}{\sqrt{y}}\,dy-\int_1^{16}\frac{1}{\sqrt{y}}\,dy\right) = \frac{1}{9}\left[\frac{y^{\frac{3}{2}}}{\frac{3}{2}}\right]_1^{16}-\frac{1}{9}\left[\frac{y^{\frac{1}{2}}}{\frac{1}{2}}\right]_1^{16} =$$

$$= \frac{1}{9}\left(\overbrace{\frac{64}{\frac{3}{2}}-\frac{1}{\frac{3}{2}}}^{63}\right)-\frac{1}{9}\left(\frac{4}{\frac{1}{2}}-\frac{1}{\frac{1}{2}}\right) = \frac{2\cdot\cancel{9}\cdot 7}{\cancel{27}_{3}}-\frac{6}{9}^{\frac{2}{3}} = \frac{12}{3} = 4$$

$$\frac{64}{\frac{3}{2}} = \frac{16^{\frac{3}{2}}}{\frac{3}{2}}$$

$y = 1+3x$

$dy = 3dx$

$dx = \frac{dy}{3}$

$x = \frac{y}{3}-\frac{1}{3}$

$y_0 = 1+3\cdot 0 = 1$

$y_1 = 1+3\cdot 5 = 16$

neurčitý

$$\int \frac{x}{\sqrt{1+3x}}\,dx = \ldots = \frac{1}{9}\,\frac{y^{\frac{3}{2}}}{\frac{3}{2}}-\frac{1}{9}\,\frac{y^{\frac{1}{2}}}{\frac{1}{2}} = \frac{1}{9}\,\frac{\sqrt{1+3x}^3}{\frac{3}{2}}-\frac{1}{9}\cdot\frac{\sqrt{1+3x}}{\frac{1}{2}}+C$$

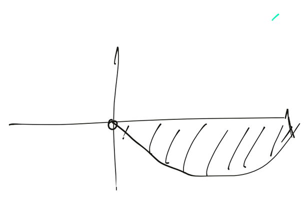
$y = 1+3x$

$$\int_0^5 \frac{x}{\sqrt{1+3x}}\,dx = \left[\frac{1}{9}\,\frac{\sqrt{1+3x}^3}{\frac{3}{2}}-\frac{1}{9}\cdot\frac{\sqrt{1+3x}}{\frac{1}{2}}\right]_0^5 = \ldots = \frac{12}{3} = 4$$

$$\int_0^1 x\cdot\ln x\,dx \;\left|\begin{matrix} u' = x & u = \frac{x^2}{2} \\ v = \ln x & v' = \frac{1}{x}\end{matrix}\right| = \left[\ln x\cdot\frac{x^2}{2}\right]_0^1-\int_0^1\frac{x^{\cancel{2}}}{2}\cdot\frac{1}{\cancel{x}}\,dx$$

$$\underset{\ln 1}{0}\cdot\frac{1}{2}-\underbrace{\lim_{x\to 0^+}\left(\ln x\cdot\frac{x^2}{2}\right)}-\frac{1}{2}\int_0^1 x\,dx = 0-\frac{1}{2}\left[\frac{x^2}{2}\right]_0^1 = -\frac{1}{4}$$

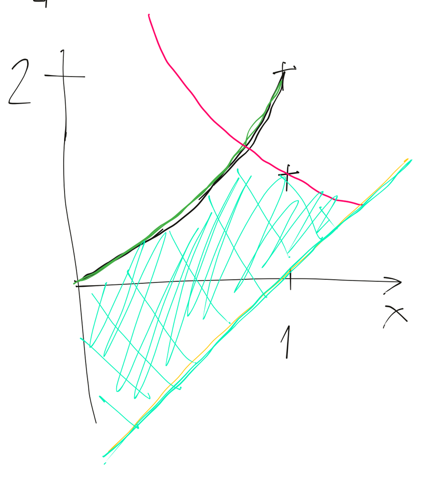
$$\int_0^1 x\cdot\ln x \approx \text{spočtěte} \lim_{a\to 0^+}\int_a^1 x\cdot\ln x\,dx$$



$$\lim_{x\to 0^+}\left(\ln x\cdot\frac{x^2}{2}\right) = \text{"}-\infty\cdot 0\text{"} = \text{"}\frac{-\infty}{\infty}\text{"}$$

$$= \lim_{x\to 0^+}\left(\frac{\ln x}{\frac{2}{x^2}}\right) \overset{LH}{=} \lim_{x\to 0^+}\frac{\frac{1}{x}}{-2\frac{2}{x^3}} = \lim_{x\to 0^+} -\frac{1}{4}x^2 = 0$$

Příklad 2



$y = 2x^3 \qquad y = \frac{1}{x} \qquad y = x-1$

x_0: $2x^3 = \frac{1}{x}$

$2x^4 = 1$

$x^4 = \frac{1}{2}$

$x_0 = \sqrt[4]{\frac{1}{2}}$

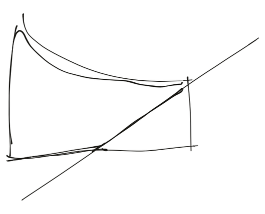
x_1: $\frac{1}{x} = x-1$

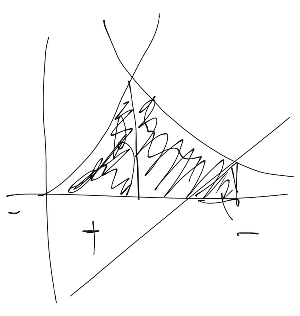
$1 = x^2-x$

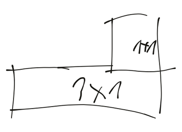
$x^2-x-1 = 0$

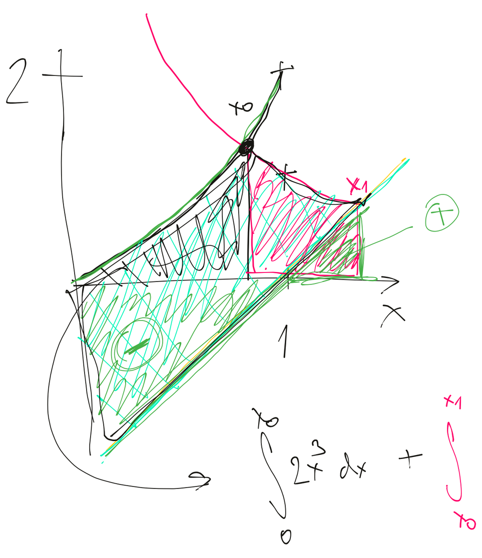
$x = \frac{+1\pm\sqrt{1-4\cdot 1\cdot -1}}{2}$

$x_1 = \frac{1+\sqrt{5}}{2}$









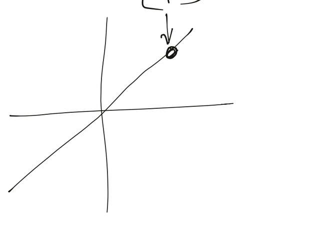
$$\int_0^{x_0} 2x^3\,dx + \int_{x_0}^{x_1}\frac{1}{x}\,dx - \int_0^{x_1} x-1\,dx$$

$x_0 = \sqrt[4]{\frac{1}{2}} \quad x_1 = \frac{1+\sqrt{5}}{2}$

$$2\left[\frac{x^4}{4}\right]_0^{x_0}+\left[\ln x\right]_{x_0}^{x_1}-\left[\frac{x^2}{2}\right]_0^{x_1}+\left[x\right]_0^{x_1} = \text{dosaďte si sami}$$

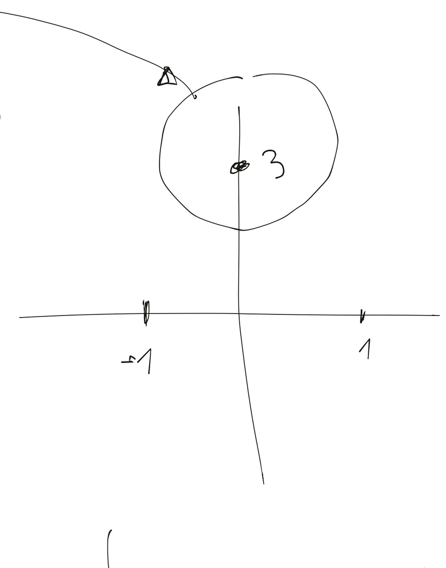
Příklad 4 objem anuloidu

$$\boxed{x^2+(y-3)^2 = 1}$$

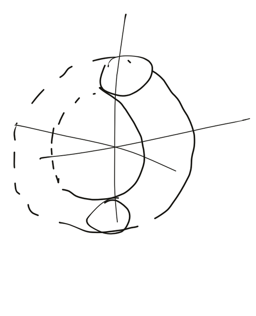


$[a,a] \to a-a = 0$

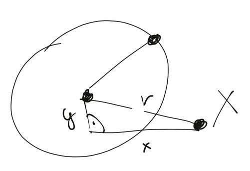
$y-x = 0$



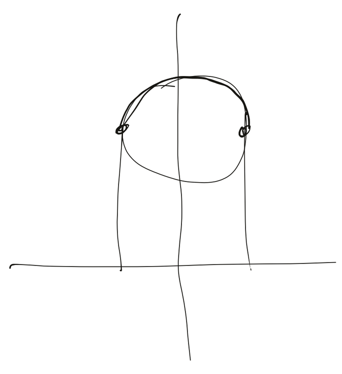
rotujeme



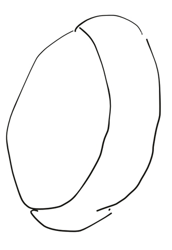
$x^2+y^2 = 1$



$r^2 = x^2+y^2$



nejdřív orotujeme vrchní oblouk



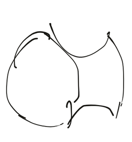
Vzorec γ: $y = f(x)$

$$V_\gamma = \pi\int_{x_0}^{x_1}(f(x))^2\,dx =$$

křivka $x^2+(y-3)^2 = 1$

$y = \pm\sqrt{1-x^2}+3$

pak odečteme vnitřek = rotace spodní části



$-2\cdot 3\cdot\sqrt{1-x^2}$

$$\Rightarrow V_{\text{anuloidu}} = \pi\int_{-1}^{1}\left(\sqrt{1-x^2}+3\right)^2 dx - \pi\int_{-1}^{1}\left(-\sqrt{1-x^2}+3\right)^2 dx$$

$$= \pi\left(\int_{-1}^{1}\left(\cancel{1-x^2}+2\sqrt{1-x^2}\cdot 3+\cancel{9}-(\cancel{1-x^2})+6\cdot\sqrt{1-x^2}\,\cancel{-9}\right)dx\right)$$

$$= \pi\int_{-1}^{1} 12\sqrt{1-x^2}\,dx$$

$$= \pi\, 12\int_{-1}^{1}\sqrt{1-x^2}\,dx = \pi\cdot 12\cdot\frac{\pi}{2} = 6\pi^2$$

$$\int_{-1}^{1}\sqrt{1-x^2}\,dx = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\underbrace{\sqrt{1-\sin^2 t}}_{\cos^2 t}\cos t\,dt = \int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\cos^2 t\,dt$$

$x = \sin t$

$dx = \cos t\,dt$

$-1 = \sin t_0 \rightsquigarrow t_0 = -\frac{\pi}{2}$

$1 = \sin t_1 \rightsquigarrow t_1 = \frac{\pi}{2}$

$$\| \qquad \int_0^\pi \sin^2 x\,dx = \frac{\pi}{2}$$

$\cos t = \sin(t+\frac{1}{2})$

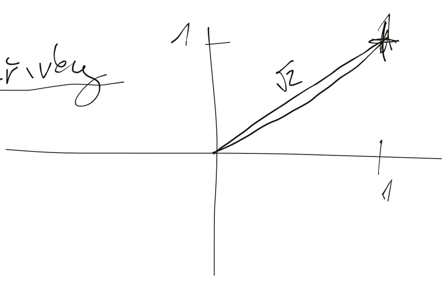
$y = x^2$

$$V = \pi\int_0^1 (x^2)^2\,dx = \pi\frac{1^5}{5}$$

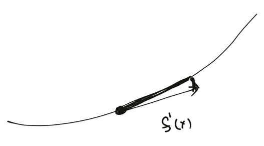
Příklad 6

$y = x^{\frac{3}{2}}$ délka křivky

na $[0,1]$



vzoreček $D = \int_0^1\sqrt{1+(f'(x))^2}$



$$\Rightarrow \quad f'(x) = \left(x^{\frac{3}{2}}\right)' = \frac{3}{2}x^{\frac{1}{2}}$$

$$(f'(x))^2 = \frac{9}{4}x$$

$$D = \int_0^1\sqrt{1+\frac{9}{4}x}\,dx$$