Výpočet délky astroidy

$$L = 4\int_0^{\frac{\pi}{2}} \sqrt{(-3\sin t - 3\sin(3t))^2 + (3\cos t - 3\cos(3t))^2}\,\mathrm{d}t =$$

$$= 4\int_0^{\frac{\pi}{2}} \sqrt{9\sin^2 t + 18\sin t\sin(3t) + 9\sin^2(3t) + 9\cos^2 t - 18\cos\cos(3t) + 9\cos^2(3t)} =$$

$$= 4\int_0^{\frac{\pi}{2}} \sqrt{18\,(1 + \sin t\sin(3t) - \cos t\cos(3t))} =$$

$$= 4\int_0^{\frac{\pi}{2}} \sqrt{18\left(1 + \sin t(3\sin t\cos^2 t - \sin^3 t) - \cos t(\cos^3 t - 3\sin^2 t\cos t)\right)} =$$

$$= 4\int_0^{\frac{\pi}{2}} \sqrt{18\left(1 + 6\sin^2 t\cos^2 t - \sin^4 t - \cos^4 t\right)}$$

Zde je asi nejlepší použít vzorce $a^2 - b^2 = (a+b)(a-b)$ následujícím způsobem

$$1 - \sin^4 t = (1 - \sin^2 t)(1 + \sin^2 t) = \cos^2 t(1 + \sin^2 t)$$

V integrálu pak dostaneme

$$4\int_0^{\frac{\pi}{2}} \sqrt{18\left(6\sin^2 t\cos^2 t + \cos^2 t + \cos^2 t\sin^2 t - \cos^4 t\right)} = 4\int_0^{\frac{\pi}{2}} \sqrt{18\left(7\sin^2 t\cos^2 t + \cos^2 t(1 - \cos^2 t)\right)} =$$

$$= 4\int_0^{\frac{\pi}{2}} \sqrt{18\left(8\sin^2 t\cos^2 t\right)} = 4\int_0^{\frac{\pi}{2}} \sqrt{36\sin^2(2t)} = 24\int_0^{\frac{\pi}{2}} \sin(2t) = 24\left[-\frac{1}{2}\cos(2t)\right]_0^{\frac{\pi}{2}} = 24$$