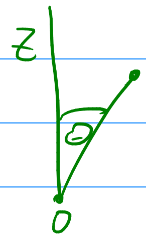
$S[0,0,0]\ r=A$

① $\iiint\limits_V f(x,y,z)\,dx\,dy\,dz = I \qquad V: \boxed{x^2+y^2+z^2 \le A^2},\ x\ge 0,\ y\ge 0,\ z\ge 0,\ A>0$

$x = \rho\,\boxed{\cos\varphi}\,\overset{\ge 0}{\sin\theta} \ge 0$

$y = \rho\,\boxed{\sin\varphi}\,\overset{\ge 0}{\sin\theta} \ge 0$

$z = \rho\cos\theta \ge 0$



$\theta \in [0,\pi]$

$\varphi \in [0, 2\pi]$

$\rho \ge 0$

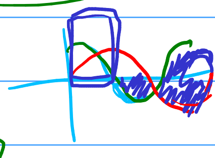
$x^2+y^2+z^2=\rho^2$

$\rho^2 \le A^2$

$\boxed{0 \le \rho \le A}$

$\cos\theta \ge 0$

$\boxed{\theta \in \left[0, \frac{\pi}{2}\right]}$



$\boxed{\varphi \in \left[0, \frac{\pi}{2}\right]}$

$\cos\varphi \ge 0$

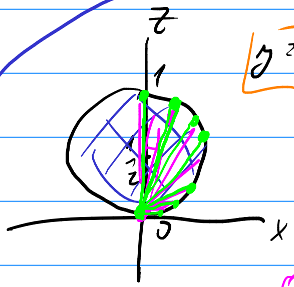
$\sin\varphi \ge 0$

$$I = \int_0^A \int_0^{\frac{\pi}{2}} \int_0^{\frac{\pi}{2}} f(\rho\cos\varphi\sin\theta, \dots)\cdot \rho^2 \sin\theta \, d\theta\, d\varphi\, d\rho$$

② $\iiint\limits_V \sqrt{x^2+y^2+z^2}\,dx\,dy\,dz \qquad V:\ x^2+y^2+z^2 \le z$

$\cancel{x^2+y^2+z^2=\rho^2}$

NE



$\boxed{y \geq 0}$

$x^2+y^2+z^2-z \le 0$

$x^2+y^2+\left(z-\frac{1}{2}\right)^2 \le \frac{1}{4}$

KOULE $S:[0,0,\frac{1}{2}]$, $r=\frac{1}{2}$

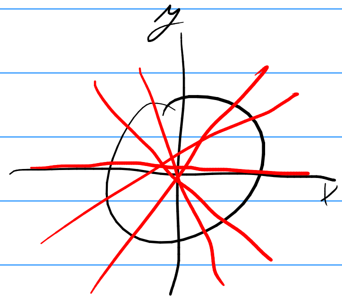
$\Theta \in [0,\frac{\pi}{2}]$ $\qquad \varphi \in [0,2\pi]$

$x = \rho \cos\varphi \sin\Theta$

$y = \rho \sin\varphi \sin\Theta$

$z = \rho \cos\Theta \cancel{+\frac{1}{2}}$

$\cancel{\varphi\in[0,2\pi],\ \Theta\in[0,\pi],\ \rho\in[0,\tfrac{1}{2}]}$



$\rho \in [0, \cos\Theta]$

$\rho^2 \le \rho\cos\Theta$

$\rho \le \cos\Theta$

$$I = \int_0^{\frac{\pi}{2}}\int_0^{2\pi}\int_0^{\cos\Theta} \underbrace{\sqrt{\rho^2}\cdot\rho^2}_{\rho^3}\sin\Theta\,d\rho\,d\varphi\,d\Theta =$$

$$= 2\pi\int_0^{\frac{\pi}{2}}\left[\frac{\rho^4}{4}\right]_0^{\cos\Theta}\sin\Theta\,d\Theta =$$

$$= \frac{\pi}{2}\int_0^{\frac{\pi}{2}}\cos^4\Theta\,\boxed{\sin\Theta\,d\Theta} = \left|\begin{array}{l} t=\cos\Theta \\ dt = -\boxed{\sin\Theta\,d\Theta} \\ 0 \to 1 \\ \frac{\pi}{2} \to 0 \end{array}\right| = +\frac{\pi}{2}\int_0^1 t^4\,dt =$$

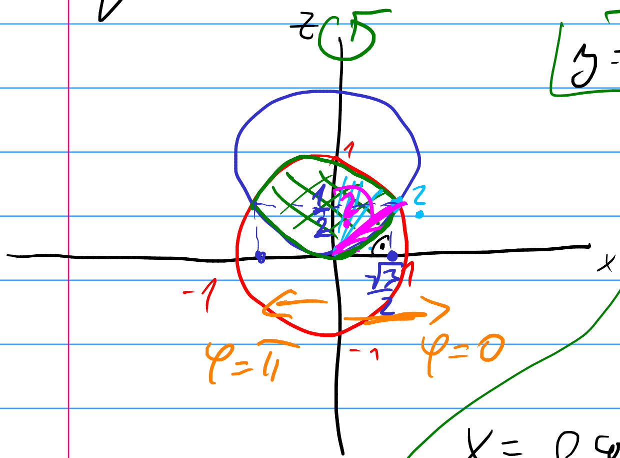
$$= \frac{\pi}{2}\left[\frac{t^5}{5}\right]_0^1 = \frac{\pi}{10}$$

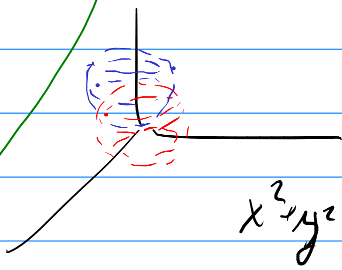
③ $\iiint\limits_V x^2+y^2\,dx\,dy\,dz = I$ $\quad V: x^2+y^2+z^2\le 1$ KOULE STŘED $[0,0,0]$

$\boxed{y=0}$ $\quad \boxed{x^2+y^2+(z-1)^2\le 1}$ KOULE

$s: [0,0,1]$

$r=1$





$x^2+y^2 = \rho^2\cos^2\varphi\sin^2\theta+\rho^2\sin^2\varphi\sin^2\theta = \rho^2\sin^2\theta$

$x = \rho\cos\varphi\sin\theta$

$y = \rho\sin\varphi\sin\theta$

$z = \rho\cos\theta$

$\varphi\in[0,2\pi]$

1) $\theta\in\left[0,\frac{\pi}{3}\right]$ $\quad \rho\in[0,1]$

2) $\theta\in\left[\frac{\pi}{3};\frac{\pi}{2}\right]$ $\quad \rho\in[0,2\cos\theta]$

$x^2+y^2+z^2-2z+1\le 1$

$x^2+y^2+z^2\le 2z$

$\rho^2\le 2\rho\cos\theta$

$\rho\le 2\cos\theta$

z

$x^2+z^2=1 \;\cap\; x^2+z^2-2z=0$

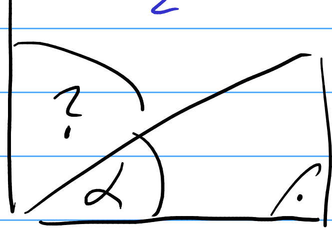
$1-2z=0$

$z=\frac{1}{2}$

$x^2 = 1-\left(\frac{1}{2}\right)^2 = 1-\frac{1}{4}$

$x^2=\frac{3}{4}$

$x=\pm\frac{\sqrt{3}}{2}$



$\operatorname{tg}\alpha = \frac{\frac{1}{2}}{\frac{\sqrt{3}}{2}} = \frac{1}{\sqrt{3}}$

$\alpha + ? = \frac{\pi}{2}$

$? = \frac{\pi}{2}-\alpha$ $\qquad \alpha=\frac{\pi}{6}$

$? = \frac{\pi}{2}-\frac{\pi}{6} = \frac{3-1}{6}\pi = \frac{2}{6}\pi = \frac{\pi}{3}$

$$I=\int_0^1\int_0^{\frac{\pi}{3}}\int_0^{2\pi}\rho^4\sin^3\theta\, d\varphi\, d\theta\, d\rho+\int_0^{2\pi}\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\int_0^{2\cos\theta}\rho^4\sin^3\theta\, d\rho\, d\theta\, d\varphi=$$

$$=2\pi\underbrace{\int_0^1\rho^4 d\rho}\int_0^{\frac{\pi}{3}}\underbrace{\overbrace{\sin^3\theta}^{\sin\theta(1-\cos^2\theta)}\,d\theta}_{s=\cos\theta}+2\pi\int_{\frac{\pi}{3}}^{\frac{\pi}{2}}\underbrace{\overbrace{\sin^3\theta}^{(1-\cos^2\theta)\sin\theta}}_{s=\cos\theta}\,\frac{2^5\cos^5\theta}{5}\,d\theta=$$

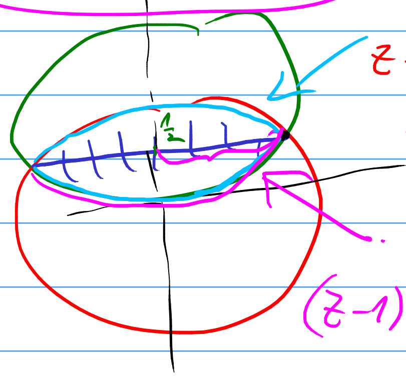
$$=-\frac{2\pi}{5}\int_1^{\frac{1}{2}}1-s^2\,ds-\frac{2^6\pi}{5}\int_{\frac{1}{2}}^{0}(1-s^2)s^5\,ds=$$

$$=\frac{2\pi}{5}\left[s-\frac{s^3}{3}\right]_{\frac{1}{2}}^{1}+\frac{2^6\pi}{5}\left[\frac{s^6}{6}-\frac{s^8}{8}\right]_0^{\frac{1}{2}}=\ldots\ldots$$

$$\iiint x^2+y^2\,dx\,dy\,dz$$

$x^2+y^2+z^2=1$

$x^2+y^2+(z-1)^2=1$



$z=\sqrt{1-x^2-y^2}$

$z=\sqrt{1-\rho^2}$

$(z-1)^2=1-x^2-y^2$

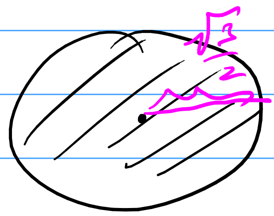
$z=1-\sqrt{1-x^2-y^2}$

$x=\rho\cos\varphi \qquad \varphi\in[0,2\pi]$

$y=\rho\sin\varphi \qquad \rho\in\left[0,\frac{\sqrt{3}}{2}\right]$

$z=z$

$$1-\sqrt{1-\rho^2}\le z\le\sqrt{1-\rho^2}$$



$$I=\int_0^{2\pi}\int_0^{\frac{\sqrt{3}}{2}}\int_{1-\sqrt{1-\rho^2}}^{\sqrt{1-\rho^2}}\rho^2\cdot\rho\,dz\,d\rho\,d\varphi=2\pi\int_0^{\frac{\sqrt{3}}{2}}\rho^3\left(2\sqrt{1-\rho^2}-1\right)d\rho=$$

$$= 4\pi \int_0^{\frac{\sqrt{3}}{2}} \rho^3 \sqrt{1-\rho^2}\, \boxed{d\rho} - 2\pi \int_0^{\frac{\sqrt{3}}{2}} \rho^3\, d\rho =$$

$\rho^2 \cdot \rho$

$\rho = \sin t$

$$\left| \begin{array}{l} t^2 = 1-\rho^2 \\ 2t\,dt = -2\rho\, d\rho \end{array} \right|$$

$$= -4\pi \int_1^{\frac{1}{4}} (1-t^2) \underbrace{\sqrt{t^2}}_{t}\, dt - 2\pi \left[\frac{\rho^4}{4} \right]_0^{\frac{\sqrt{3}}{2}} =$$

$$= 4\pi \left[\frac{t^2}{2} - \frac{t^4}{4} \right]_{\frac{1}{4}}^{1} - 2\pi \ldots$$