$P\,dx + Q\,dy$ $\quad F = \begin{pmatrix} P \\ Q \end{pmatrix} = \begin{pmatrix} 0 \\ 3 \end{pmatrix}$

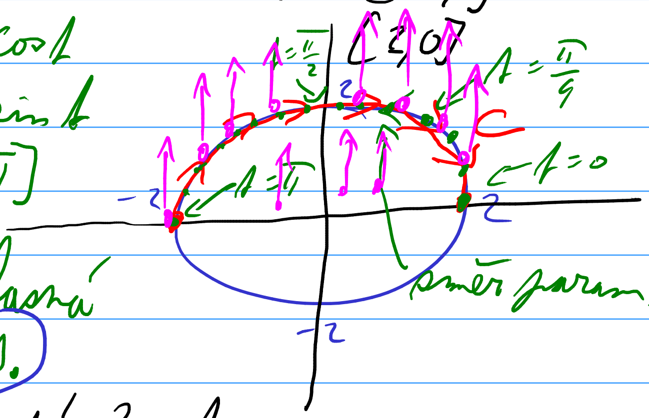
1) $C: x^2+y^2=4,\ y\geq 0$, orientovaná od bodu $[-2,0]$ do bodu $[2,0]$

$$\int_C 3\,dy = -\int_0^{\pi} 0\cdot x' + 3\cdot y'\,dt$$

$x = 2\cos t$

$y = 2\sin t$

$t \in [0,\pi]$



nesúhlasná s orient.

$y' = 2\cos t$

$$= -\int_0^{\pi} 3\cdot 2\cos t\,dt = $$

$$= -6\left[\sin t\right]_0^{\pi} = 0$$

$P_y = 0 = Q_x$ ✓ F potenciálové

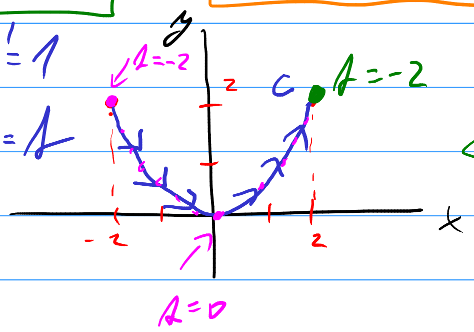
NENÍ NUTNÉ

② $I = \int_C \overset{P}{x}\,dx + \overset{Q}{y}\,dy$ C: $\boxed{y = \frac{x^2}{2}}$ od $\boxed{[-2,2]$ k $[2,2]}$

① $x = t \rightsquigarrow x' = 1$

$y = \frac{t^2}{2} \rightsquigarrow y' = t$

$t \in [-2,2]$ souhlasná orientace



$$= +\int_{-2}^{2} \overset{x\cdot x' + y\cdot y'}{t\cdot 1 + \frac{t^2}{2}\cdot t}\,dt =$$

$$= \left[\frac{t^2}{2} + \frac{t^4}{8}\right]_{-2}^{2} = \frac{4}{2} + \frac{16}{8} - \left(\frac{4}{2} + \frac{16}{8}\right) = 0$$

② $x = -t$

$y = \frac{t^2}{2}$

$t \in [-2,2]$ nesouhlasná

$t = -2 : [2,2]$

$t = 2 : [-2,2]$

$\boxed{K'_x = P}$, $K'_y = Q$

$$I = K(2,2) - K(-2,2) = \frac{4}{2} + \frac{4}{2} - \left(\frac{4}{2} + \frac{4}{2}\right) = 0$$

$K'_x = P = x$

$K = \int x\,dx = \frac{x^2}{2} + C(y)$

$K_y = Q$

$C' = Q = y$

$C' = y \to C = \int y\,dy = \frac{y^2}{2} + D$, $D \in \mathbb{R}$

$$K = \frac{x^2}{2} + \frac{y^2}{2} + D$$

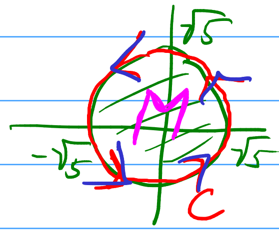
kdyby F bylo potenciálové $\leadsto I = 0$

C je uzavřená!

(3) $I = \int\limits_C \underbrace{\overbrace{(x^2+y^2)}^{P}dx + \overbrace{(xy-y^2)}^{Q}dy}_{\text{diferenciál?}}$

C: je hranice $x^2+y^2 \le 5$ orientovaná kladně

$$= + \int_0^{2\pi} 5\cdot(-\sqrt{5})\sin t + (5\sin t\cos t - 5\sin^2 t)\cdot\sqrt{5}\cos t \, dt$$



$x = \sqrt{5}\cos t$

$y = \sqrt{5}\sin t$

$t \in [0, 2\pi]$

souhlasná

$$= -5\sqrt{5}\int_0^{2\pi} \sin t - \overbrace{\cos^2 t \sin t}^{\cos^3 t} + \sin^2 t \cos t \, dt$$

$x' = -\sqrt{5}\sin t$

$y' = \sqrt{5}\cos t$

$$= -5\sqrt{5}\left[-\cos t + \frac{\cos^3 t}{3} + \frac{\sin^3 t}{3}\right]_0^{2\pi} =$$

$$= -5\sqrt{5}\left[-1 + \frac{1}{3} + 0 - \left(-1 + \frac{1^3}{3} + 0\right)\right] = 0$$

$P_y \overset{?}{=} Q_x \leadsto Q_x - P_y = y - 2y = -y$

$(x^2+y^2)'_y \overset{?}{=} (xy-y^2)'_x$

$2y \overset{?}{=} y \leadsto$ NENÍ POTENCIÁLOVÉ

$$I = \iint\limits_{x^2+y^2\le 5} -y \, dx dy \underset{\substack{\uparrow \\ \text{polární} \\ \text{souř.}}}{=} -\int_0^{2\pi}\int_0^{\sqrt{5}} \rho^2 \sin\varphi \, d\rho \, d\varphi =$$

$$= -\left[\frac{\rho^3}{3}\right]_0^{\sqrt{5}} \cdot \left[-\cos\varphi\right]_0^{2\pi} = 0$$

$= -1 + 1 = 0$

$x(x^2+y^2+z^2)^{-\frac{1}{2}}$

$$I=\int_C \underbrace{\frac{x}{\sqrt{x^2+y^2+z^2}}}_{P}\,dx+\underbrace{\frac{y}{\sqrt{\boxed{x^2}+y^2+\boxed{z^2}}}}_{Q}\,dy+\underbrace{\frac{z}{\sqrt{x^2+y^2+z^2}}}_{R}\,dz$$

C začíná na sféře $x^2+y^2+z^2=A^2$

končí —||— $x^2+y^2+z^2=B^2$, $A>0, B>0$

$$P_y=-\frac{xy}{\sqrt{(x^2+y^2+z^2)^{\frac{3}{2}}}}=Q_x \checkmark$$

$$P_z=-\frac{xz}{\sqrt{(x^2+y^2+z^2)^{\frac{3}{2}}}}=R_x \checkmark \qquad \boxed{\operatorname{rot} F=\vec{0}}$$

$$Q_z=-\frac{yz}{\sqrt{(x^2+y^2+z^2)^{\frac{3}{2}}}}=R_y \checkmark$$

vekt. pole je potenciální

$$K_x=\frac{x}{\sqrt{x^2+y^2+z^2}} \rightsquigarrow K=\int\frac{x}{\sqrt{x^2+y^2+z^2}}\,dx=\left|\begin{matrix}t=x^2+y^2+z^2\\ dt=2x\,dx\end{matrix}\right|$$

$$=\frac{1}{2}\int\frac{dt}{\sqrt{t}}=\cancel{\frac{1}{2}}\,\frac{t^{\frac{1}{2}}}{\cancel{\frac{1}{2}}}+C(\dots)=$$

$$=\sqrt{x^2+y^2+z^2}+\underbrace{C(y,z)}_{D(z)}$$

$K_y=Q$

$$K_y=\cancel{\frac{1}{2\sqrt{x^2+y^2+z^2}}}\cdot 2y+C_y=\cancel{\frac{y}{\sqrt{x^2+y^2+z^2}}}$$

$$C_y=0 \Rightarrow C(y,z)=\int 0\,dy=D(z)$$

$K_z=R$

$$K_z=\cancel{\frac{1}{2\sqrt{x^2+y^2+z^2}}}\cdot 2z+D'=\cancel{\frac{z}{\sqrt{x^2+y^2+z^2}}}$$

$$D'=0 \rightsquigarrow D(z)=E\in\mathbb{R}$$

$$K=\sqrt{x^2+y^2+z^2}+E$$

$$I = K\left(x^2+y^2+z^2=B^2\right) - K\left(x^2+y^2+z^2=A^2\right) =$$

$$= \sqrt{B^2} - \sqrt{A^2} = \underline{B-A}$$

$$I = \int_C \underbrace{-x^2 y}_{P}\,dx + \underbrace{xy^2}_{Q}\,dy$$

C: $x^2+y^2=A^2$, $A>0$

orient. v kladném směru

$$Q_x = y^2$$

$$P_y = -x^2$$

$$Q_x - P_y = x^2+y^2$$

$$I = \iint_{x^2+y^2\le A^2} x^2+y^2 \,dx\,dy = \int_0^{2\pi}\int_0^A \rho^3 \,d\rho\, d\varphi = 2\pi\left[\frac{\rho^4}{4}\right]_0^A =$$

$$= \frac{\pi A^4}{2}$$