(1) $\sum_{n=1}^{\infty} \underbrace{\frac{(n!)^2}{(2n)!}}\ (x-0)^n \qquad R = ? \qquad \sum_{n=0}^{\infty} \underbrace{a_n}\ (x-x_0)^{\boxed{n}}$

$$a_n = \frac{(n!)^2}{(2n)!} \qquad x_0 = 0 \qquad L = \lim_{n\to\infty} \sup \sqrt[n]{|a_n|}$$

$$L = \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{[(n+1)!]^2}{(2n+2)!} \cdot \frac{\cancel{(2n)!}}{(\cancel{n!})^2} =$$

$(n+1)\cdot\cancel{n!}$

$(2n+2)(2n+1)\cancel{2n!}$

$$= \lim_{n\to\infty} \frac{(n+1)^2}{(2n+2)\cdot(2n+1)} = \lim_{n\to\infty} \frac{n^2+\cancel{2n}+\cancel{1}}{4n^2+\cancel{6n}+\cancel{2}} = \lim_{n\to\infty} \frac{1}{4} = \frac{1}{4}$$

$\in (0,\infty)$

$$R = \frac{1}{L} = \underline{\underline{4}}$$

② $\sum_{n=1}^{\infty} \underbrace{\frac{(-1)^{n+1}}{n}}_{a_n = \frac{(-1)^{n+1}}{n}} (x-1)^n \qquad I = ?$

$x_0 = 1$

$$L = \lim_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{n\to\infty} \sqrt[n]{\frac{1}{n}} = \lim_{n\to\infty} \frac{1}{\sqrt[n]{n}} = 1 \in (0,\infty)$$

$\sqrt[n]{n} \to 1$

$R = \frac{1}{1} = 1$

$? x_0 - R, x_0 + R ? = (0, 2]$

$$x = 2 \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \underbrace{(2-1)^n}_{1^n = 1} = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \quad \text{KONV.}$$

$2 \in I$

$$x = 0 \quad \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n} \underbrace{(0-1)^n}_{(-1)^n} = \sum_{n=1}^{\infty} \frac{(-1)^{2n+1}}{n} = \sum_{n=1}^{\infty} \frac{-1}{n} \quad \text{DIV}$$

$0 \notin I$

obor konvergence $I = (0, 2]$

(3) $\sum_{n=2}^{\infty} \frac{4^n}{n^6}(x-0)^n \qquad x_0 = 0$

$a_n = \frac{4^n}{n^6}$

$L = \lim_{n\to\infty} \sqrt[n]{\frac{4^n}{n^6}} = \lim_{n\to\infty} \frac{4}{\sqrt[n]{n^6}} = \lim_{n\to\infty} \frac{4}{(\sqrt[n]{n})^6} = \frac{4}{1^6} = 4 \in (0,\infty)$

$R = \frac{1}{4} \qquad ?\ -\frac{1}{4}, \frac{1}{4}\ ?$

$x = \frac{1}{4}$

$\sum_{n=2}^{\infty} \frac{\cancel{4^n}}{n^6}\underbrace{\left(\frac{1}{4}\right)^n}_{\cancel{\frac{1}{4^n}}} = \sum_{n=2}^{\infty} \frac{1}{n^6}$ K

$\frac{1}{4} \in I$

$x = -\frac{1}{4}$

$\sum_{n=2}^{\infty} \frac{\cancel{4^n}}{n^6}\frac{(-1)^n}{\cancel{4^n}} = \sum_{n=2}^{\infty} \frac{(-1)^n}{n^6}$ K

$\sum_{n=2}^{\infty} \left|\frac{(-1)^n}{n^6}\right| = \sum_{n=2}^{\infty} \frac{1}{n^6}$ K

$-\frac{1}{4} \in I$

$I = \left[-\frac{1}{4}, \frac{1}{4}\right]$

4

$$\sum_{n=1}^{\infty} (-7x)^n \ln^n n \qquad I = ?$$

$$\sum_{n=1}^{\infty} \underbrace{(-7\ln n)^n}_{a_n} \cdot \underbrace{x^n}_{x-x_0} \qquad \underline{x_0 = 0}$$

$$L = \lim_{n\to\infty} \sqrt[n]{|(-7\ln n)^n|} = \lim_{n\to\infty} 7\ln n = 7\cdot\infty = \infty$$

$$R = \left|\frac{1}{\infty}\right| = 0 \qquad I = \{x_0\}$$

$$\sum a_n \underbrace{(x-x_0)^n}_{0^n} = \sum a_n \cdot 0 = 0$$

obor konvergence $I = \{0\}$

⑤ $\sum_{n=1}^{\infty} \frac{n+3}{n!}(2x-1)^n \qquad I = ?$

$x_0 = \frac{1}{2}$

$\sum_{n=1}^{\infty} \underbrace{\frac{n+3}{n!}2^n}_{a_n}\left(x-\frac{1}{2}\right)^n$

$L = \lim \frac{n+4}{\cancel{(n+1)!}_{(n+1)}} 2^{\cancel{n+1}} \cdot \frac{\cancel{n!}}{(n+3)\cancel{2^n}} = \lim_{n\to\infty} \frac{2(n+4)}{(n+1)(n+3)} =$

$= \lim_{n\to\infty} \frac{2n+\cancel{4}}{n^2+\cancel{4n}+\cancel{3}} = \lim_{n\to\infty} \frac{2\cancel{n}}{n^{\cancel{2}}} = 0$

$R = "\left|\frac{1}{0^+}\right|" = \infty$

$\left(\frac{1}{2}-\infty, \frac{1}{2}+\infty\right) = (-\infty, \infty)$

$I = \mathbb{R}$

⑥ $\sum_{n=1}^{\infty}\left(\frac{A^n}{n^2}+\frac{B^n}{n^3}\right)x^n \qquad R=?$

$A>0, B>0$

$\lim \frac{\sqrt[n]{2(\mathrm{MAX}\{A,B\})^n}}{\sqrt[n]{n^2}} \to M$

$\lim_{n\to\infty}\sqrt[n]{\frac{A^n+B^n}{n^3}} \le \lim_{n\to\infty}\underbrace{\sqrt[n]{\frac{A^n}{n^2}+\frac{B^n}{n^3}}}_{\to M} \le \lim_{n\to\infty}\sqrt[n]{\frac{A^n+B^n}{n^2}} = \mathrm{MAX}\{A,B\}$

$\mathrm{MAX}\{A,B\} \quad \sqrt[n]{A^n+B^n} = \sqrt[n]{B^n\left(\left(\frac{A}{B}\right)^n+1\right)} = B\sqrt[n]{\underbrace{\left(\frac{A}{B}\right)^n}_{\to 0}+1} \to B$

$\lim_{n\to\infty}\frac{\sqrt[n]{M^n}}{\sqrt[n]{n^3}} \to M$

$B>A$

$B<A \qquad \to A$

$A=B \quad \sqrt[n]{2A^n} = A\cdot\underbrace{\sqrt[n]{2}}_{\to 1} = A = B$

$$R=\frac{1}{\mathrm{MAX}\{A,B\}}=\mathrm{MIN}\left\{\frac{1}{A},\frac{1}{B}\right\}$$

(7) $\sum_{n=1}^{\infty} 6^n x^{2n}$ $\qquad I = ?$

$x_0 = 0$

(($\sum a_n x^n = a_1 x + a_2 x^2 + a_3 x^3 + \dots$

$6^1 x^2 + 6^2 x^4 + 6^3 x^6$

$0 \cdot x^0 + 0 \cdot x^1 + 6 x^2 + 0 x^3 + 36 x^4 + 0 x^5 + 216 x^6 + 0 x^7 + \dots$

$a_n = \begin{cases} 0, & n \text{ liche} \\ 6^k & n = 2k \rightsquigarrow \frac{n}{2} \end{cases}$

$\qquad = \sqrt{6^n}$

$L = \limsup_{n\to\infty} \sqrt[n]{|a_n|} = \lim_{k\to\infty} \sqrt[2k]{6^k} = \lim_{k\to\infty} 6^{\frac{k}{2k}} = \lim_{k\to\infty} 6^{\frac{1}{2}} = \sqrt{6}$

$\in (0, \infty)$

$R = \frac{1}{\sqrt{6}}$

$\left(-\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$?

$x = \frac{1}{\sqrt{6}}$ $\qquad\qquad x = -\frac{1}{\sqrt{6}}$

$\sum_{n=1}^{\infty} 6^n \left(\left(\frac{1}{\sqrt{6}}\right)^2\right)^n = \sum_{n=1}^{\infty} \cancel{6^n} \frac{1}{\cancel{6^n}} = \sum_{n=1}^{\infty} 1$ D

$\pm \frac{1}{\sqrt{6}} \notin I$

$I = \left(-\frac{1}{\sqrt{6}}, \frac{1}{\sqrt{6}}\right)$

$nx^{n-1} = (x^n)'$

(8) $\sum_{n=1}^{\infty} n x^{n+1} = f(x) \qquad R = 1$

$$\sum_{n=1}^{\infty} n x^{n-1} \cdot x^2 = x^2 \sum_{n=1}^{\infty} n x^{n-1} = x^2 \sum_{n=1}^{\infty} (x^n)' =$$

$$= x^2 \Big(\underbrace{\sum_{n=1}^{\infty} x^n}_{\substack{\text{GEOM ŘADA } q = x \\ x + x^2 + x^3 + \dots}}\Big)' = x^2 \left(\frac{x}{1-x}\right)' = x^2 \, \frac{1 - \cancel{x} + \cancel{x} \cdot (+1)}{(1-x)^2} =$$

$$= \frac{x^2}{(1-x)^2}$$

$$\hookrightarrow \sum_{n=1}^{\infty} n \cdot \left(\frac{1}{2}\right)^{n+1} = \frac{x^2}{(1-x)^2}\bigg|_{x=\frac{1}{2}} = \frac{\left(\frac{1}{2}\right)^2}{\left(\frac{1}{2}\right)^2} = 1$$

(9) $\sum_{n=2}^{\infty} \frac{1}{n 4^n} \longleftrightarrow \sum_{n=2}^{\infty} \frac{1}{n} x^n$

$x = \cancel{4}$ $\frac{1}{4}$ $\rightsquigarrow$ $\cancel{\sum^{\infty} \frac{1}{n x^n}}$

x^{n-1}

$$\sum_{n=2}^{\infty} x^n = \frac{x^2}{1-x} \quad /:x \qquad \int x^{n-1} dt = \frac{x^n}{n}$$

$$\sum_{n=2}^{\infty} x^{n-1} = \frac{x}{1-x} \quad / \int dt$$

$$\sum_{n=2}^{\infty} \frac{x^n}{n} = \int \frac{x-1+1}{1-x} dt = \int -1 - \frac{1}{1-x} dt =$$

$$= -x - \ln|1-x| + C$$

$x=0$

$$\sum \frac{0}{n} = 0 \quad = -0 - \ln 1 + c = c$$

$$C = 0$$

$$\sum_{n=2}^{\infty} \frac{x^n}{n} = -x - \ln|1-x|$$

$$\sum_{n=2}^{\infty} \frac{1}{n 4^n} = -\frac{1}{4} - \ln\frac{3}{4} = -\frac{1}{4} + \ln\frac{4}{3}$$

(10) $\sum_{n=1}^{\infty} n(n+3)x^n$ $R=1$

$$\sum_{n=1}^{\infty} x^n = \frac{x}{1-x} \quad \Big/ \frac{d}{dx}$$

$$\sum_{n=1}^{\infty} n x^{n-1} = \left(\frac{x}{1-x}\right)' = \frac{1-x-x(-1)}{(1-x)^2} = \frac{1}{(1-x)^2} \quad \Big/ \cdot x^4$$

$$\sum_{n=1}^{\infty} n x^{n+3} = \frac{x^4}{(1-x)^2} \quad \Big/ \frac{d}{dx}$$

$$\sum_{n=1}^{\infty} n(n+3)x^{n+2} = \frac{4x^3(1-x)^{\cancel{2}\,1} - x^4 2\cancel{(1-x)}(-1)}{(1-x)^{\cancel{4}\,3}}$$

$$\sum_{n=1}^{\infty} n(n+3)x^{n+2} = \frac{x^3(4-4x+2x)}{(1-x)^3} \quad \Big/ : x^2$$

$$\sum_{n=1}^{\infty} n(n+3)x^n = \boxed{\frac{x(4-2x)}{(1-x)^3}}$$

(11) $\sum_{n=1}^{\infty} \frac{n}{n+2} x^{n-1}$ $\qquad R = 1 \qquad \int x^{n+1}\,dt = \frac{x^{n+2}}{n+2}$

$\cancel{1} + x + x^2$

$$= \sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \qquad \Big| \frac{d}{dt}$$

$$\sum_{n=1}^{\infty} n \cdot x^{n-1} = -\frac{1}{(1-x)^2} \cdot (-1) = \frac{1}{(1-x)^2} \qquad \Big| \cdot x^2$$

$$\sum_{n=1}^{\infty} n x^{n+1} = \frac{x^2}{(1-x)^2} \qquad \Big| \int dt$$

$$\sum_{n=1}^{\infty} \frac{n}{n+2} x^{n+2} = \int \frac{x^2}{(1-x)^2}\,dt = \int 1 + \frac{2x-2+1}{x^2-2x+1}\,dt$$

$1 - 2x + x^2$

$$= \int 1 + \frac{2x-2}{x^2-2x+1} + \frac{1}{(x-1)^2}\,dt$$

$x = 0$

$$= x + \ln|(x-1)^2| - \frac{1}{x-1} + C \qquad \Big| : x^3$$

$\underset{-1}{\shortparallel}$

$$0 = 0 + \ln 1 - \frac{1}{0-1} + C$$

$$0 = 1 + C$$

$$C = -1$$

$$\sum_{n=1}^{\infty} \frac{n}{n+2} x^{n-1} = \frac{1}{x^2} + \frac{\ln(x-1)^2}{x^3} - \frac{1}{x^3(x-1)} - \frac{1}{x^3}$$

(12) $\sum_{n=1}^{\infty} (-1)^n n x^{n+1}$ $\qquad q = -x$

$$\sum_{n=1}^{\infty} (-1)^n x^n = \frac{-x-1+1}{1+x} \quad \Big| \frac{d}{dx}$$

$$\sum_{n=1}^{\infty} (-1)^n n x^{n-1} = \frac{-(1+x)+x}{(1+x)^2} = \frac{-1}{(1+x)^2} \quad \Big| \cdot x^2$$

$$\sum_{n=1}^{\infty} (-1)^n n x^{n+1} = \boxed{-\frac{x^2}{(1+x)^2}}$$

(13) $$\sum_{n=2}^{\infty} (2n+1) n \cdot x^{2n-1}$$

$$x^4 + x^6 + x^8 + \dots$$

$$\sum_{n=2}^{\infty} x^{2n} = \frac{x^4}{1-x^2} \quad \Big| \frac{d}{dx}$$

$$\sum_{n=2}^{\infty} 2 n x^{2n} = \frac{4x^3(1-x^2) + 2x \cdot x^4}{(1-x^2)^2} \quad \Big| : 2$$

$$\sum_{n=2}^{\infty} n x^{2n+1} = \frac{x^3(4-4x^2+2x^2)}{2(1-x^2)^2} = \frac{x^{\cancel{3}\,4}(4-2x^2)}{2(1-x^2)^2} \quad \Big| \frac{d}{dx}$$

$$\sum_{n=2}^{\infty} (2n+1) n x^{2n-1} = \boxed{\frac{4x^{\cancel{3}\,2}(4-2x^2) + x^{\cancel{4}\,3} \cdot (-4x) 2 (1-x^2)^2 - 2x^{\cancel{4}\,3}(4-2x^2) 2 (1-x^2) \cdot (-2x)}{4(1-x^2)^4}}$$