De Morganovy vzorce

(1) $\overline{\bigcup_{n=1}^{\infty} A_n} = \bigcap_{n=1}^{\infty} \overline{A_n}$ (2) $\overline{\bigcap_{n=1}^{\infty} A_n} = \bigcup_{n=1}^{\infty} \overline{A_n}$

D.: (1) $w \in \overline{\bigcup_{n=1}^{\infty} A_n} \Leftrightarrow w \notin \bigcup_{n=1}^{\infty} A_n \Leftrightarrow \forall n \; w \notin A_n \Leftrightarrow w \in \overline{A_n} \; \forall n$

$\Leftrightarrow w \in \bigcap_{n=1}^{\infty} \overline{A_n}$

D.V.2. 1) $\emptyset \in \mathcal{A}$; víme $\Omega \in \mathcal{A} \overset{(iii)}{\Rightarrow} \overline{\Omega} = \emptyset \in \mathcal{A}$

2) $A_1, A_2 \in \mathcal{A}$; $A_1 = A_1$, $A_2 = A_2$, $A_3 = A_4 = \dots = A_2 = \bigcup_{n=1}^{\infty} A_n = A_1 \cup A_2 \in \mathcal{A}$

$A_1 \cap A_2 = \overline{\overline{A_1 \cap A_2}} = \overline{\underbrace{\overline{A_1} \cup \overline{A_2}}_{\in \mathcal{A}}} \in \mathcal{A}$

$A_1 - A_2 = A_1 \cap \overline{A_2} \in \mathcal{A}$

3) $\bigcap_{n=1}^{\infty} A_n = \overline{\bigcup_{n=1}^{\infty} \overline{A_n}} \in \mathcal{A}$

$$\liminf_{n\to\infty} A_n \subseteq \limsup_{n\to\infty} A_n$$

<u>Př.</u> $\Omega = \{0,1\}$

$A_1 = \{0\}$

$A_2 = \{1\}$

$A_3 = \{0\}$

$A_4 = \{1\}$

$\vdots$

$\limsup_{n\to\infty} A_n = \{0,1\}$

$\liminf_{n\to\infty} A_n = \emptyset$

<u>D.Vě.</u> (1) $\omega \in \liminf_{n\to\infty} A_n \Leftrightarrow \exists n \ \omega \in A_k, k \geq n \Leftrightarrow \exists n \ \omega \in \bigcap_{k=n}^{\infty} A_k$

$\Leftrightarrow \omega \in \bigcup_{n=1}^{\infty} \bigcap_{k=n}^{\infty} A_k$

(2) $\omega \in \limsup_{n\to\infty} A_n \Leftrightarrow \forall n \ \exists k \geq n \ \omega \in A_k \Leftrightarrow \forall n \ \omega \in \bigcup_{k=n}^{\infty} A_k$

$\Leftrightarrow \omega \in \bigcap_{n=1}^{\infty} \bigcup_{k=n}^{\infty} A_k$

(3) $\overline{\limsup_{n\to\infty} A_n} = \overline{\bigcap_{n=1}^{\infty}\bigcup_{k=n}^{\infty} A_k} = \bigcup_{n=1}^{\infty}\overline{\bigcup_{k=n}^{\infty} A_k} = \bigcup_{n=1}^{\infty}\bigcap_{k=n}^{\infty}\overline{A_k} = \liminf_{n\to\infty}\overline{A_n}$

<u>D.V.6.</u> $A = \lim_{n\to\infty} A_n = \bigcup_{n=1}^{\infty}\bigcap_{k=n}^{\infty} \underbrace{A_k}_{\in \mathcal{A}} \in \mathcal{A}$

<u>D.V.7.</u> 1) $\ldots \subseteq A_n \subseteq A_{n+1} \subseteq \ldots$

<u>Horní limita:</u> $\limsup_{n\to\infty} A_n = \bigcap_{n=1}^{\infty} \underbrace{\bigcup_{k=n}^{\infty} A_k}_{B_n} = \bigcap_{n=1}^{\infty} B_1 = B_1 = \boxed{\bigcup_{k=1}^{\infty} A_k}$

$B_1 = B_2 = B_3 = \ldots$

<u>Dolní limita:</u> $\liminf_{n\to\infty} A_n = \bigcup_{n=1}^{\infty} \underbrace{\bigcap_{k=n}^{\infty} A_k}_{C_n} = \boxed{\bigcup_{n=1}^{\infty} A_n}$

$C_1 = A_1 \quad C_2 = A_2 \quad C_3 = A_3$

2) $\ldots \supseteq A_n \supseteq A_{n+1} \supseteq \ldots$

$\ldots \subseteq \overline{A_n} \subseteq \overline{A_{n+1}} \subseteq \ldots$ de Morgan