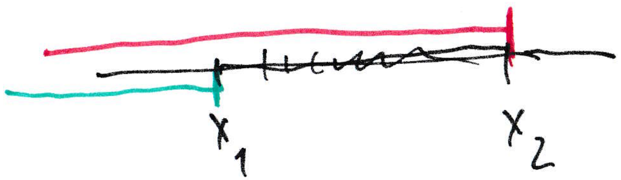
D.V.6.

(7) $P(x_1 < X \le x_2) = P(\{\omega \in \Omega;\ X(\omega) \in (x_1, x_2\rangle\})$

$\underbrace{x_1 < x_2}$



$$= P(\{\omega \in \Omega;\ X(\omega) \in (-\infty, x_2\rangle - (-\infty, x_1\rangle\})$$

$$= P(\{X \le x_2\} - \{X \le x_1\}) \quad \Big| \ \{X \le x_1\} \subseteq \{X \le x_2\}$$

$$\{\omega \in \Omega;\ X(\omega) \le x_1\}$$

$$= P(X \le x_2) - P(X \le x_1)$$

$$= F(x_2) - F(x_1).$$

(1) $F(x_2) - F(x_1) \overset{(7)}{=} P(x_1 < X \le x_2) \ge 0 \Rightarrow F$ nekles.

(2) $x_1 \ge x_2 \ge x_3 \ge \dots \ge x_0$; $\lim\limits_{n\to\infty} x_n = x_0$

$$\{X \le x_1\} \supseteq \{X \le x_2\} \supseteq \{X \le x_3\} \supseteq \dots$$

$$\{X \le x_0\} = \bigcap_{n=1}^{\infty} \{X \le x_n\} = \lim_{n\to\infty} \{X \le x_n\}$$

$$\lim_{n\to\infty} F(x_n) = \lim_{n\to\infty} P(X \le x_n) = P(\lim_{n\to\infty} \{X \le x_n\}) = P(X \le x_0) = F(x_0)$$

(3) $x_1 < x_2 < \ldots\ldots \qquad \lim_{n\to\infty} x_n = \infty$

$$\{X \le x_1\} \subseteq \{X \le x_2\} \ldots \qquad \lim_{n\to\infty} \{X \le x_n\} = \bigcup_{n=1}^{\infty} \{X \le x_n\} = \Omega$$

$$\lim_{n\to\infty} F(x_n) = \lim_{n\to\infty} P(X \le x_n) = P\left(\bigcup_{n=1}^{\infty} \{X \le x_n\}\right) = P(\Omega) = 1$$

$x_1 > x_2 > \ldots\ldots \qquad \lim_{n\to\infty} x_n = -\infty$

$$\{X \le x_1\} \supseteq \{X \le x_2\} \supseteq \ldots \qquad \lim_{n\to\infty} \{X \le x_n\} = \bigcap_{n=1}^{\infty} \{X \le x_n\} = \emptyset$$

(4) $F(x) = P(X \le x)$

(5) $P(X = x) \qquad x_1 < x_2 < \ldots ; \quad \lim_{n\to\infty} x_n = x$

$$\{x_n < X \le x\} \supseteq \{x_{n+1} < X \le x\} \supseteq \ldots \qquad \lim_{n\to\infty} \{x_n < X \le x\} = \bigcap_{n=1}^{\infty} \{x_n < X \le x\} = \{X = x\}$$

$$P(X = x) = P\left(\lim_{n\to\infty} \{x_n < X \le x\}\right) = \lim_{n\to\infty} P(x_n < X \le x)$$

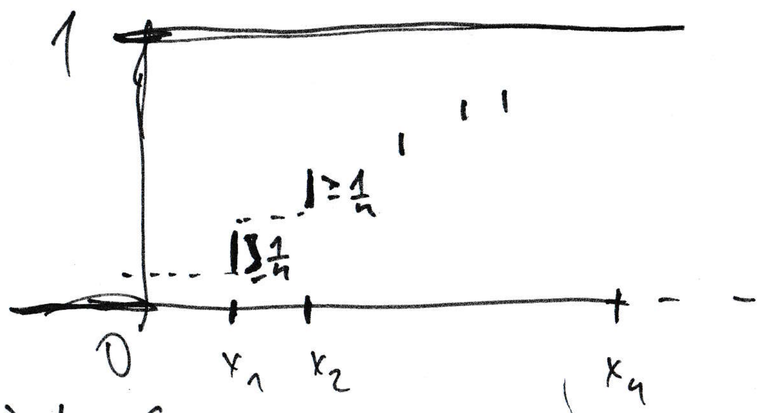
$$= \lim_{n\to\infty} [F(x) - F(x_n)] = F(x) - \lim_{n\to\infty} F(x_n)$$

$$= F(x) - \lim_{y\to x^-} F(y)$$

(6) $C_n = \{x \in \mathbb{R};\ P(X=x) \geq \frac{1}{n}\}$

$|C_n| \leq n$

$n \in \mathbb{N}$ $\quad C = \bigcup_{n=1}^{\infty} C_n$ ← nejvýše spočetné



D.V.8. μ_F je pst na $(\mathbb{R}, \mathcal{B})$?

1) $\mu_F(\mathbb{R}) \overset{?}{=} 1$

$$\mu_F((-\infty,\infty)) = \mu_F((-\infty; x_0\rangle \cup (x_0,\infty)) =$$

$x_0 \in \mathbb{R}$

$$= \mu_F((-\infty; x_0\rangle) + \mu_F((x_0,\infty)) = \cancel{F(x_0)} - F(-\infty) + F(\infty) - \cancel{F(x_0)}$$

$$= -0 + 1 = 1$$

2) $\mu_F(B) \geq 0 \quad \forall B \in \mathcal{B}$ $\qquad a_i < b_i$

$$\mu_F(B) = \mu_F\left(\bigcup_{i=1}^{\infty} (a_i, b_i\rangle\right) = \sum_{i=1}^{\infty} \mu_F((a_i, b_i\rangle) = \sum_{i=1}^{\infty} \underbrace{F(b_i) - F(a_i)}_{\geq 0} \geq 0$$

3) $B_1, \dots, B_n, \dots \in \mathbb{B}$, $B_i \cap B_j = \emptyset$ $i \neq j$

$$\mu_F\left(\bigcup_{i=1}^{\infty} B_i\right) = \mu_F\left(\bigcup_{i=1}^{\infty} \bigcup_{j=1}^{\infty} (a_{ij}, b_{ij}\rangle\right) = \sum_{i=1}^{\infty} \underbrace{\sum_{j=1}^{\infty} F(b_{ij}) - F(a_{ij})}$$

$$= \sum_{i=1}^{\infty} \mu_F(B_i)$$