$$f(\mathbf{x}) = (2\pi)^{-\frac{n}{2}} |\Sigma|^{-\frac{1}{2}} e^{-\frac{1}{2}(\mathbf{x}-\boldsymbol{\mu})^T \Sigma^{-1} (\mathbf{x}-\boldsymbol{\mu})} \quad ; \quad n=2$$

$$\Rightarrow f(x_1,x_2) = \frac{1}{2\pi\sigma_1\sigma_2\sqrt{1-\rho^2}} \cdot e^{-\frac{1}{2(1-\rho^2)}} \cdot [\,\cdot\,/\,\cdot\,]$$

$$\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \quad \boldsymbol{\mu} = \begin{pmatrix} \mu_1 \\ \mu_2 \end{pmatrix} \quad \Sigma = \begin{pmatrix} \sigma_1^2 & \rho\sigma_1\sigma_2 \\ \rho\sigma_1\sigma_2 & \sigma_2^2 \end{pmatrix}$$

$$|\Sigma| = \sigma_1^2\sigma_2^2 - \rho^2\sigma_1^2\sigma_2^2 = \underline{\sigma_1^2\sigma_2^2(1-\rho^2)} \Rightarrow |\Sigma|^{-\frac{1}{2}} = \frac{1}{\sigma_1\sigma_2\sqrt{1-\rho^2}}$$

$$\Sigma^{-1} = \begin{pmatrix} \sigma_2^2 & -\rho\sigma_1\sigma_2 \\ -\rho\sigma_1\sigma_2 & \sigma_1^2 \end{pmatrix} \cdot \frac{1}{\sigma_1^2\sigma_2^2(1-\rho^2)}$$

$$(\mathbf{x}-\boldsymbol{\mu})^T \Sigma^{-1} (\mathbf{x}-\boldsymbol{\mu}) = (x_1-\mu_1 ; x_2-\mu_2) \cdot \begin{pmatrix} \sigma_2^2 & -\rho\sigma_1\sigma_2 \\ -\rho\sigma_1\sigma_2 & \sigma_1^2 \end{pmatrix} \cdot \begin{pmatrix} x_1-\mu_1 \\ x_2-\mu_2 \end{pmatrix} \cdot \frac{1}{\sigma_1^2\sigma_2^2(1-\rho^2)}$$

$$= (x_1-\mu_1 ; x_2-\mu_2) \cdot \begin{pmatrix} \sigma_2^2(x_1-\mu_1) - \rho\sigma_1\sigma_2(x_2-\mu_2) \\ \sigma_1^2(x_2-\mu_2) - \rho\sigma_1\sigma_2(x_1-\mu_1) \end{pmatrix} \cdot \frac{1}{\sigma_1^2\sigma_2^2(1-\rho^2)}$$

$$= \underline{\sigma_2^2(x_1-\mu_1)^2} - \rho\sigma_1\sigma_2(x_1-\mu_1)(x_2-\mu_2) + \underline{\sigma_1^2(x_2-\mu_2)^2} - \rho\sigma_1\sigma_2(x_1-\mu_1)(x_2-\mu_2) \cdot \frac{1}{\sigma_1^2\sigma_2^2(1-\rho^2)}$$

$$= \frac{1}{1-\rho^2} \cdot \left[\left(\frac{x_1-\mu_1}{\sigma_1}\right)^2 - 2\rho\frac{x_1-\mu_1}{\sigma_1} \cdot \frac{x_2-\mu_2}{\sigma_2} + \left(\frac{x_2-\mu_2}{\sigma_2}\right)^2 \right]$$