

M6140 Topology Exercises - 7th Week (2020)

1 Fundamental Groups

Exercise 1. Suppose that $x \in A \subseteq X$, where A is a connected component of a topological space X . Prove that $\pi_1(A, x) \cong \pi_1(X, x)$.

Exercise 2. Show that the fundamental group at each point of a discrete space is trivial.

Exercise 3. Prove that the fundamental group at each point of an indiscrete space is trivial.

Exercise 4. Show that each star-shaped set is simply connected.

Exercise 5. Prove that if a two-point space is path-connected, then it is simply connected.

Exercise 6. Show that for each topological group G the set of loops in G based at the neutral element is a group.

Exercise 7. The binary group operation from the previous exercise induces a binary operation on $\pi_1(G, 1)$. Prove that this binary operation coincides with the binary group operation $*$ on $\pi_1(G, 1)$.

Exercise 8. Show that the fundamental group $\pi_1(G, 1)$ is abelian for each topological group G .