The 1804 examination for the chair of Elementary Mathematics at the University of Prague

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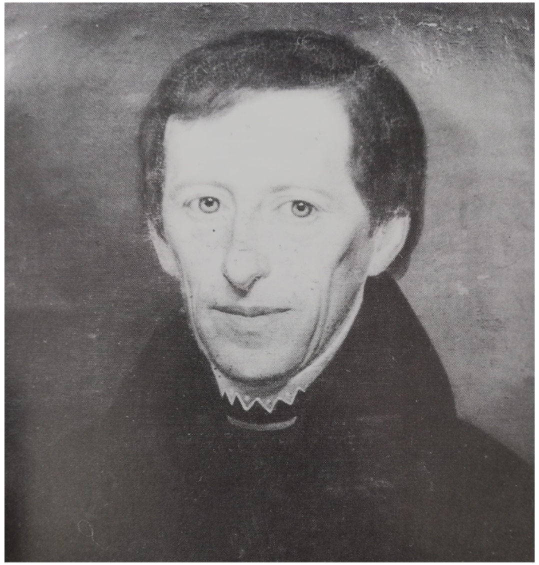
History of mathematics in the Czech lands: from the Jesuit teaching to Bernard Bolzano



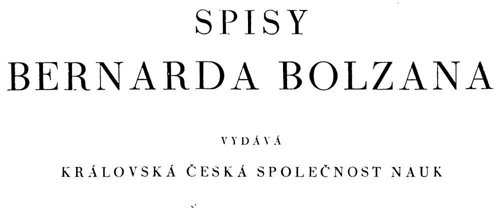
Bolzano’s mathematical works

- Praise for his early (1804-10) “ideas and innovations in methodology” (Russ).
- “Epoch-making [1816-17 works] on the foundations of real analysis” (Ewald).
- *Reine Zahlenlehre* (1830s) is “a surprising achievement for its time” (Russ).
- He has an “imputed role as pathfinder for the later work on sets by Cantor and Dedekind” (Simons).

The general impression is that we “can only speculate on the direction the history of mathematics might have taken” if he had become professor of mathematics and that he remained “isolated” (Rusnock and Šebestík).

“a man who had a great influence on the occupancy of these chairs, and because he was fond of me, sent for me especially to ask me which of the two I would prefer[;] I replied that I could hope with some confidence to be of more meaningful use by lecturing on Religion than it would ever be possible [for me to be] by lecturing on Mathematics.” (Bolzano, 1836)

The 1804 *Concursprüfung* and Bolzano's early manuscripts constitute valuable sources of information on his knowledge of mathematics at the time and his early mathematical practices.



SPISY
BERNARDA BOLZANA
VYDÁVÁ
KRÁLOVSKÁ ČESKÁ SPOLEČNOST NAUK

Our research sheds light on the mathematical practices in the teaching and learning of mathematics in Prague that were common at the turn of the 19th century, as well as on the institutional context within which Bolzano lived.

Bolzano’s education

1784: teaching in “all educational institutions above elementary level” should be in German (Armour); at gymnasia and universities, mathematics, among other subjects, was emphasised.

1791-1796: Roman Catholic Piarist gymnasium in Prague’s New Town.

- The Piarists cooperated in the school reform after the Pope’s 1773 suppression of the Society of Jesus, which until then had played a key role in education throughout much of Catholic Europe.

1796: Faculty of Philosophy (three-year studies for a decade; plus 1 year).

1800-1804: Faculty of Theology.

Three chairs and their context

In 1804 the chair of Elementary Mathematics at the University of Prague (Philosophy) fell vacant and two new chairs of Religious Doctrine were established in Prague, one there and one at the Old Town gymnasium.

The chairs of Religious Doctrine were part of the Holy Roman Emperor Franz II's efforts "to educate obedient citizens of the State" and avoid replication of the French Revolution: "I do not need scholars, but obedient citizens. It is your obligation thus to form our youth" (Rusnock & Šebestík).

Theology — Medicine — Law

Philosophy
(propaedeutic studies; mathematics)

Mathematics in Prague

During the last decades of the 18th century the number of hours devoted to natural sciences and mathematics increased and new chairs were created in fields which were considered apt to play a key role in the modernisation of Bohemia, such as physical geography, astronomy, higher mathematics and practical and applied mathematical sciences (Tomek).

In 1804 there were 3 chairs in mathematics:

- Elementary Mathematics: Stanislav Vydra (since 1772).
- Practical Mathematics: Franz Anton Herget (since 1784).
- Higher Mathematics: Franz Joseph Gerstner (since 1788-89).

Teaching of mathematics c. 1800

	Mathematical subjects	
Math chairs	Pure	Applied
Elementary	Arithmetic, geometry, trigonometry. From Kästner's *Anfangsgründe*.	Mechanics, optics, astronomy. Mainly from Kästner's *Anfangsgründe*.
Higher	Introduction to the analysis of finite and infinite quantities, and differential and integral calculus (1st year). From Euler.	Higher mechanics and hydraulics with application to mechanical engineering (2nd year); optics and theoretical astronomy, with application to gnomonics, chronology, geography and nautics (3rd year). From Karsten and de la Lande.
Practical	Various arts and crafts. Land-surveying, practical arithmetic, nautical arts, civil and hydraulic engineering, etc.	

Elementary Mathematics

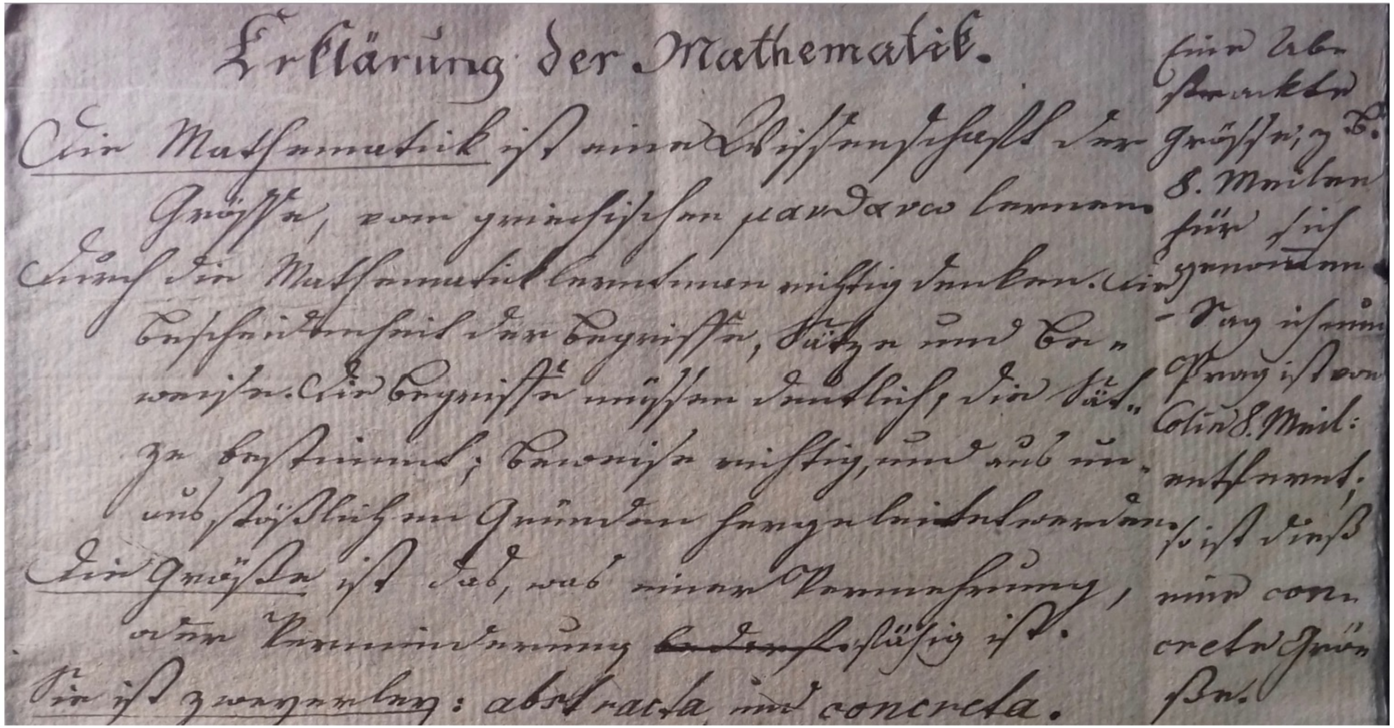
He was one of the few professors who, despite being members of the Jesuit Order, were not dismissed after its suppression.

- Among other things, he wrote *Historia Matheseos In Bohemia Et Moravia Cvltae* (1778) and *Počátkowé Arytmetyky* (1806), the first published textbook of elementary mathematics written in Czech.

He followed Kästner's work (imperial order of 1784) but not in a strict manner, as Bolzano's notes on his lectures reveal.

- E.g., he began with a general introduction to mathematics similar to that by Kästner, but included a section in Napier's bones (not in Kästner's).

Notes by Bolzano



Erklärung der Mathematik.

Die Mathematik ist eine Wissenschaft der Größe, [illegible] durch die Mathematik [illegible] richtig denken. Die Beschaffenheit der Begriffe, Sätze und Beweise. Die Begriffe müssen deutlich, die Sätze zu bestimmt; Beweise richtig, und aus ur[illegible] Gründen hergeleitet werden.

Die Größe ist das, was einer Vermehrung oder Verminderung fähig ist.

Sie ist zweyerley: abstracta und concreta.

[illegible]

“*Anfangsgründe*”

Anfangsgründe
der
angewandten
Mathematik

abgefaßt
von
Abraham Gotthelf Kästner
Der Math. und Naturl. Prof. der Kön. Ges. der Wiss. zu Göttingen; der Kön. Schwed. und Preuss. Akad. der Wiss. der Erfurtischen Churf. Ges. nützl. Wiss. des Bononischen Instituts; der Perusinischen Academiae Augustae, der Jenaischen latein. und teutschen und der Leipziger deutschen Gesellschaft, und Ges. der freyen Künste Mitgliede.

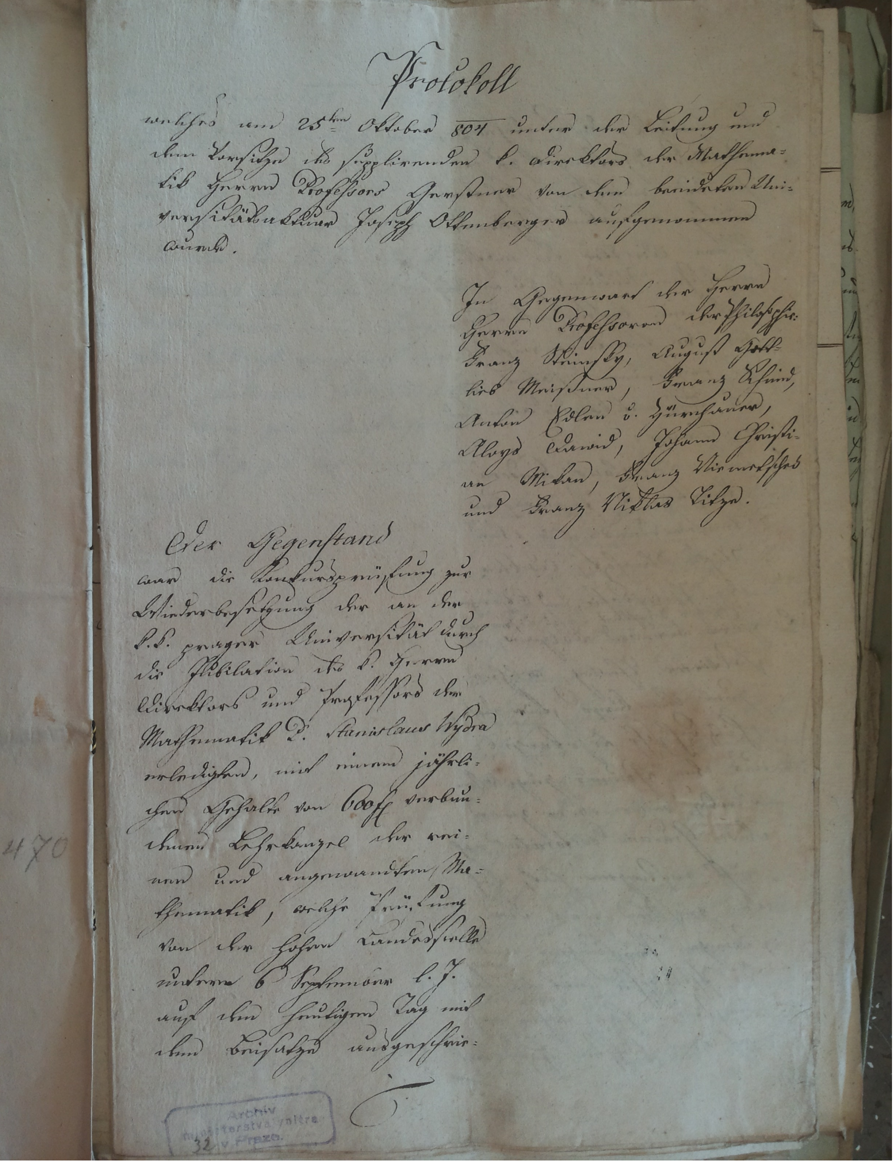
Der mathematischen Anfangsgründe
zweyter Theil.

Göttingen
In Verlag der Wittwe Vandenhoek
1759.

The learner should start from the foundations and go upwards (elementary vs higher)

Mathematical sciences should be divided according to their object (pure/applied).

“Kästner proved that which is generally passed over because everybody already knows it. That is, he attempted to make clear to the reader the reasons upon which his judgment rested – this I liked the best of all.” (Rusnock and Šebestík; Bolzano, 1836)



The 1804 *Concurs*

Two candidates took part in a written and oral examination for the chair of elementary maths: Bernard Bolzano and Ladislav Joseph Jandera (1776-1857).

Written exam 25th October 1804, 9am till the afternoon.

Oral exam: 26th October 1804, half an hour (“trial lesson”?)

Committee and questions

Examination board: F. A. Steinský (Diplomatics, Heraldry, Antiquity Studies and Numismatics), F. Schmidt (Natural Sciences), A. David (Practical Astronomy), A. G. Meißner (Classical Ancient Authors and Aesthetics), A. von Zürchauer (Economics), J. K. Mikan (General Natural History and Technology), F. N. Titze (General World History) and F. Něměcek (Theoretical and Practical Philosophy), J. F. Gerstner (higher mathematics and director of the philosophical faculty for the math. and phys. studies).

Nine questions submitted to a higher commission, 3 were chosen and opened during the examination.

The 1804 *Concurs*: Questions

I. Calculate both the surface area and the volume of a spherical cap from its given height and the diameter of the sphere.

II. What proofs are known for the theorem on the equilibrium of the lever? And what does in particular speak for or against each of them?

III. At what speed does the water flow from small outlets of large vessels? And how can this speed be established on a theoretical basis?

I. Aus der gegebenen Höhe des Abschnittes und dem Durchmesser der Kugel sowohl seine Oberfläche als auch seine kubischen Inhalt zu berechnen.

II. Welche Beweise sind für den Satz über des Gleichgewicht am Hebel bekannt ? Und was hat jeder Vorziegliches für oder gegen sich ?

III. Mit welche Geschwindigkeit fliesst das Wasser aus kleinen Oeffnungen großer Behälter?Und wie kann diese Geschwindigkeit aus theoretischen Gründen dargethan werden ?

The 1804 *Concurs*: Oral question

(on 26th October) “Shall the 14 proposition from Kästners *Geometrie* be chosen, that they should explain orally: “parallelograms included between the same parallels and having the same base have equal area” (Gerstner, *Protokoll*)

Expectations from the committee

"The objects of these questions were chosen in such as way that their answers would not only prove a complete familiarity with the adopted textbook, but also be the occasion [for the candidate] to distinguish himself thanks to his superior knowledge (*durch höhere Kenntnisse aus zuzeichnen*)."

Gerstner, Protokoll 38039/3376

Bolzano: Question 1

How can one calculate a curved surface, or a plane one bounded by curved lines, as well as a solid confined within such a surface?

Archimedes formulated postulates that should not be placed among the "Grundsatzen".

Infinitesimals remove the difficulty "for the eye, but not for the understanding."

Use of Lagrange's method that has the advantage of brevity, uniformity and generality: it applies algebraic methods to geometric investigations.

Theorem I

"If the quantity M should always lie within the two quantities N and $N + iP$, however small i is taken, then it must be [that] $M = N$."

The theorem is not Lagrange's, but uses Lagrange's symbolism.

Let $M - N > 0$, and $iP - (M - N) \geq 0$.

Let $M - N = n^2$ (a positive quantity)

Then $iP - n^2 \geq 0$ (for every i)

Hence $i \geq \frac{n^2}{P}$, against the assumption.

Bolzano gives general theorems about arc-length, area and volumes (also in Lagrange), and applies them to the case presented in the problem. For exemple:

Theorem II

"An arc of a curved line, for which the equation between the orthogonal coordinates is $y = fx$, is a primitive function of the abscissa x, whose first derivative function is expressed by the formula

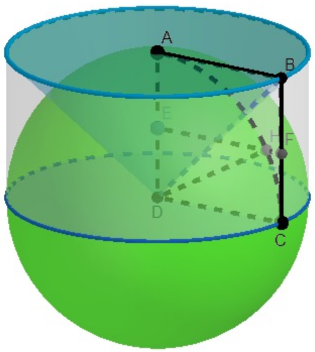
$$F'x = [1 + f'(x)^2]^{\frac{1}{2}}$$

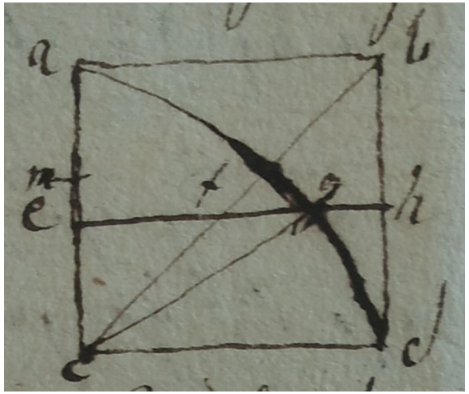
Positive evaluation of Bolzano's approach

"He showed the difficulties, that are inherent to the postulates of Archimedes and the recent differential calculus, and thus chose the newest, and most solid of all know methods (*die neunste und strengste unter allen bekanntne Methoden*), that is the theory of functions of Lagrange, and by it he solved completely and correctly the given problems"

(Protokoll B - written exam)

Question 1: Jandera





Jandera's solution reproduces Kästner's method and order of propositions **("Methoden und Ordnung wie Kästner in dem vorgeschriebenen Lehrbuche behandelt"), consisting in dissecting an area or surface into infinitesimal strips. Jandera makes a mistake ("den Inhalt des ganzen Abschnitts unrichtig berechnet hat")!**

“This would be a simple way, among the various possible ones, to respond to this question. The simplest would be through differential calculus (I have intentionally refrained myself from using this last method, **because it did not seem to me adequate for the present circumstances**), although I think it will be a useless digression.”

"Es wäre ein Leichtes, die vorgelegte Frage auf mehreren ander[e]n Wegen zu beantworten, am leichtesten durch die Differenzialrechung, (mit Vorsatz vermied ich diesen letzeren Weg, weil er mir unter diesen Umständen nicht zweckmässig schien), doch ich glaube, es wäre eine nutzenlose Weitläufigkeit."

Opinion about the oral examination

"The unanimous opinion of the Professors was as follows: that although the first one, namely **Bernard Bolzano, went deeper into the subject matter, he deviated somewhat from Kästner** in the order of the assumptions by sending the theory of the similarity of triangles on ahead, which in [the work of] Kästner is addressed later. **However, the other one, namely Mr. L. Jandera, stood out due to a clearer analysis and a more comprehensible lecture for the students, maintained the order of Kästner and not only exhaustively proved that proposition in the same period of time, but also drew from it a few subsequent propositions**."

*CG Publicum*1796-1805 98/755, *Protokoll* A

Summary Report

Bolzano's talent:
"Thorough reflection and erudition"

Jandera's talent:
multiplicity of topics (probably referred to the oral question), respect of the order, a stronger voice

"er [Bolzano] mehr durch gründliche Behandlung und Erudition, als durch Vielheit der Gegenstände, und ängstliche Ordungsabmessung sein Talent hat geltend machen wollen"

Gerstner: it was “regrettable” that there was no chair available in Higher Mathematics in addition to that of Elementary Mathematics, **“the unanimous testimony of all the professors” would be to appoint Bolzano to the former and Jandera to the latter**.

The "official" story

6/12/1804: Final decision of the commission: Bolzano was recommended for the chair, but Jandera should not be completely ruled out, as he had shown a "great deal of knowledge of mathematics" and a familiarity with the chair due to the couple of years he had spent as Vydra's assistant.

Administrative motivations: Vienna urgently needed a teacher of religion, and Bolzano, who had taken part in the examination for the chair of religion was the best, and only available candidate.

I. Univerſitäten und andere öffentliche Lehranſtalten.

Prag.

Am 19 April wurde Hr. *Bernhard Bolzano*, von Kronſtätten, zum Katecheten für die Zuhörer der Philoſophie feyerlich im Collegio Clementino inſtallirt. Der Director des philoſ. Studiums, Hr. Prof. *Ignaz Sinke*, hielt dabey eine Rede, worin er zeigte, welche Vortheile daraus entſprängen, wenn die ſtudirende Jugend durch die Religionslehre immer gröſsere Fortſchritte in der Bildung des Herzens mache. Hierauf bewies Hr. *Bolzano* in ſeiner Antrittsrede die Nützlichkeit und Nothwendigkeit des Religionsunterrichts für die Zuhörer der Philoſophie.

Jenaische allgemeine literatur-zeitung: Intelligenzblatt, 1805