C7790 Introduction to Molecular Modelling
TSM Modelling Molecular Structures

Lesson 18
Quantum chemistry II

PS/2020 Distant Form of Teaching: Rev1

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Context

macroworld **microworld**

states

(thermodynamic properties, G, T,...)

phenomenological thermodynamics

equilibrium (equilibrium constant)
kinetics (rate constant)

free energy
(Gibbs/Helmholtz)

partition function

statistical thermodynamics

microstates

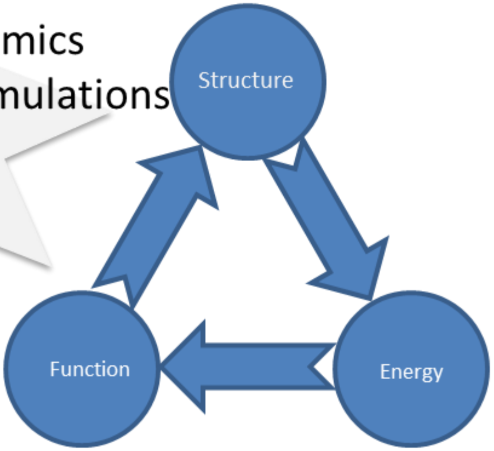
(mechanical properties, E)

Description levels (model chemistry):

- quantum mechanics
    - semiempirical methods
    - ab initio methods
    - post-HF methods
    - DFT methods
- molecular mechanics
- coarse-grained mechanics

Simulations:

- molecular dynamics
- Monte Carlo simulations
- docking
- ...



microstate ≠ microworld

Revision: Variational Theory

Search for a solution of SR using the variational method.

$$E_k = E[\Psi_k] = \frac{\int\limits_{\mathbf{\Omega}} \Psi_k^* \hat{H} \Psi_k d\boldsymbol{\tau}}{\int\limits_{\mathbf{\Omega}} \Psi_k^* \Psi_k d\boldsymbol{\tau}} = \min!$$

The wave function, which provides the minimum value of the integral, is a solution of the Schrödinger equation. **The global minimum of a functional is the energy of the ground state**, which implies:

$$\boxed{\begin{array}{c} \Psi \neq \Psi_0 \\ E > E_0 \end{array}}$$

The inaccurate wave function always provides a higher value of energy.

Revision: HF Method

One-electron approximation:

$$\Psi(r_1, r_2, \ldots, r_n, \sigma_1, \sigma_2, \ldots, \sigma_n) = \sum_P sign(P)\{\varphi_1(r_1)\chi_1(\sigma_1)\varphi_2(r_2)\chi_2(\sigma_2)\ldots\varphi_n(r_n)\chi_n(\sigma_n)\}$$

One-electron functions expressed using basis functions:

$$\varphi_i(\mathbf{r}_i) = \sum_{j=1}^{m} \boxed{c_{ij}}\,\chi_j(\mathbf{r}_i)$$

Roothan's equations lead to the solution of a generalized eigen-problem.

one-electron orbitals and their energies

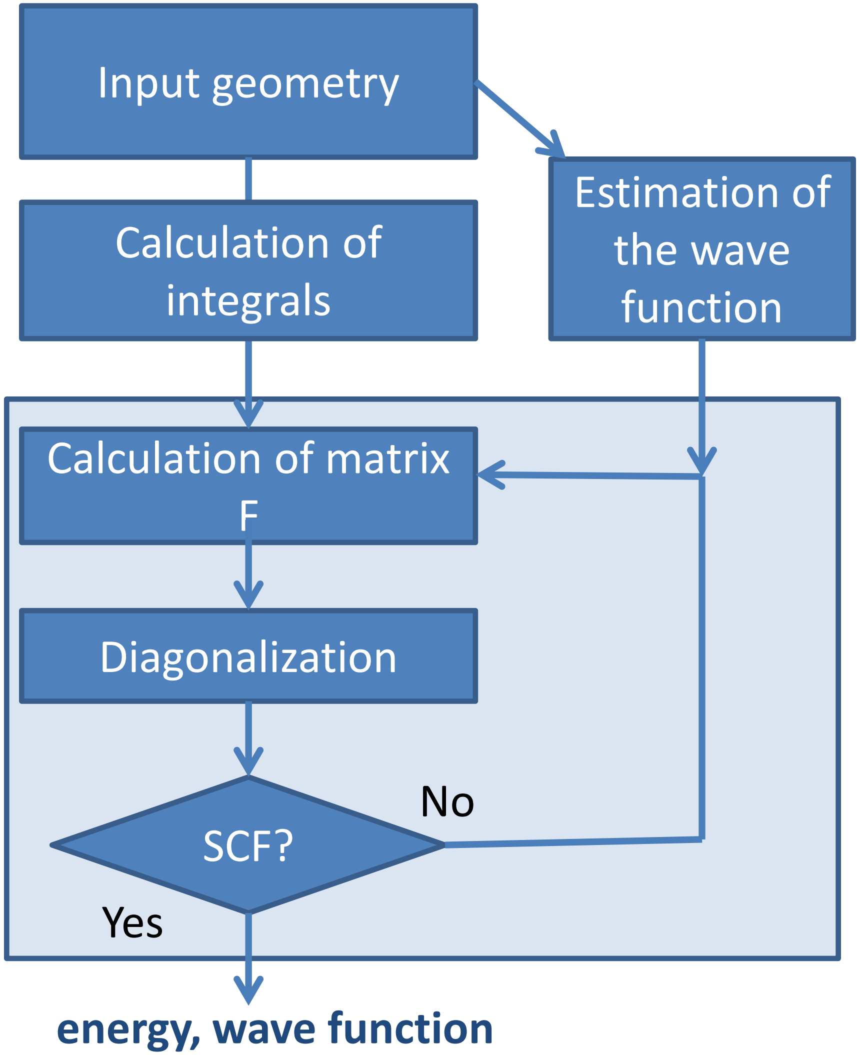
$$\mathbf{F}\mathbf{c}_i = \varepsilon_i \mathbf{S}\mathbf{c}_i$$

Minor problem: The Fock matrix (**F**) calculation requires the solution ($\mathbf{c}_i$) of the Roothan's equations => **SCF method**.

Revision: HF Method - SCF

HF method

- **solution using SCF (self consistent field)**



Convergence criteria:

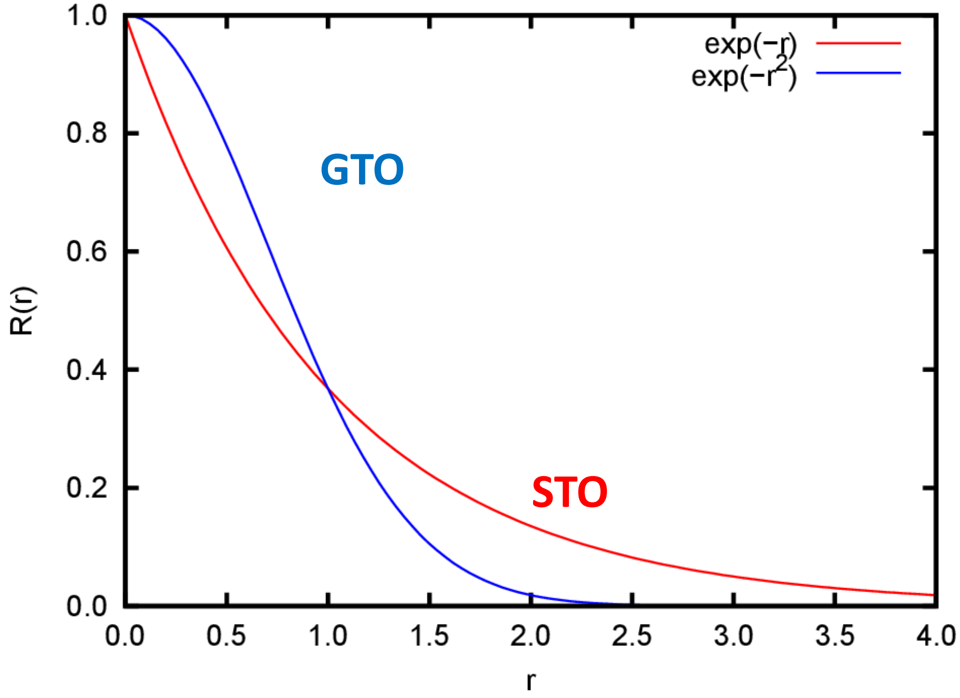
- total energy does not change
- wave function (development ***c*** coefficients does not change

Revision: Basis Sets

$$\varphi_i(\mathbf{r}_i) = \sum_{j=1}^{m} c_{ij} \boxed{\chi_j(\mathbf{r}_i)}$$

predefined basis functions (basis set)

Basis functions: comparison of STO (Slater Type Orbital) and GTO (Gaussian Type Orbitals)



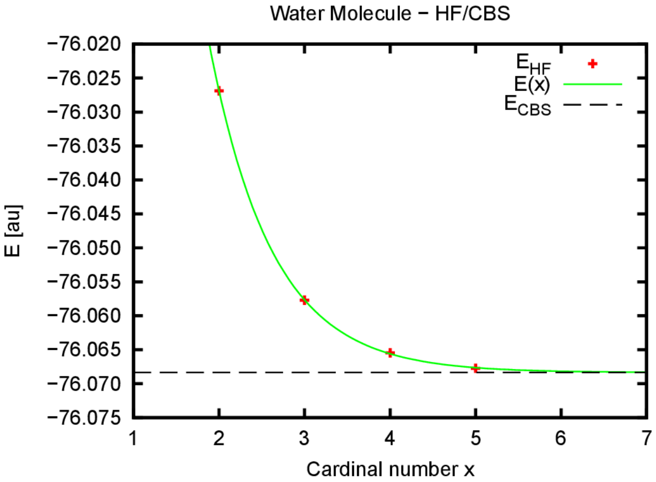
Disadvantage of GTO:

- they incorrectly describe situation near the core and for greater distances from the core.

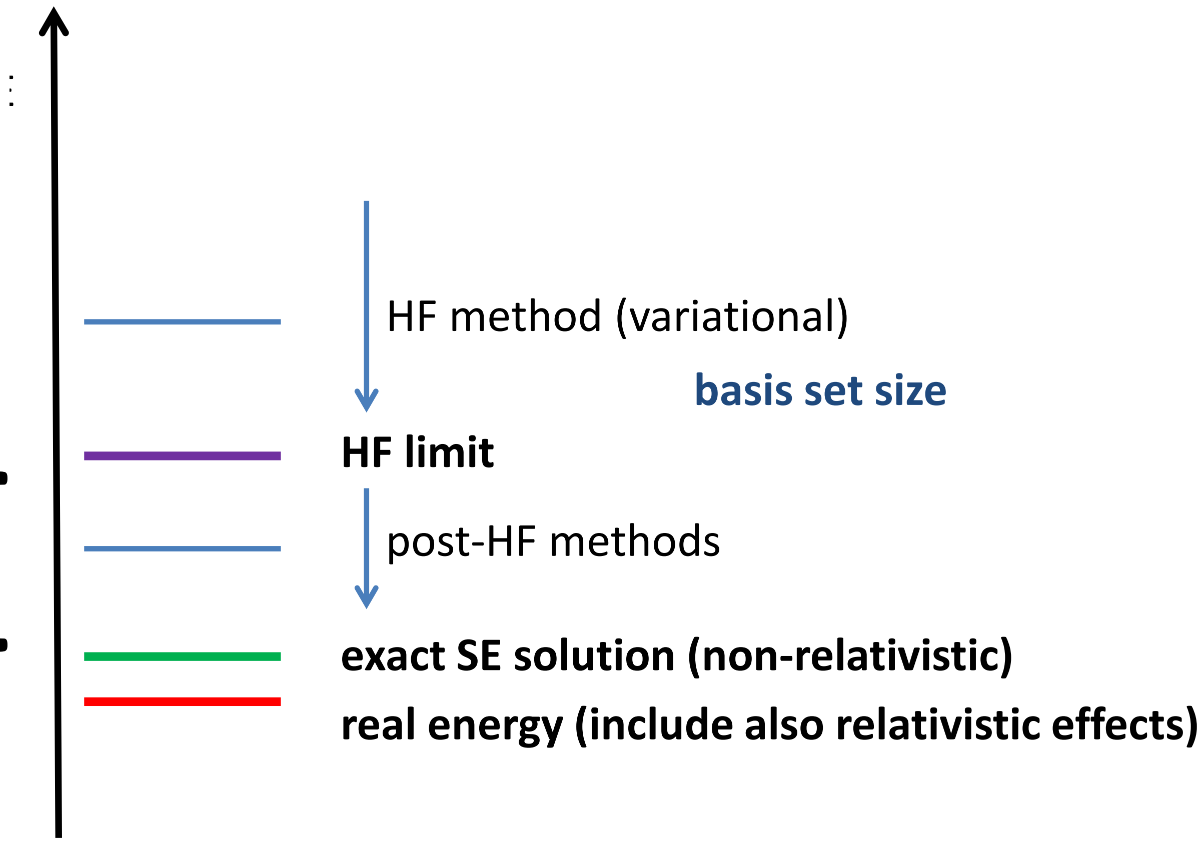
Advantage of GTO:

- more convenient numerical work
- simpler form of some integrals

Revision: Correlation energy



E_c



HF method (variational)

basis set size

HF limit

post-HF methods

exact SE solution (non-relativistic)

real energy (include also relativistic effects)

correlation energy – it is not included in the HF method **because of one-electron approximation**

correlation energy is always negative because electron repulsion is overestimated by the HF method

QM method overview

Classification by theoretical approaches and approximations:

- empirical methods
  - extended Hückel method (EHT)
  - ....
- semi-empirical methods
  - AM1
  - PM3, PM6, PM7
  - ...
- ***ab initio* methods**
  - Hartree-Fock (HF) method
  - post-HF methods
    - Møller-Plesset method (MP2, MP3, ...)
    - coupled-clusters method (CC)
    - ...
- **density functional theory (DFT)**
  - LDA
  - GGA (BLYP, TPSS, PBE, ...)
  - hybrid (B3LYP, M06-2X, ...)

they include the correlation energy

Software for QM calculations

Overview:
http://en.wikipedia.org/wiki/List_of_quantum_chemistry_and_solid-state_physics_software

Paid (commercial, academic license with fee):

- Gaussian (http: // www.gaussian.com/)
- Turbomole (http: // www.cosmologic.de/)
- ADF (http: // www.scm.com/)
- Schrödinger (http: // www.schrodinger.com/)
- Spartan (http: // www.wavefun.com/)
- Hyperchem (http: //www.hyper.com/)

Freely available (free licenses, academic licenses free of charge):

- mopac (http://openmopac.net/)
- orca (https://orcaforum.cec.mpg.de/)
- MPQC (http: // www.mpqc.org/)
- GAMESS-US (http: // www.msg.ameslab.gov/ GAMESS /)
- GAMESS-PC (http://classic.chem.msu.su/gran/gamess/index.html)
- cpmd (http: // www.cpmd.org/)
- cp2k (http://cp2k.berlios.de/)

System properties

Overview

Wave function

- population analysis
- electron density
- electrostatic potential
- electric multipole moments (monopole, dipole, quadrupole, ...)
- partial atomic charges
- magnetic properties of molecules (chemical shift, spin-spin interaction constant, ...)

PES curvature and wave function

- vibrations (IR and Raman transitions)

Wave functions of electronic states

- electronic transition (UV/VIS transitions)

Property calculation methods given here are general (they are in no way limited to the HF method). The quantum chemical method need only provide the wave function of the ground and possibly excited states and possibly the curvature of the potential energy surface.

Wave function population analysis

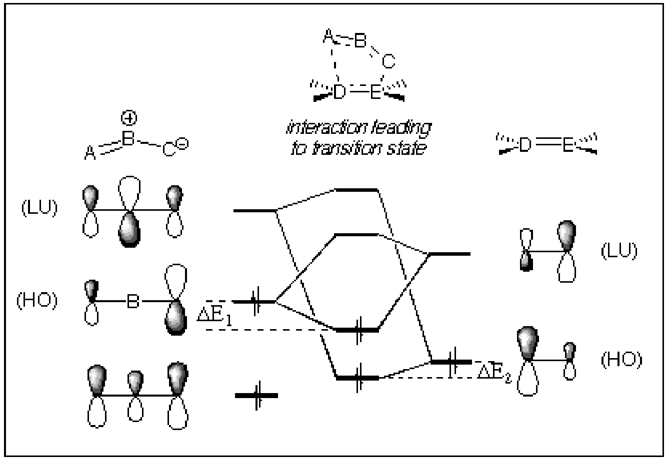
Population analysis is a way of studying the wave function that provides the following information:

- partial contributions of atomic orbitals to molecular orbitals, which is especially important for frontier orbitals, which are important for the assessment of reactivity
  - **HOMO** - highest occupied molecular orbital (nucleophilic properties)
  - **LUMO** - lowest occupied molecular orbital (electrophilic properties)
- visualization of molecular orbitals
- interaction diagrams
- description of bonds and their quantification (order of bonds)
- partial atomic charges

The most used types of population analyzes:

- **Mulliken population analysis (MPA)**
- **Natural population analysis (NPA)**
- **Hirshfeld population analysis**

More detailed description in specialized lectures.



http://euch6f.chem.emory.edu/13dipolar.html

Electron density

Electron density indicates the density of electrons at a point with coordinates x, y, z. It is calculated from **wave function**.

$$\rho(x,y,z) = \int_{\boldsymbol{\Omega}} \Psi^*(\mathbf{r}_1,\ldots,\mathbf{r}_n,\mathbf{R})\Psi(\mathbf{r}_1,\ldots,\mathbf{r}_n,\mathbf{R})d\boldsymbol{\tau}$$

$$d\boldsymbol{\tau} = d\mathbf{r}_2 \ldots d\mathbf{r}_n$$

it is integrated over all but one electron coordinate (electrons are indistinguishable particles)

The electron density can be employed in the calculation of **partial atomic charges**, it can be used to define the envelope of a molecule (**molecular surface**), etc.

The integral of the electron density over the whole space is equal to the number of electrons that are present in the system.

$$n = \int_{\boldsymbol{\Omega}} \rho(x,y,z)d\boldsymbol{\tau}$$

Electron density is an important property of the system and plays a central role in DFT methods (Density Functional Theory, density functional methods).

Electrostatic potential

Electrostatic potential indicates the value of the electrostatic potential at a point with coordinates x, y, z, which is caused by the electrostatic action of the atomic nuclei of the molecule and the effective field of electrons. It is calculated from **wave function**.

$$V(x,y,z)=\underbrace{\sum_{i=1}^{N}\frac{Z_i}{\left|\mathbf{r}_{xyz}-\mathbf{R}_i\right|}}_{\text{action of nuclei}}-\underbrace{\frac{\int_{\mathbf{\Omega}}\Psi^*(\mathbf{r},\mathbf{R})\hat{V}\Psi(\mathbf{r},\mathbf{R})d\boldsymbol{\tau}}{\int_{\mathbf{\Omega}}\Psi^*(\mathbf{r},\mathbf{R})\Psi(\mathbf{r},\mathbf{R})d\boldsymbol{\tau}}}_{\text{action of electrons}}$$

$$d\boldsymbol{\tau}=d\mathbf{r}_1 d\mathbf{r}_2 d\mathbf{r}_3 d\mathbf{r}_4 \ldots d\mathbf{r}_n$$

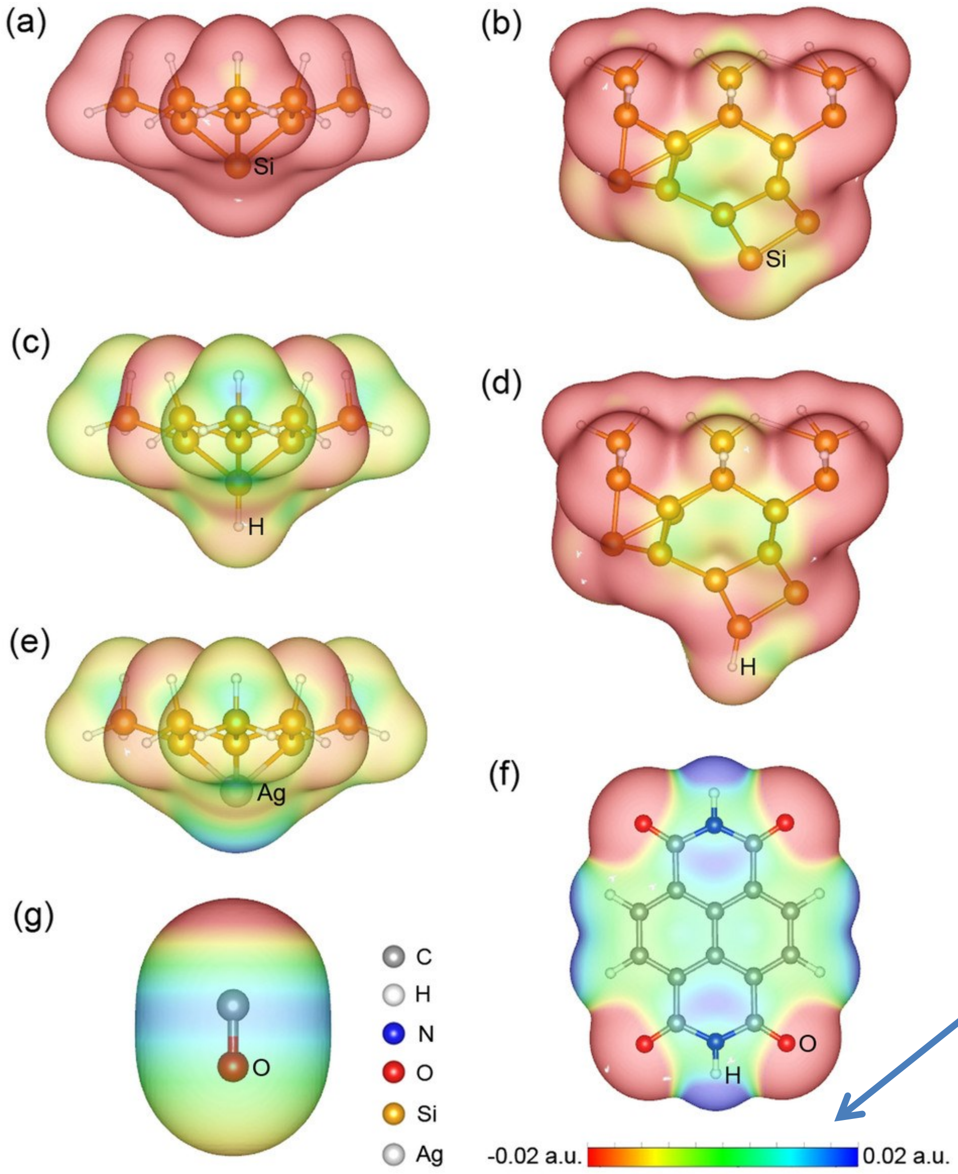
it integrates over all electrons and the entire space $\mathbf{\Omega}$

Electrostatic potential operator:

$$\hat{V}=\sum_{i=1}^{n}\frac{1}{\left|\mathbf{r}_{xyz}-\mathbf{r}_i\right|}$$

Electrostatic potential provides the assessment of **electrostatic properties of molecules** and can serve for the calculation of **partial atomic charges**.

Visualization of ρ and V



Electrostatic potential is mapped onto isosurface of electron density (molecular envelope).

convention in chemistry

http://www.nature.com/srep/2014/141020/srep06678/fig_tab/srep06678_F3.html

Electric multipole moments

Electric multipole moments are quantities describing **electric charge distribution** in a system. They are calculated from **wave function**.

Component of moment:

$$M_{x^k y^l z^m} = \underbrace{\sum_{i=1}^{N} Z_i R^k_{x,i} R^l_{y,i} R^m_{z,i}}_{\text{contribution of nuclei}} - \underbrace{\frac{\int_{\Omega} \Psi^*(\mathbf{r},\mathbf{R}) \widehat{Q} \Psi(\mathbf{r},\mathbf{R}) d\boldsymbol{\tau}}{\int_{\Omega} \Psi^*(\mathbf{r},\mathbf{R}) \Psi(\mathbf{r},\mathbf{R}) d\boldsymbol{\tau}}}_{\text{electron contribution}}$$

Operator of electric multipole:

$$\widehat{Q} = \sum_{i=1}^{n} r^k_{x,i} r^l_{y,i} r^m_{z,i}$$

The sum of k+l+m determines the type of multipole moment component:

- **monopole** (0) - scalar number, total charge of the molecule
- **dipole** (1) - vector (three components)
- **quadrupole** (2) - tensor (3x3 components)

Electric dipole moment

Electric dipole moment describes the asymmetrical distribution of the electric charge in the system. It is a vector quantity.

$$\boldsymbol{\mu} = \langle M_x, M_y, M_z \rangle \qquad \mu = \sqrt{M_x^2 + M_y^2 + M_z^2}$$

Size of the vector is independent of the position and rotation of the system of atoms only in electrically neutral systems!

For electrically charged systems, the value must be calculated for the system geometry in **standard orientation** (may depend on the program used).

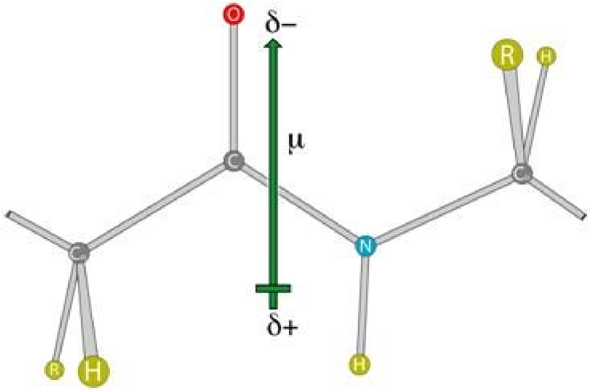
Unit: Debye (**D**), a unit of dipole moment not belonging to the SI system, is defined as 10^{-18} sC·cm, where sC is a statcoulomb. Relationship of debye to the SI unit is

$$1\ \mathrm{D} = 3.33564\times10^{-30}\ \mathrm{C{\cdot}m}$$

Attention (sign convention):

physics

chemistry (sometimes)



Loren Williams, Georgia Tech

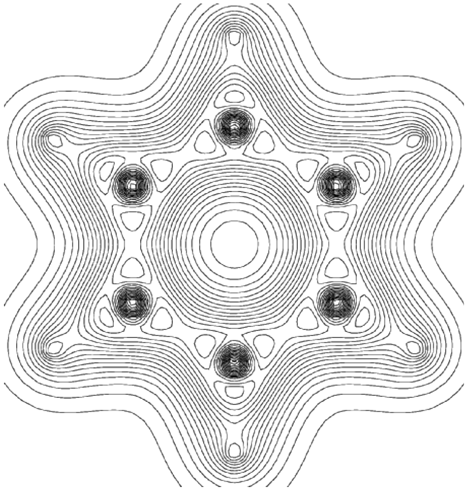
Partial atomic charges

Partial atomic charge expresses the property of the atom that is caused **by uneven distribution of electrons** in molecules.

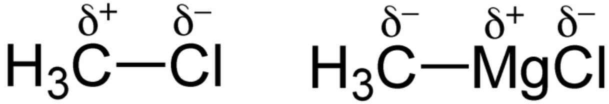
For **qualitative expression,** the δ^+ and δ^- designation is used.

Quantitative calculation of partial atomic charges is possible, however there is no uniform or best approach or method for the calculation. Calculation and most importantly **analysis of calculated charges** must always be done **in the context of the used method**.

Electron density:



Use of partial atomic charges:



$H_3\overset{\delta+}{C}-\overset{\delta-}{Cl}$ $\quad$ $H_3\overset{\delta-}{C}-\overset{\delta+}{Mg}\overset{\delta-}{Cl}$

- estimation of chemical reaction mechanism

Partial atomic charges - classes

Classes of charges:[1]

- **I. class** (class I charges) - charges are not determined from quantum chemical calculations but based on intuitive or other approaches using experimental data (electronegativity, dipole moments).
- **II. class** (class II charges) - charges are derived based on the distribution of a wave function using some scheme based on orbitals
- **III. class** (class III charges) - charges are derived based on the distribution of a physically observable quantity derived from a wave function (e.g., electron density)
- **IV. class** (class IV charges) - charges are derived based on semiempirical mapping of precursors of charges from II. or III. classes in order to reproduce experimental data (electronegativity, dipole moments)

(1) Cramer, CJ Essentials Of Computational Chemistry: Theories And Models; John Wiley & Sons, 2004.

Most often used types of charges

Population analysis of the wave function (Class II):

- **Mulliken charges** from Mulliken population analysis (MPA - Mulliken Population Analysis)
  - their values greatly depend on the size and type of basis set
  - they do not have a clear CBS limit
- **natural charges** from natural population analysis (NPA - Natural Population Analysis)

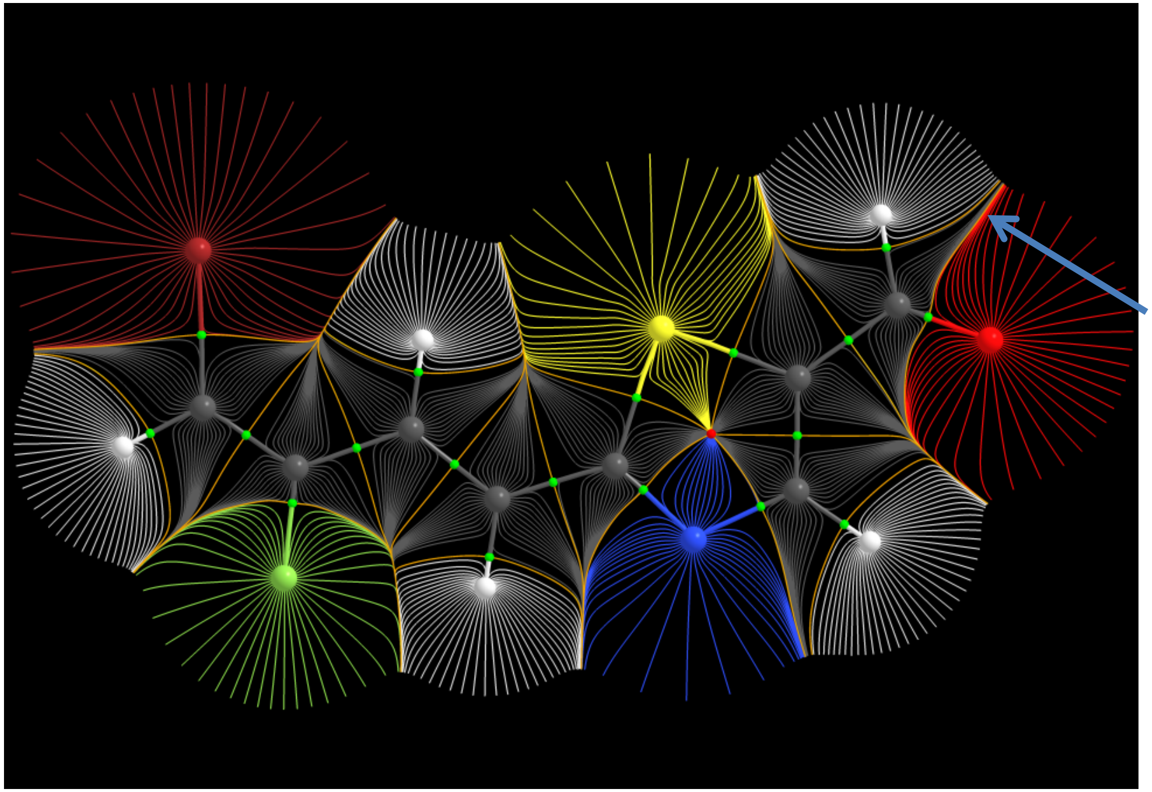
Electron density analysis (class III):

- **Bader charges** (from AIM analysis [Atoms In Molecules])
- **Hirschfeld charges**
- **VDD charges** (Voronoi Deformation Density)

Charges derived from electrostatic potential (Class III):

- **ChelpG charges**
- **Merz-Singh-Kollman charges** (MSK or only Merz-Kollman [MK])

Bader charges (AIM charges)



Derived from electron density analysis (wave functions).

surface delimiting individual atoms

$$\nabla\rho.\mathbf{n}=0$$

normal surface vector

electron density gradient

Partial charge: $$q_k = Z_k - \int_{\Omega_k} \rho(\mathbf{r})d\mathbf{r}$$

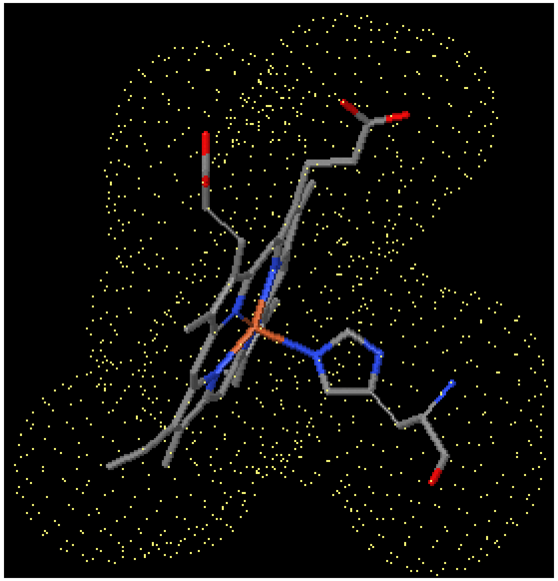
integration over the space of the atom defined by surfaces

http://aim.tkgristmill.com/screenshots1aaa.html

ESP charges

ESP charges (ElectroStatic Potential) are charges derived from electrostatic potential. The principle of charge calculation consists of two steps:

1. calculation of electrostatic potential V_{QM} from wave function on discretized molecular envelope (set of points)
2. finding point atomic charges that create electrostatic potential V_{PC} which is in the best agreement with the quantum mechanical potential (least squares method)



$$\Psi \rightarrow V_{QM,p} \qquad V_{PC,p} \leftarrow \boxed{q_k}$$

$$\sum_p \left(V_{QM,p} - V_{PC,p}\right)^2 = \min!$$

is searched by least squares method

ESP charges and their derivatives are used in **molecular mechanics** because, by their nature, they describe well **electrostatic properties** of molecules/system.

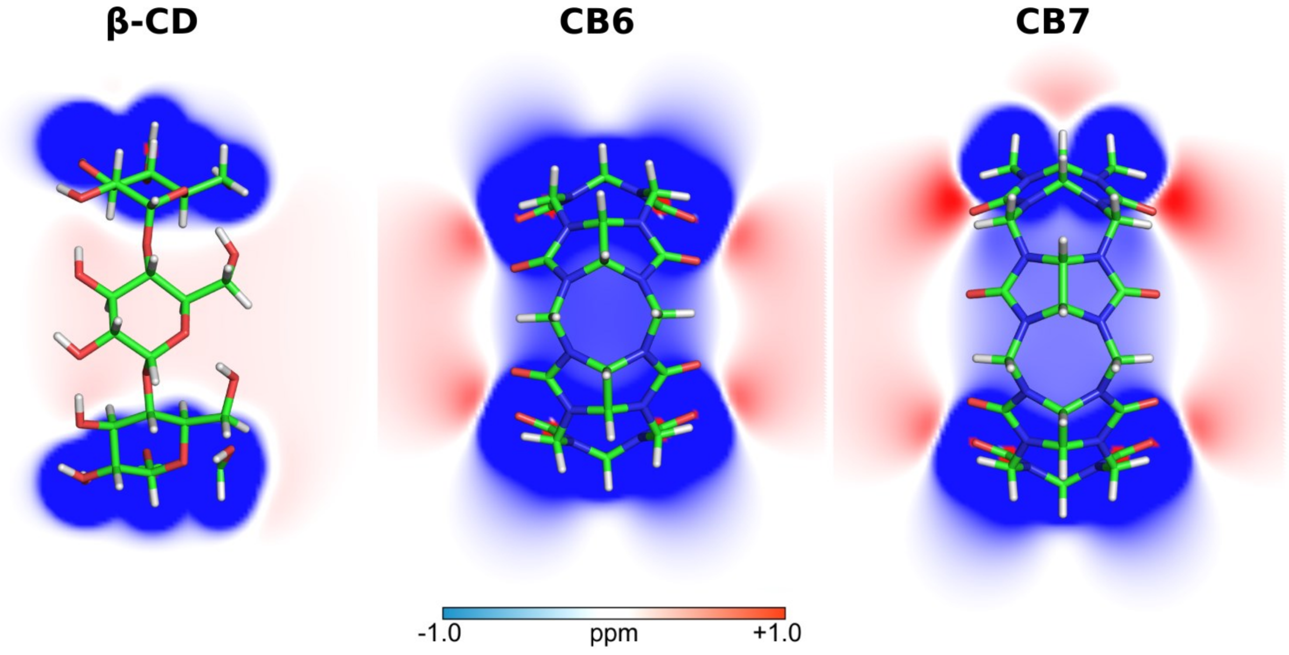
http://biomodel.uah.es/Jmol/surfaces/inicio.htm

Magnetic properties

Calculated properties:

- hyperfine splitting constants (EPR)
- chemical shift (NMR)
- spin-spin interaction constant (NMR)
- NICS (Nuclear Independent Chemical Shielding) (NMR)

NICS



It is necessary to use special methods and analyzes.

Vibrations

When searching for vibrational states, the following SR is solved:

$$\hat{H}_R = -\frac{\hbar^2}{2}\sum_{i=1}^{N}\frac{1}{M_i}\nabla_i^2 + E_e(R)$$

GF method (Wilson's method) solves the above equation in the limit of classical mechanics in the harmonic approximation (rotation and translation are not considered).
The method considers only **normal vibration modes** when all atoms move with the same frequency and phase. The atomic system has 3N-6 (5) linearly independent normal modes of frequency ν_k.

$$\mathbf{GFc}_k = \lambda_k \mathbf{c}_k \qquad \nu_k = \sqrt{\lambda_k} \qquad E_{v,0} = \frac{1}{2}\sum_{k=1}^{3N-6} h\nu_k$$

mass of atoms

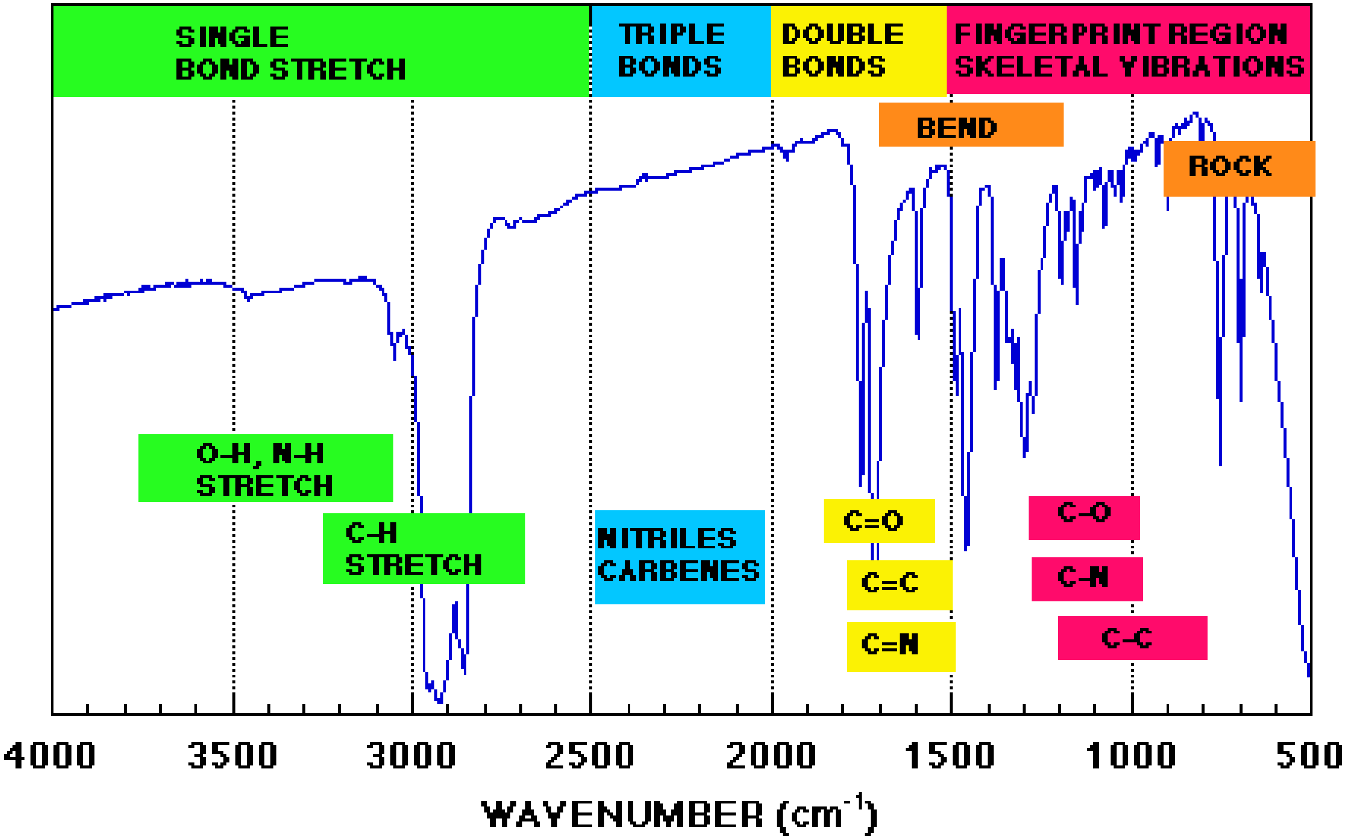
matrix of force constants
Hessian

zero-point vibrational energy (ZPVE)

!!!! analyzed geometry must be a stationary point !!!

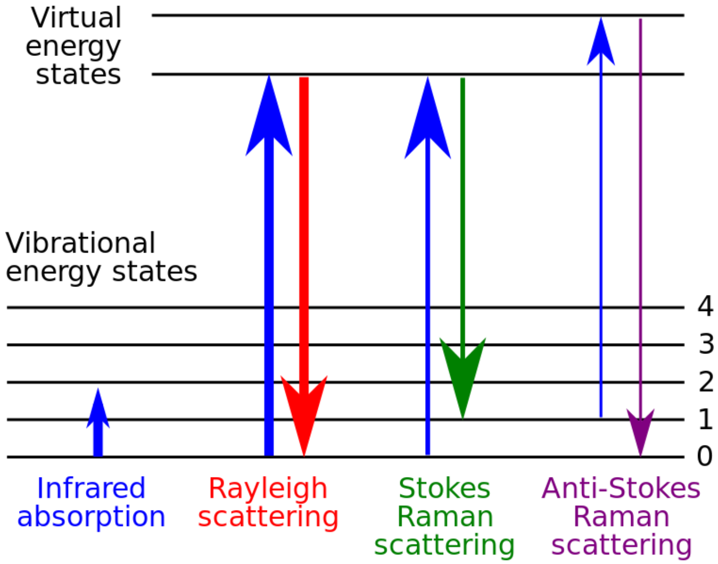
Infrared spectroscopy

Infrared Spectroscopy (IR) - the absorption of infrared radiation is measured. Absorption leads to excitation between adjacent vibrational levels. Active are only those transitions which results into the change of the dipole moment of the molecule (this can be determined from the analysis of the wave function).

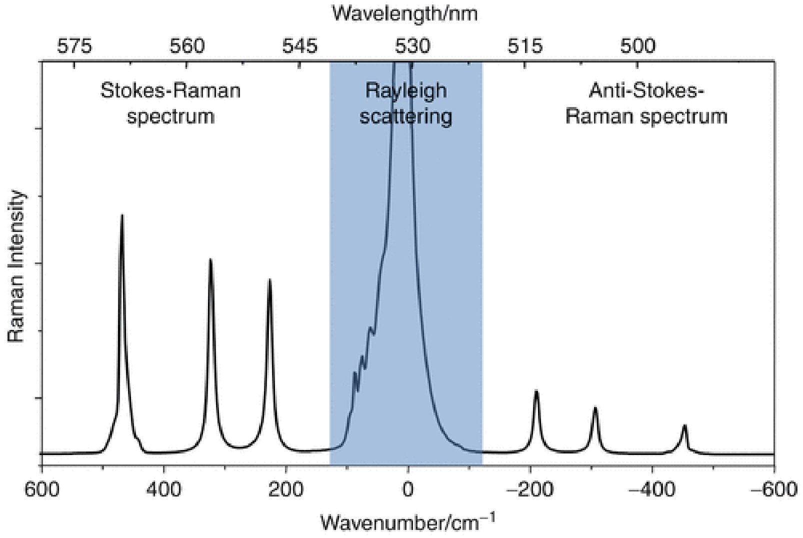


http://www.chm.bris.ac.uk/webprojects1997/RogerEC/welcome.htm

Raman spectroscopy



Active are only those transitions which leads to the change of the polarization of the molecule (this can be determined from the analysis of the wave function).



(1) Dietzek, B .; Deckert, V .; Popp, J. Raman Spectroscopic Instrumentation, Experimental Considerations. In Encyclopedia of Biophysics; Roberts, GCK, Ed .; Springer Berlin Heidelberg: Berlin, Heidelberg, 2013; pp 2173–2178.

Type of stationary point on PES

Analysis of Hessian

$$\mathbf{H}\mathbf{c}_k = \lambda_k \mathbf{c}_k$$

eigenvalues

eigenvectors

$$\mathbf{H} = \mathbf{F}$$

$$k = 1,\ldots,3N$$

Vibrational analysis

$$\mathbf{G}\mathbf{F}\mathbf{c}_k = \lambda_k \mathbf{c}_k$$

eigenvalues

$$\nu_k = \sqrt{\lambda_k}$$

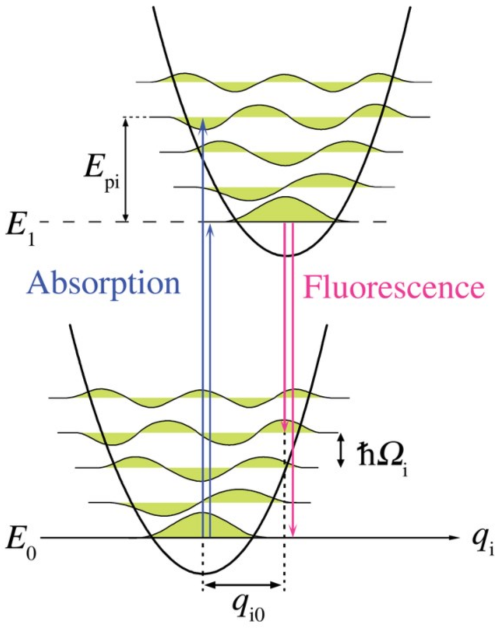
eigenvectors

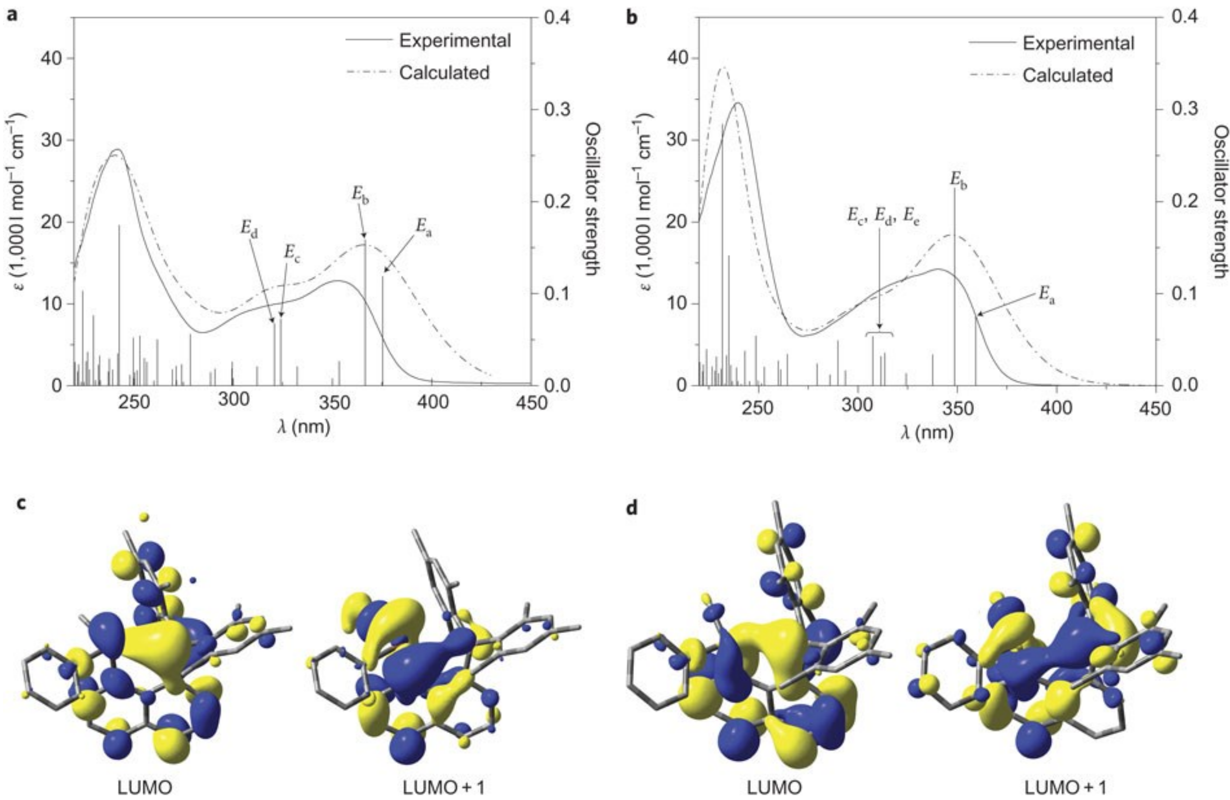
- 6 (5) eigenvalues are zero - this corresponds to the translation and rotation of the system
- remaining eigen numbers:
  - **all positive** - **local minimum**
  - **one negative, others positive** - **first order saddle point**
  - two negative, the other positive - saddle point of the second order
  - .....
  - all negative - local maximum

- 6 (5) frequencies are zero - this corresponds to the translation and rotation of the system
- remaining frequencies:
  - **all positive** - **local minimum**
  - **one imaginary, others positive** - **first order saddle point**
  - two imaginary, the others positive - a saddle point of the second order
  - .....
  - all imaginary - local maximum

!!!! analyzed geometry must be a stationary point !!!

Electronic transitions





It is necessary to use special methods (time-dependent DFT or time-dependent HF; CI; CASSCF). The transition probability is given by the strength of the oscillator.

http://www.nature.com/nchem/journal/v2/n11/fig_tab/nchem.838_F4.html

Summary

- Quantum chemical methods provide powerful tools to study small to middle size models.
- QM describes behavior of electrons and nuclei. QM is then suitable for studying all possible chemical transformations (conformational changes, interactions, reactivity, etc.)
- Solution of Schrodinger equation (SR) is not only potential energy but also wavefunction (WF).
- WF can provide additional properties of system, which can further improve our knowledge about the system.