

Optická emisní spektroskopie atomů

Diagnostické metody 1

Zdeněk Navrátil

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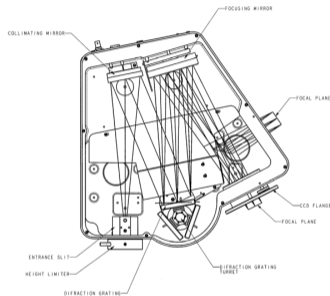
Outline

- 1 OES
- 2 CR modelling
- 3 CR model for neon discharge
- 4 Examples
 - DC
 - RF
 - MW
- 5 Measurement of densities by self-absorption methods

Instrumentation

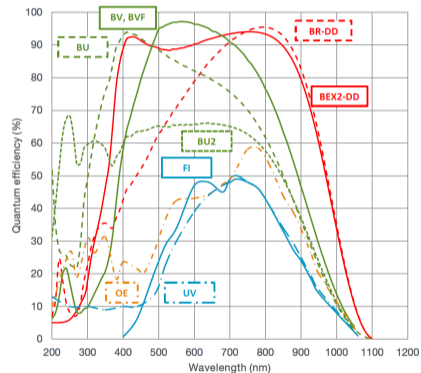
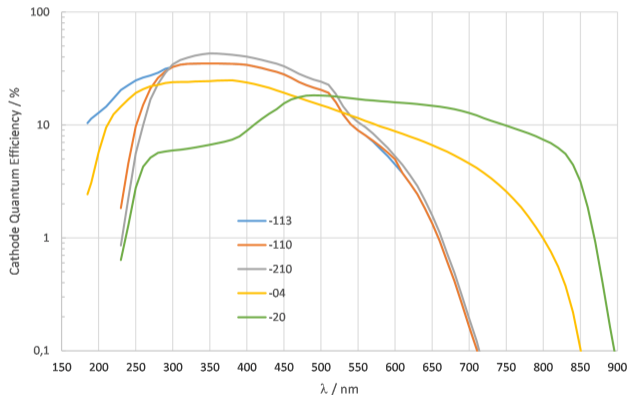
- typically grating spectrometer of Czerny-Turner mounting equipped with CCD/ICCD detector
- typical spectral range 190–1100 nm
- sensitivity of detectors (silicon CCD, photocathode of PMT), grating efficiency
- resolution: number of illuminated grating grooves, slit width, pixel size

$$R = \lambda / \Delta\lambda = mN$$



Sensitivities

PMC-150: Cathode Quantum Efficiency



- grating efficacy, fibre efficacy, windows

Technique overview – how we measure

- collecting the light emitted by plasma (optical emission spectroscopy, OES):
 - non-intrusive
 - sensing the light at the plasma boundary
 - optical probes
- sending the light through the plasma (optical absorption spectroscopy):
 - based on Lambert-Beer law
 - can disturb the plasma, two ports
 - white light, hollow cathode lamps, lasers
- collecting the light emitted and reabsorbed by the plasma (self-absorption methods of OES)

Technique overview – how we measure

- collecting the light emitted by plasma (optical emission spectroscopy, OES):
 - non-intrusive
 - sensing the light at the plasma boundary → self-absorption can play a role
 - optical probes
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 - white light, hollow cathode lamps, lasers
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Technique overview – what we look at

- line positions = wavelengths: electric, magnetic fields, atom velocities (Stark, Zeeman, Doppler effect)
- lineshapes and linewidths: electron density, gas pressure, density, temperatures (Stark, van der Waals, resonance, Doppler line broadening)
- line intensities: . . . all

Technique overview – what we look at

- line positions = wavelengths: electric, magnetic fields, atom velocities (Stark, Zeeman, Doppler effect)
- lineshapes and linewidths: electron density, gas pressure, density, temperatures (Stark, van der Waals, resonance, Doppler line broadening)
- line intensities: . . . all
 - relative – instrument spectral sensitivity is taken into account, no absolute intensity calibration is performed
output: relative populations of excited states, excitation temperatures etc.
 - absolute – access to absolute densities of excited states, electron density etc.

Absolute intensity measurement

- radiant flux/zářivý tok – energy emitted/incident on surface per unit time

$$\Phi = \frac{d\mathcal{E}}{dt}, \quad \text{W} \quad (1)$$

- irradiance – flux density (per unit surface)

$$I = \frac{d\Phi}{dS} = \frac{d^2\mathcal{E}}{dt dS}, \quad \text{W m}^{-2} \quad (2)$$

- specified during calibration of calibrated light sources (spectral irradiance)
- optical fibre is not a detector of irradiance (acceptance angle)
- radiometric irradiance probes, cosine correction diffusers, integrating spheres, ...



Absolute intensity measurement 2

- radiance (zář) – radiant flux per unit perpendicular surface and unit solid angle

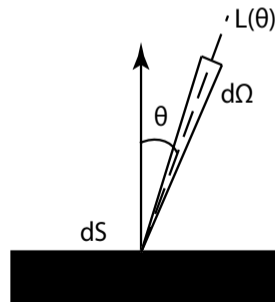
$$L = \frac{d^2\Phi}{dS \cos\theta d\Omega} = \frac{d^3\mathcal{E}}{dt dS \cos\theta d\Omega}, \quad \text{W m}^{-2} \text{sr}^{-1} \quad (3)$$

- radiance \times irradiance

$$I = \int_{\Omega} L(\theta) \cos\theta d\Omega \quad (4)$$

For constant L (Lambert) radiators $I = \pi L$.

- for description of radiating solid surfaces



Absolute intensity measurement 3

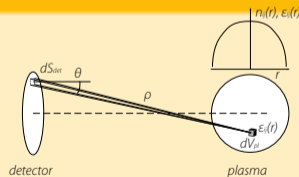
- emission coefficient – radiant power emitted by unit volume into unit solid angle

$$j = \frac{d^3 \mathcal{E}}{dt dV d\Omega} \quad (5)$$

- all quantities have their spectral densities, e.g. $j(\lambda)$

emission coefficient of transition

$$j_{ij} = \frac{1}{4\pi} n_i A_{ij} h\nu_{ij}$$



$$I_{ij} = \frac{1}{S_{\text{det}}} \int_{V_{\text{pl}}} \int_{S_{\text{det}}} \frac{j_{ij}(r)}{\rho^2} \text{Acc}(\theta) dV_{\text{pl}} dS_{\text{det}}$$

irradiation of detector for optical thin plasma condition

Electron temperature from Boltzmann plot?

- density of atoms in excited state

$$n_i = n \frac{g_i}{Q} e^{-\frac{\mathcal{E}_i}{k_b T_e}} \quad (6)$$

2 g_i – statistical weight, \mathcal{E}_i – excitation energy, n – atom density, Q – state sum, T_e excitation ($\stackrel{?}{=}$ electron) temperature

- spectral line intensity

$$I \propto n_i A_{ij} \frac{hc}{\lambda} \quad (7)$$

$$I = C \cdot \frac{g_i A_{ij}}{\lambda} e^{-\frac{\mathcal{E}_i}{k_b T_e}} \quad (8)$$

- Boltzmann plot

$$\ln \frac{I \lambda}{g_i A_{ij}} = -\frac{1}{k_b T_e} \mathcal{E}_i + \ln k_1, \quad (9)$$

Possibility of electron temperature measurement

excited level balance

- local thermodynamic equilibrium (LTE) plasma
 - LTE condition

$$n_e \gg 1.6 \cdot 10^{12} \sqrt{T_e} (\Delta E)^3 \quad (\text{cm}^{-3})$$

- electron temperature from Boltzmann plot
- non-LTE plasma
 - corona equilibrium, excitation saturation phase, ...
 - low electron density plasma
 - use of Boltzmann-plot leads to erroneous electron temperature
 - CR modelling

non-Maxwellian EDF

- inelastic collisions, beam electrons, non-local EDF

Collisional-radiative modelling

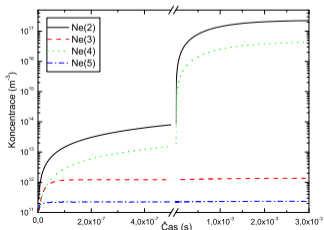
coupled DE for densities of excited states

$$\frac{\partial n_i}{\partial t} + \nabla(n_i \vec{v}) = \left(\frac{\partial n_i}{\partial t} \right)_{c,r} \quad (10)$$

population and depopulation processes are very fast:

$$\frac{\partial n_i}{\partial t} = \left(\frac{\partial n_i}{\partial t} \right)_{c,r} = 0 \quad (11)$$

not valid for ground-state atoms, ions, metastables, high pressure



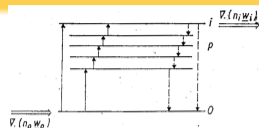
Level balance

$$\frac{\partial n_0}{\partial t} + \nabla(n_0 \vec{v}_0) = -S_{cr} n_e n_0 + \alpha_{cr} n_e n_{ion}$$

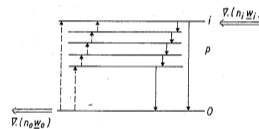
$$\frac{\partial n_{ion}}{\partial t} + \nabla(n_{ion} \vec{v}_{ion}) = +S_{cr} n_e n_0 - \alpha_{cr} n_e n_{ion}$$

classification of models (plasma state)

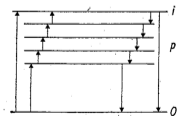
- ionizing plasma $S_{cr} n_e n_0 - \alpha_{cr} n_e n_{ion} > 0$
 - plasma conducting current, ionizing waves
- recombining plasma $S_{cr} n_e n_0 - \alpha_{cr} n_e n_{ion} < 0$
 - afterglows, outer regions of flames
- equilibrium plasma $S_{cr} n_e n_0 - \alpha_{cr} n_e n_{ion} = 0$
(ioniozation-recombination equilibrium)



ionizing plasma



recombining plasma

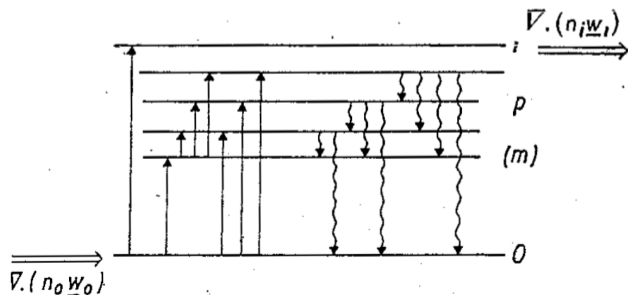


equilibrium plasma

Excitation phases: corona phase

population by electron impact excitation, radiative deexcitation

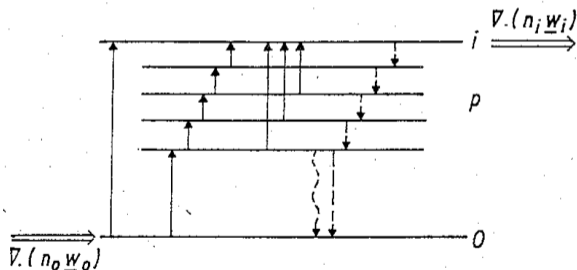
$$k_{0i}^{\text{el}} n_e n_0 + k_{mi}^{\text{el}} n_e n_m \left(+ \sum_{j>i} \Lambda_{ji} A_{ji} n_j \right) = \sum_{j<i} \Lambda_{ij} A_{ij} n_i \quad (12)$$



Excitation phases: excitation saturation phase

population and depopulation by electron impact

$$\sum_{j \neq i} k_{ji}^{\text{el}} n_e n_j + (\alpha_i n_e n_{\text{ion}}) = \sum_{j \neq i} k_{ij}^{\text{el}} n_e n_i + S_i n_e n_i$$



- saturation of the excited state densities with increased n_e
- no Saha equilibrium, $S_i n_i \gg \alpha_i n_{\text{ion}}$

Excitation phases: excitation saturation phase 2

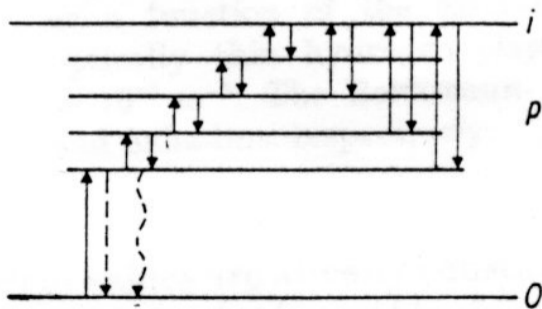
- stepwise excitation \rightarrow ladder-like excitation flow
- coefficients of upward processes are larger (closer upper levels, higher statistical weights of upper levels)

$$k_{i-1,j}n_e n_{i-1} - k_{i,i-1}n_e n_i = k_{i,j+1}n_e n_i - k_{i+1,j}n_e n_{i+1} - S_i n_e n_i$$

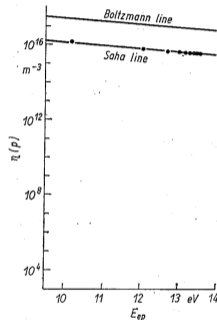
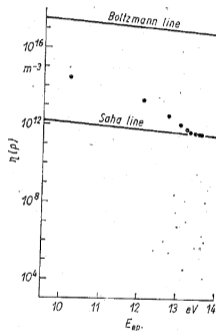
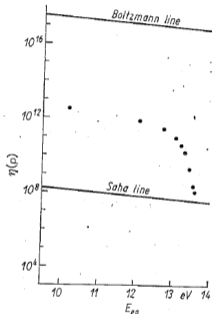
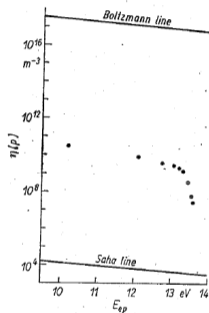
Excitation phases: partial local thermodynamic equilibrium

- 2 equilibria: excited state \times ion state, neighbouring excited states
- ionization \sim recombination \gg excitation flow

$$k_{i-1,j}n_en_{i-1} - k_{i,i-1}n_en_i = k_{i,i+1}n_en_i - k_{i+1,i}n_en_{i+1} - S_in_en_i + \alpha_in_en_{ion}$$



Role of dominant electron collisions



Boltzmann

Saha

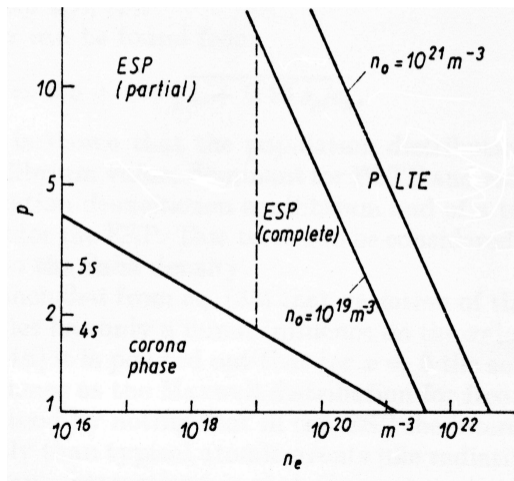
deviation from B & S

$$n_i^B = n_0 g_i / g_0 e^{-E_i / kT_e}$$

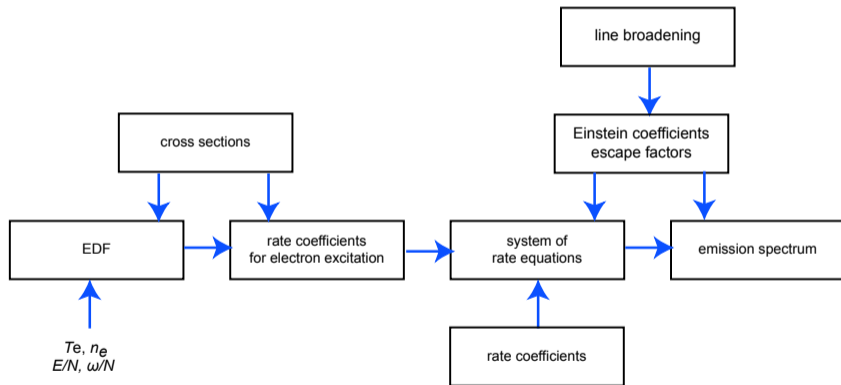
$$n_i^S = n_e n_{\text{ion}} \frac{g_i}{g_e g_{\text{ion}}} (h^2 / 2\pi m_e kT_e)^{3/2} e^{E_{\text{ion},i} / kT_e}$$

$$n_i = r_i^0 n_i^S + r_i^1 n_i^B$$

Excitation phases



Collisional-radiative model



Electron distribution function

- Maxwellian EDF
- solution of Boltzmann kinetic equation
- normalization of the EDF

$$\int_0^{\infty} f(\varepsilon) \varepsilon^{1/2} d\varepsilon = 1 \quad (13)$$

- mean electron energy

$$\langle \varepsilon \rangle = \int_0^{\infty} f(\varepsilon) \varepsilon^{3/2} d\varepsilon, \quad (14)$$

- rate coefficients k , k_{inv} of electron collision with cross section σ and of inverse process

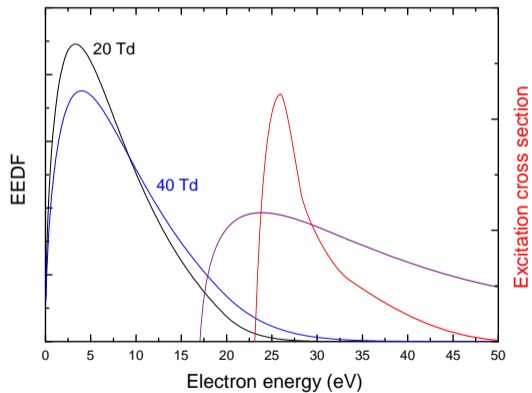
$$k = \sqrt{\frac{2e}{m_e}} \int_0^{\infty} \sigma(\varepsilon) f_0(\varepsilon) \varepsilon d\varepsilon$$

$$k_{\text{inv}} = \sqrt{\frac{2e}{m_e} \frac{g_j}{g_i}} \int_{\varepsilon_{ij}}^{\infty} \sigma(\varepsilon) f_0(\varepsilon - \varepsilon_{ij}) \varepsilon d\varepsilon$$

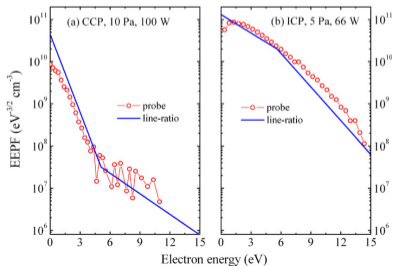
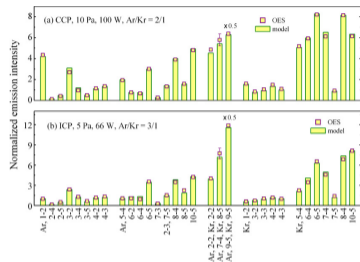
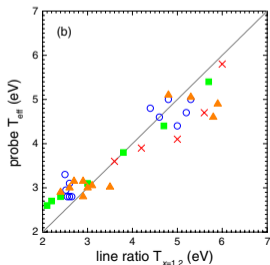
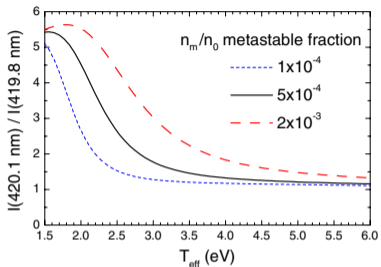
Approaches of OES data processing

- line ratio methods
 - selection of convenient line pair (sensitivity, model simplicity, ease of measurement)
 - no control of model validity
- „many line fitting“ methods

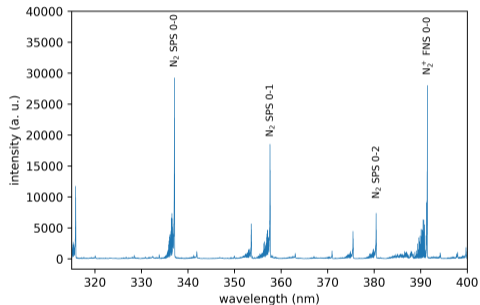
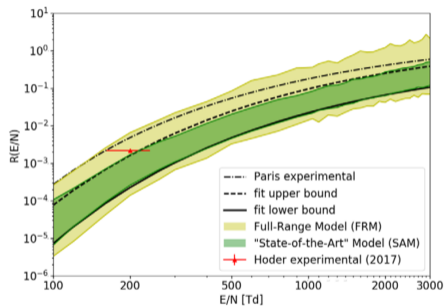
Line ratio method – ideal case



Electron temperature and EDF measurement by OES+CR



Electric field measurement in air



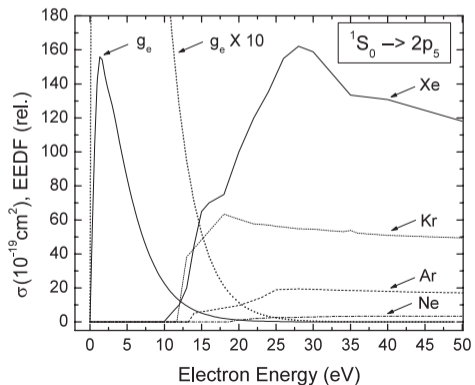
$$R(E/N) = \frac{FNS(0,0)}{SPS(0,0)}$$

Kozlov and Wagner 2001 *J. Phys. D: Appl. Phys.* **34** 3164

Bilek et al 2018 *Plasma Sources Sci. Technol.* **27** 085012

TRG spectroscopy

- based on admixing of a small amount of rare gas into plasma
- mapping EDF at specific electron energies
- low pressure, what is small amount?



Helium line ratio method

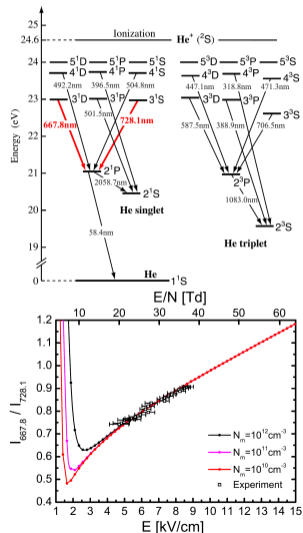
- ratio of helium singlet lines $R = I_{667}/I_{728}$
He I 667.8 nm ($2^1P - 3^1D$)
He I 728.1 nm ($2^1P - 3^1S$)

- ✓ high spectral resolution is not required
- ✓ sensitive to fields of several kV/cm
- ✓ verified at atmospheric pressure
- ✗ dependence on the gas purity
- ✗ dependence on metastable density at low field

$$E(R) = 2.224 - 20.18R + 45.07R^2 - 19.98R^3 + 3.369R^4$$

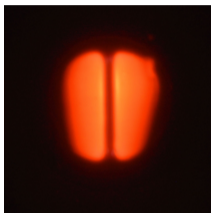
$[E] = \text{kV/cm}$, for 3–40 kV/cm, $T = 310 \text{ K}$

Ivković et al 2014 *J. Phys. D: Appl. Phys.* 47 055204



Diffuse coplanar barrier discharge in rare gases

Neon



diffuse,
high gas purity

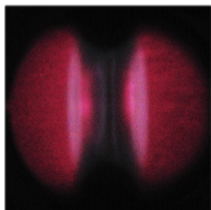


filamentary,
low gas purity

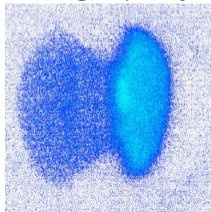


low gas purity,
higher voltage

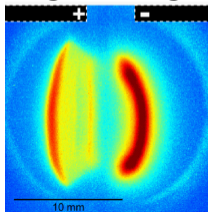
Helium



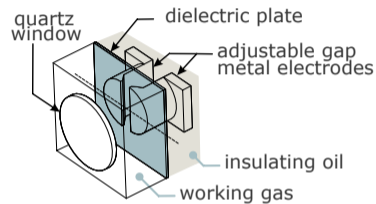
50 ms



1 μs

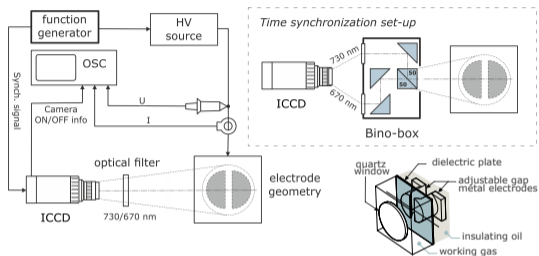


100 ns, 10 k



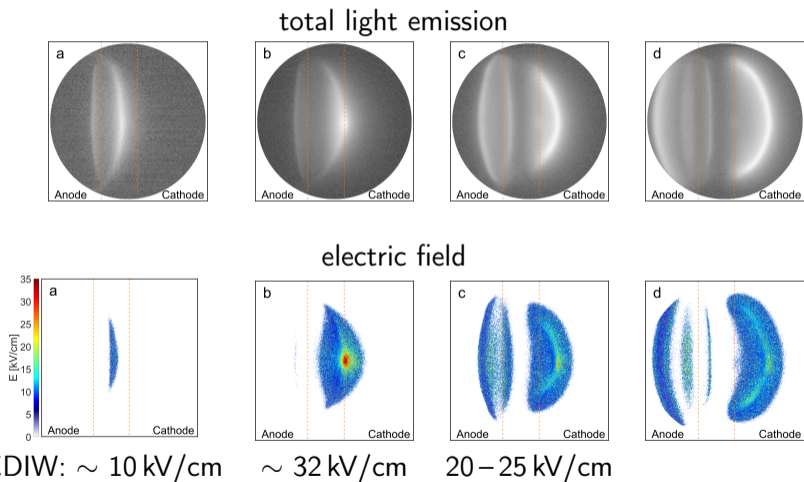
run: I.mp4

Experimental setup

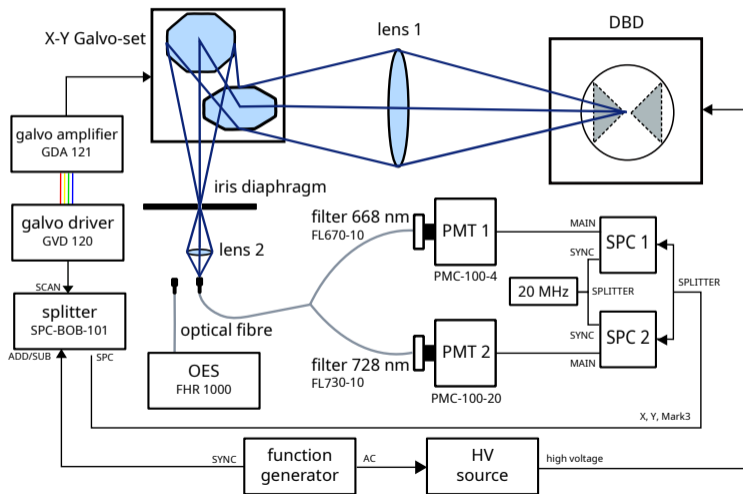


- coplanar DBD, brass electrodes covered with 96% Al_2O_3 (0.63 mm thick),
- parallel gap footprint, electrode distance 4.75 mm,
- helium 5.0 at atmospheric pressure, gas flow 550 sccm
- AC sine-wave high voltage of 1.6 kVmax, 10.3 kHz
- ICCD camera Princeton Instruments PI-MAX3 (time window 50 ns)
- bandpass filters Thorlabs FL670-10 and FL730-10 (670, 730 nm, FWHM 10 nm)

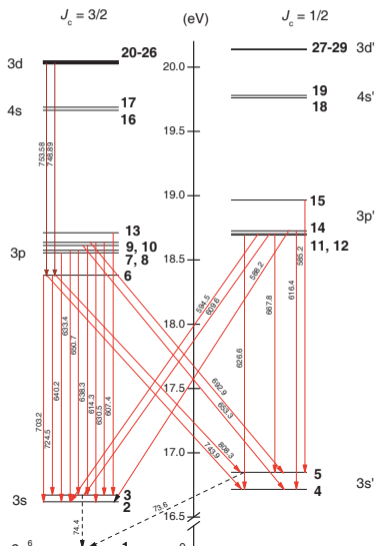
2D resolved electric field development



2D resolved simultaneous line ratio measurement



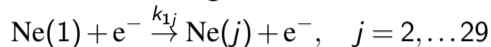
Excited levels



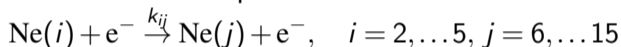
<i>i</i>	<i>nlpqr</i>	Racah	Paschen	(eV)
1	21000	2p ⁶	1p ₀	1 0.000000
2	30332	3s [3/2] ₂ ^o	1s ₅	5 16.61907
3	30331	3s [3/2] ₁ ^o	1s ₄	3 16.67083
4	30110	3s' [1/2] ₀ ^o	1s ₃	1 16.71538
5	30111	3s' [1/2] ₁ ^o	1s ₂	3 16.84805
6	31311	3p [1/2] ₁	2p ₁₀	3 18.38162
7	31353	3p [5/2] ₃	2p ₉	7 18.55511
8	31352	3p [5/2] ₂	2p ₈	5 18.57584
9	31331	3p [3/2] ₁	2p ₇	3 18.61271
10	31332	3p [3/2] ₂	2p ₆	5 18.63679
11	31131	3p' [3/2] ₁	2p ₅	3 18.69336
12	31132	3p' [3/2] ₂	2p ₄	5 18.70407
13	31310	3p [1/2] ₀	2p ₃	1 18.71138
14	31111	3p' [1/2] ₁	2p ₂	3 18.72638
15	31110	3p' [1/2] ₀	2p ₁	1 18.96596
16	40332	4s [3/2] ₂ ^o	2s ₅	5 19.66403
17	40331	4s [3/2] ₁ ^o	2s ₄	3 19.68820
18	40110	4s' [1/2] ₀ ^o	2s ₃	1 19.76060
19	40111	4s' [1/2] ₁ ^o	2s ₂	3 19.77977
20	32310	3d [1/2] ₀ ^o	3d ₆	1 20.02464
21	32311	3d [1/2] ₁ ^o	3d ₅	5 20.02645
22	32374	3d [7/2] ₄ ^o	3d' ₄	9 20.03465
23	32373	3d [7/2] ₃ ^o	3d' ₄	7 20.03487
24	32332	3d [3/2] ₂ ^o	3d' ₃	5 20.03675
25	32331	3d [3/2] ₁ ^o	3d' ₂	3 20.04039
26	32352	3d [5/2] ₂ ^o	3d'' ₁	5 20.04821
	32353	3d [5/2] ₃ ^o	3d'' ₁	7 20.04843
27	32152	3d' [5/2] ₂ ^o	3s'' ₁	5 20.13611
	32153	3d' [5/2] ₃ ^o	3s'' ₁	7 20.13630
28	32132	3d' [3/2] ₂ ^o	3s'' ₁	5 20.13751

Considered elementary processes

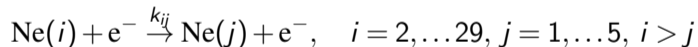
- ① Electron impact excitation out of the g.s.



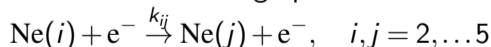
- ② Electron impact excitation out of $2p^53s$ states



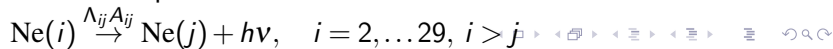
- ③ Electron impact deexcitation to the g.s. and $2p^53s$ states



- ④ Electron induced excitation transfer among $2p^53s$ states

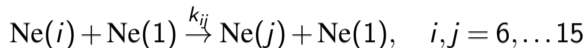


- ⑤ Spontaneous emission and absorption of radiation

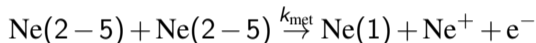


Considered elementary processes 2

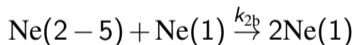
- 6 Two-body collision induced deactivation and excitation transfer among $2p^53p$ states



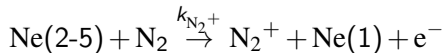
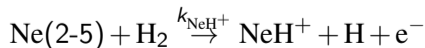
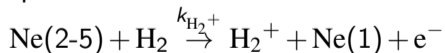
- 7 Chemoionization



- 8 Two-body collision induced deactivation

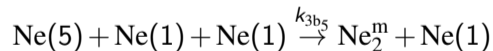
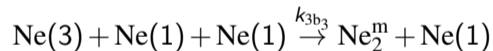
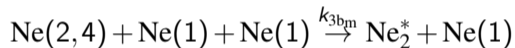


- 9 Penning ionization of impurities

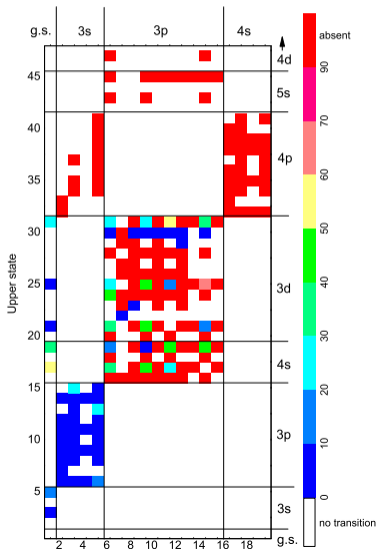


Considered elementary processes 3

- 12 Electron impact ionization of the ground-state and metastable atoms
- 13 Three-body production of dimers



Spontaneous emission



Einstein coefficient A_{ij}

$$A_{ij} = \frac{16\pi^3 \nu^3 S}{3\epsilon_0 hc^3 g_i}$$

$$A_{ij} = \frac{g_j}{g_i} \frac{2\pi e^2 \nu^2}{\epsilon_0 mc^3} f$$

effective levels

$$A_{\{i\}j} = \frac{\sum_i g_i A_{ij}}{\sum_i g_i}$$

← relative differences of two data source –
NIST and Seaton 1998

Absorption of radiation

Number of absorption transitions between states i, j (j is lower) in unit volume is

$$n_j B_{ji} \rho(\omega_0)$$

What is $\rho(\omega_0)$?

The spatial distribution of population of excited state due to the radiation propagation can be described by Holstein equation

$$\frac{\partial n(\vec{r})}{\partial t} = -An(\vec{r}) + A \int n(\vec{r}') G(\vec{r}', \vec{r}) d\vec{r}', \quad (15)$$

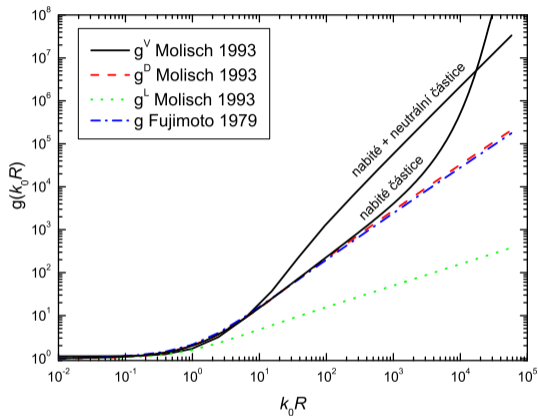
$$G(\vec{r}', \vec{r}) = -\frac{1}{4\pi\rho^2} \frac{\partial T}{\partial \rho}, \quad \rho = |\vec{r}' - \vec{r}|, \quad T(\rho) = \int f(\omega) e^{-kf(\omega)\rho} d\omega.$$

Solution of Holstein equation has a form

$$n(\vec{r}, t) = \sum_j c_j n_j(\vec{r}) e^{-A/g_j t}, \quad (16)$$

in which g_j are *trapping* factors attached to eigenfunctions n_j . Escape factor $\Lambda = 1/g_0$.

Trapping factor



Parameters of solution:

- discharge geometry
- opacity $k_0 R$
- spectral line profile

Solution of rate-equations

Initial conditions

$$n_i(t=0) = \begin{cases} N \equiv \frac{p}{k_b T_n}, & i = 1 \\ 0, & i > 1 \end{cases} \quad (17)$$

- Runge-Kutta methods
- stationary state solution: all excited states reach stationary state ($\frac{\partial n_i}{\partial t} = 0$)
- Non-linear dependence of some rate equations

$$\begin{aligned} \left(\frac{\partial n_2}{\partial t}\right)_{\text{met}} &= -4k_{\text{met}}n_2^2 - 2k_{\text{met}}n_2n_4 - \dots - (k_{\text{H}_2^+} + k_{\text{NeH}^+})[\text{H}_2]n_2 - \\ &\quad - (k_{\text{N}_2^+} + k_{\text{NeN}_2^+})[\text{N}_2]n_2 - k_{\text{O}_2^+}[\text{O}_2]n_2 - \frac{D_2}{l_D^2}n_2 - k_{\text{ionmet}}n_e n_2 \end{aligned}$$

Spectrum calculation and comparison

- measured spectral line intensities, integrated over lineshapes

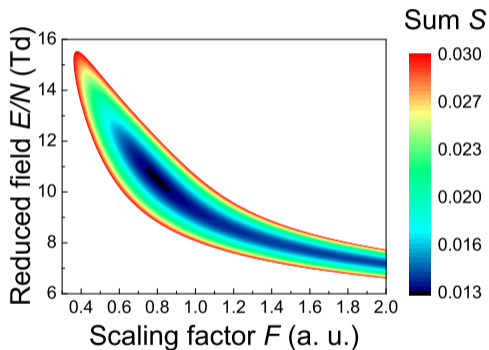
$$[\lambda_k, I_k^{\text{exp}}], k = 1, \dots, n, n = 30$$

- calculated total emission coefficients of transitions

$$I_{ij}^{\text{cr}} = \frac{1}{4\pi} n_i \Lambda_{ij} A_{ij} h\nu_{ij}$$

- comparison of spectra by least squares method

$$\mathcal{S} = \sum_{k=1}^n \frac{(\mathcal{F} \cdot I_k^{\text{cr}}(T_e, n_e, n_{1s3}, n_{1s5}) - I_k^{\text{exp}})^2}{I_k^{\text{exp}}}$$

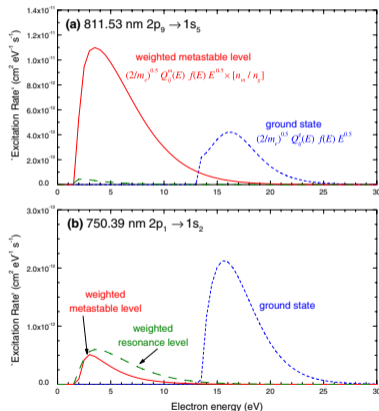
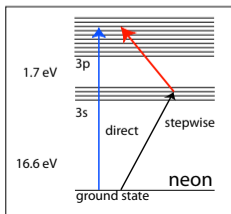


Role of metastables

- simplified 0D scheme is not valid

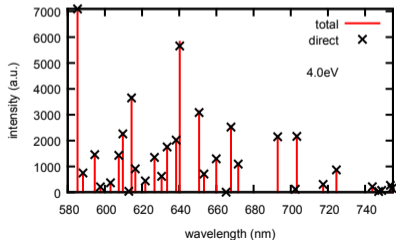
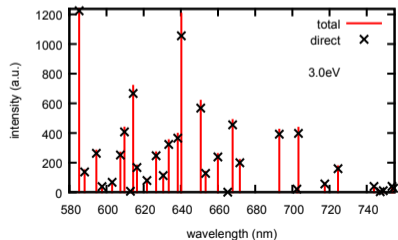
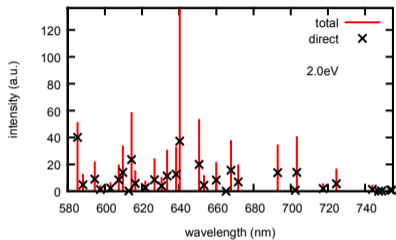
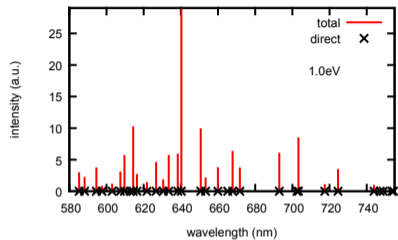
$$\frac{\partial n_i}{\partial t} + \nabla \cdot (n_i \vec{v}) = \left(\frac{\partial n_i}{\partial t} \right)_{c,r} \quad (18)$$

- longer computational times
- increased sensitivity at low electron energies



argon

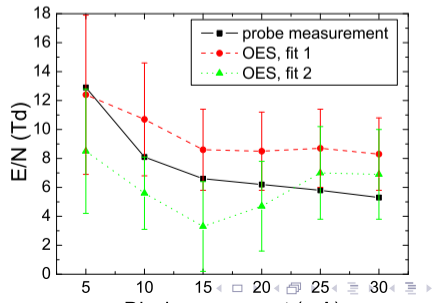
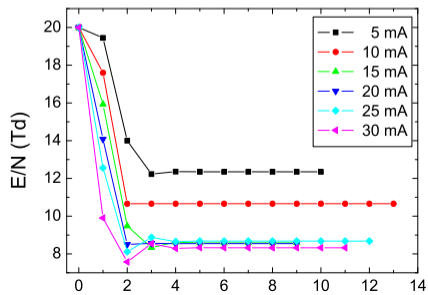
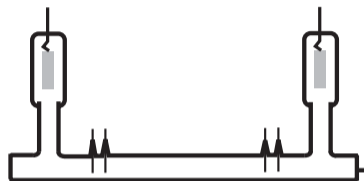
Direct and stepwise excitation



Maxwellian EDF, gas temperature 300 K, fixed densities of all $1s_i$ levels

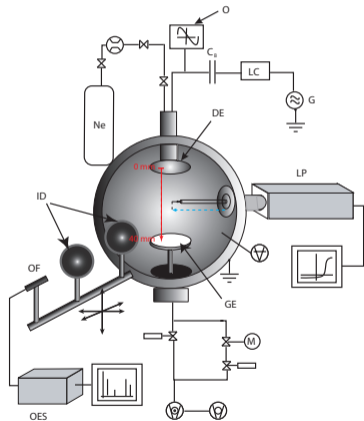
DC glow discharge in neon

- positive column of DC glow discharge at 1.1 Torr
- OES in spectral range 300–850 nm
- CR model with stationary BKE solver
- probe measurement

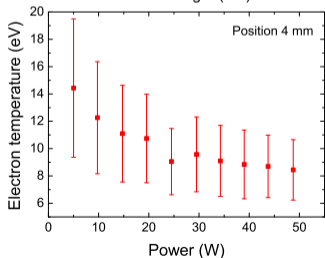
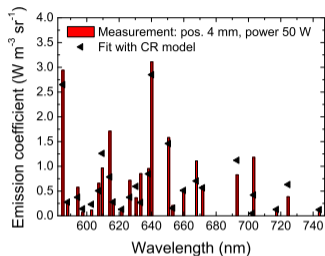
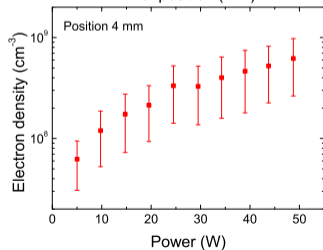
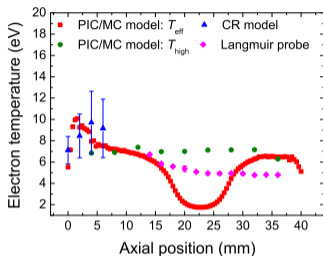


Radio-frequency discharge in neon

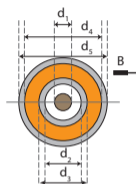
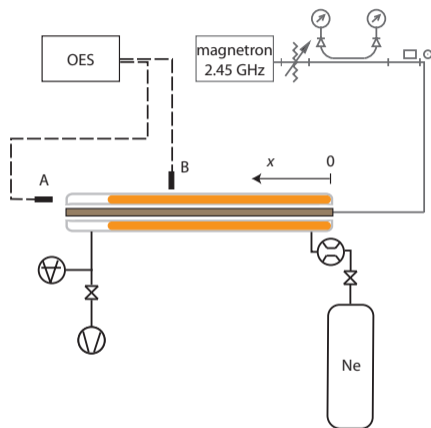
- capacitively coupled RF discharge in neon (13.56 MHz)
- low pressure (10 Pa)
- reactor R3 “Temelín”, inner diameter 33 cm, discharge gap 40 mm, electrodes 8 cm in diameter
- studied by OES/CR, OAS, PIC/MC, Langmuir probe
- absolute intensity measurement



RF (13.56 MHz) capacitive discharge in neon at 10 Pa



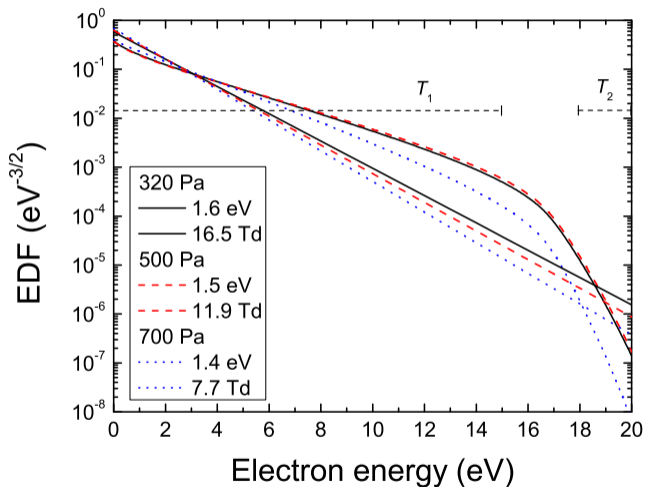
MW surface-wave driven discharge in neon in coaxial configuration



argon

- two-cylinder quartz tube with copper rod antenna, length 320 mm, dimensions $d_1 = 5$ mm, $d_2 = 7$ mm, $d_3 = 11$ mm, $d_4 = 20$ mm and $d_5 = 24$ mm
- microwave power 60 W
- pressure 300 – 700 Pa of neon with research purity 99.9999% flow rate 6 – 30 sccm

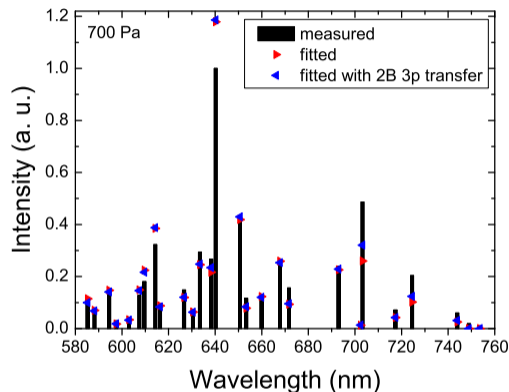
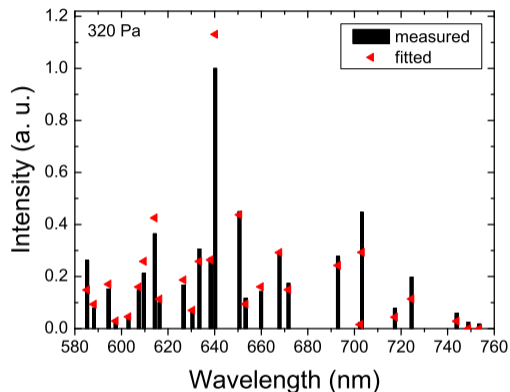
Electron distribution function



$$T_1 = 2.1 - 2.5 \text{ eV}, \quad \mathcal{E} < 15 \text{ eV}$$

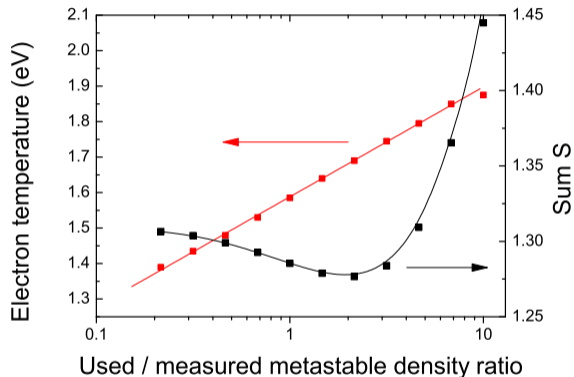
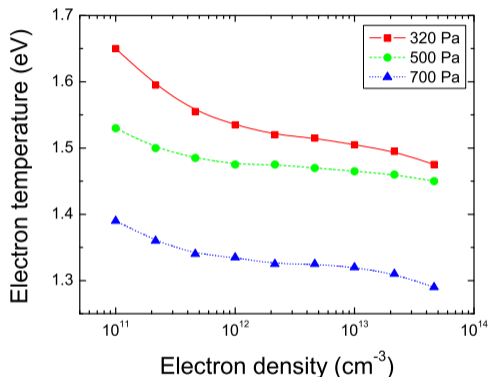
$$T_2 = 0.33 - 0.43 \text{ eV}, \quad 18 \text{ eV} < \mathcal{E} < 20 \text{ eV}$$

Spectra fit



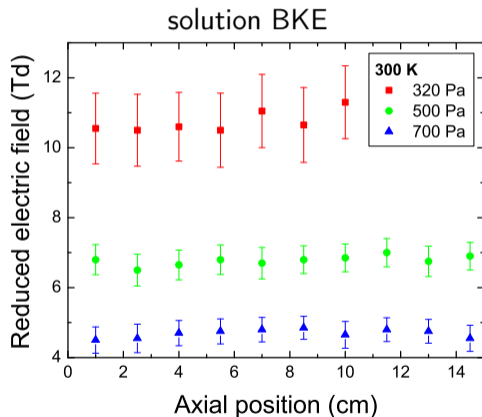
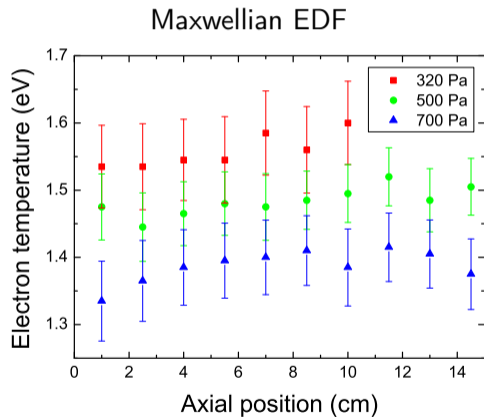
- using BKE solver
- effect of deactivation by heavy particles on spectra under studied conditions is small

Sensitivity to electron density and metastable density

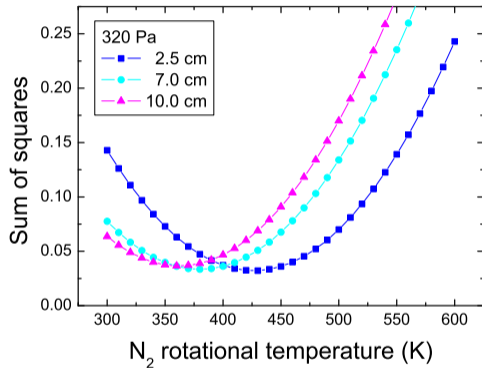
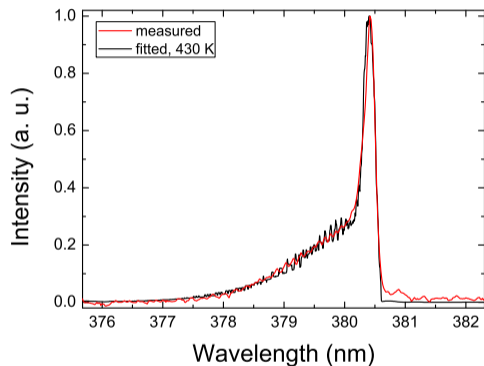


sensitivity to metastables: 0.3 eV or 2 Td per order of density

Axial dependencies for $T = 300$ K

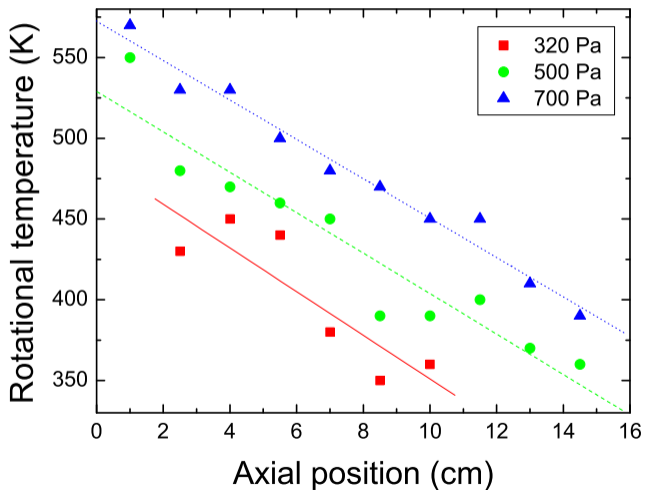


N_2 rotational temperature in $C^3\Pi_u$ state



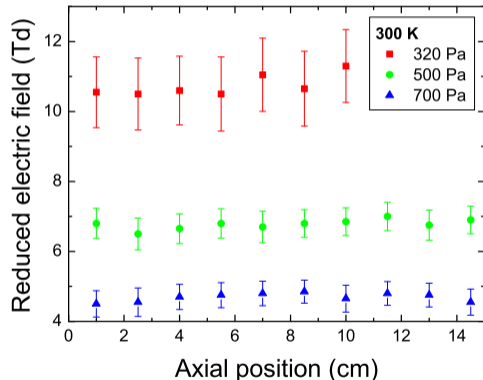
Program Specair. Laux C O 2002. In Fletcher D, Charbonnier J M, Sarma G S R and Magin T, eds., *von Karman Institute Lecture Series 2002-07, Physico-Chemical Modeling of High Enthalpy and Plasma Flows* Rhode-Saint-Genese, Belgium.

N_2 rotational temperature in $C^3\Pi_u$ state

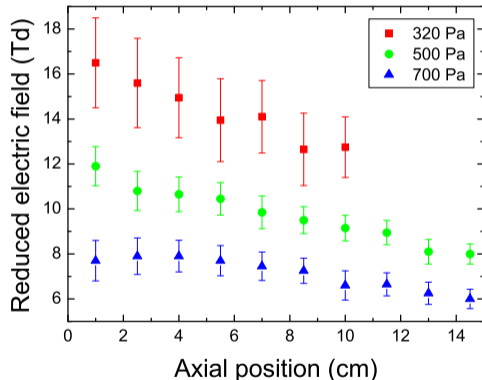


Effect of gas temperature

300 K

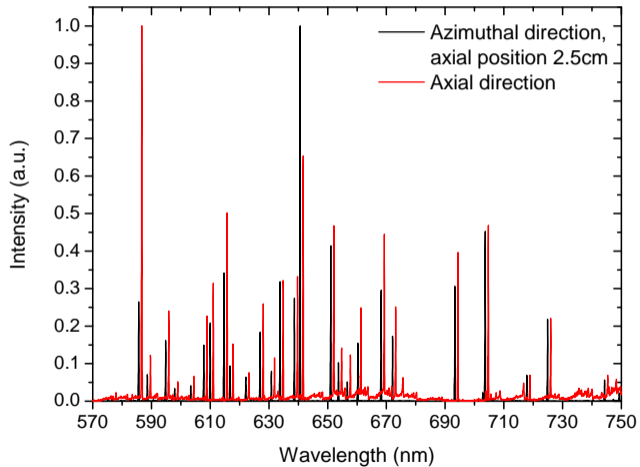


N₂ rotational temperature



- heating by oscillating field is governed by E/N and ω/N , elastic collisions enhance heating
- ω/N is not constant along the column

Self-absorption



Effective branching fractions Γ_{ij}

- isolated atom

$$\Gamma_{ij} = \frac{A_{ij}}{\sum_l A_{il}}$$

- plasma

$$\Gamma_{ij}^{\text{eff}} = \frac{g(k_{ij}^0 L) A_{ij}}{\sum_l g(k_{il}^0 L) A_{il}}$$

- absorption coefficient

$$k_{ij}^0 = \frac{\lambda_{ij}^3}{8\pi^{3/2}} \sqrt{\frac{m_0}{2k_b T} \frac{g_i}{g_j}} A_{ij} n_j$$

- measured

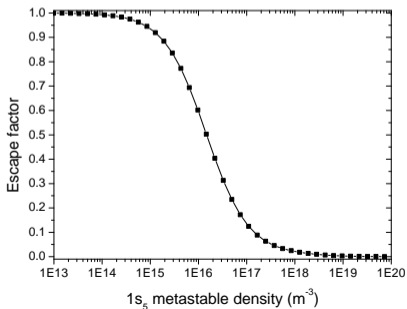
$$\Gamma_{ij}^{\text{exp}} = \frac{I_{ij}/h\nu_{ij}}{\sum_l I_{il}/h\nu_{il}}$$

Escape factor

- Mewe approximate expression

$$g(k_{ij}^0 L) = \frac{2 - e^{-k_{ij}^0 L / 1000}}{1 + k_{ij}^0 L}$$

- assumption of homogeneous distribution of atoms
- e.g. Ar $2p_6 \rightarrow 1s_5$ (763.5 nm), $\rho = 10$ cm



Example – density of Ti and Ti^+ in magnetron discharge

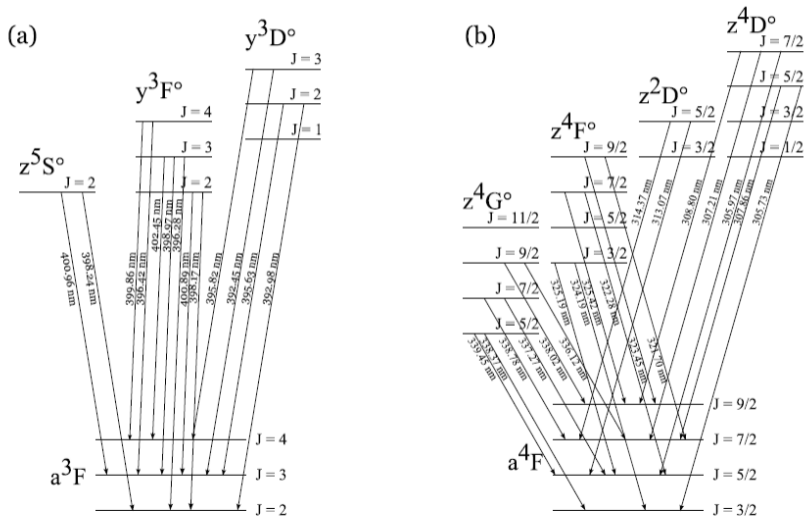
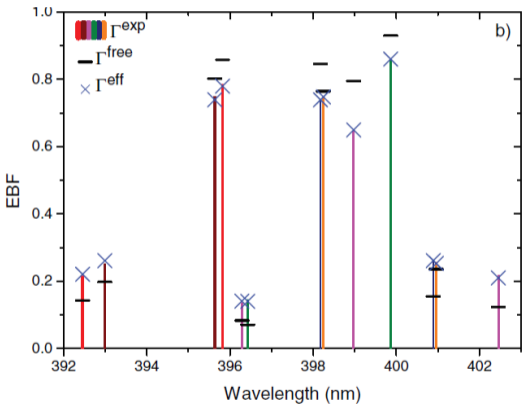
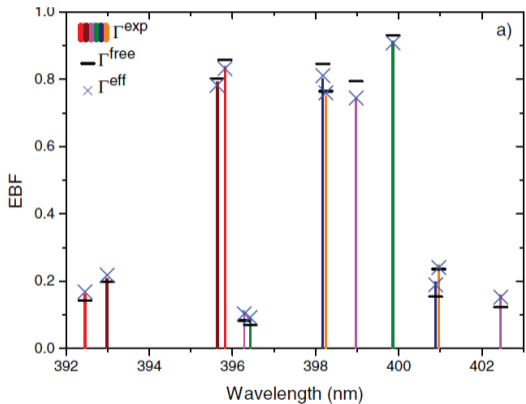


Figure 1. Energy levels and selected transitions for density measurement of (a) Ti neutral atom and (b) Ti ion.

Example – density of Ti and Ti^+ in magnetron discharge



Example – density of Ti and Ti^+ in magnetron discharge

