Paraxial image formation

1. Partikulární řeš. vlnové rovnice

$$\psi(x,y,z) = e(z)\ \exp\left(\frac{i}{\hbar}\, S(x,y,z)\right)$$

S je eikonál, jeho paraxiální tvar:

$$S(\vec{q}_0, \vec{q}) = \frac{g_0}{2h_p}\left(\frac{g(z)}{g_0}\, h_p'(z)\, \vec{q}^{\,2} - 2\vec{q}\vec{q}_0 + g_p(z)\, \vec{q}_0^{\,2}\right) \qquad \underline{(1)}$$

g_0 – kinetický impulz v předmětu
$g(z)$ – — " — v z
$g_p(z)$ paraxiální traj. g: $g_p(z_0) = 1$, $g_p'(z_0) = 0$
$h_p(z)$ — " — h: $h_p(z_0) = 0$, $h_p'(z_0) = 1$

2. Formování obrazu

Eikonál ~~rovnice~~ (1) diverguje pokud $h_p(z) = 0$ – rovina předmětu a obrazu. Lze obejít pomocí vytvoření vlnového klubka:

$$\psi = \iint e(\vec{q}_0)\, e^{\frac{i}{\hbar} S(\vec{q}, \vec{q}_0)}\, \frac{d\vec{q}_0^{\,2}}{h_p(z)}$$

$$S = \frac{g\, g_p'}{2 g_p}\, \vec{q}^{\,2} + \frac{g_0 g_p}{2 h_p}\left(\vec{q}_0 - \frac{\vec{q}}{g_p}\right)^2$$

$$\psi(\vec{q}, z) = \frac{1}{h_p} \exp\left\{ \frac{i g g_p'}{2\hbar g_p} \vec{q}^{\,2} \right\} \iint c(\vec{q}_0)\, e^{\frac{i g_0 g_p \xi}{2\hbar h_p} \left(\vec{q}_0 - \frac{\vec{q}}{g_p}\right)^2} d^2\vec{q}_0$$

$h_p \to 0$ $\quad e^{\frac{i g_0 g_p \xi}{2\hbar h_p}\left(\vec{q}_0 - \frac{\vec{q}}{g_p}\right)}$ rychle osc. funkce, jediný nenul.

příspěvek $\vec{q}_0 - \dfrac{\vec{q}}{g_p} = 0$

$$\Rightarrow \iint c(\vec{q}_0)\, e^{\frac{i g_0 g_p \xi}{2\hbar h_p}\left(\vec{q}_0 - \frac{\vec{q}}{g_p}\right)} d^2\vec{q}_0 \overset{h_p \to 0}{=} c\left(\frac{\vec{q}}{g_p}\right) \int e^{\frac{i g_0 g_p \xi}{2\hbar h_p}\left(\vec{q}_0 - \frac{\vec{q}}{g_p}\right)^2} d^2\vec{q}_0$$

$$\int e^{i \frac{g_0 g_p}{2\hbar h_p}\left(\vec{q}_0 - \frac{\vec{q}}{g_p}\right)} d^2\vec{q}_0 = \frac{2\pi i}{g_0 g_p} \hbar h_p$$

$$\psi = \frac{2\pi i \hbar}{g_0 g_p} c\left(\frac{\vec{q}}{g_p}\right) e^{\frac{i g g_p'}{2\hbar g_p} \vec{q}^{\,2}}$$

object plane: $g_p = 1$, $g_p' = 0$

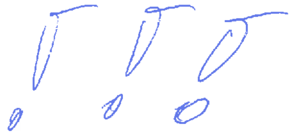
$$\psi(\vec{q}_0) = \frac{2\pi i \hbar}{g_0} c(\vec{q}_0) = i\lambda\, c(\vec{q}_0)$$

image plane: $g_p = M$

$$\psi = \frac{2\pi i \lambda}{M g_0} c(\vec{q}_0)\, e^{\frac{i g_i g_{pi}'}{2\hbar M} \vec{q}_i^{\,2}} = \frac{i\lambda_0}{M} c(\vec{q}_0)\, e^{\frac{i\pi g_{pi}'}{\lambda M} \vec{q}_i^{\,2}}$$

$$|\psi(\vec{q}_i, z_i)| = M^{-2} \left|\psi\left(\frac{\vec{q}_i}{M}, z_0\right)\right|^2$$

- Nezahrnuje efekt difrakce
- Nezahrnuje aberace



$$n^{(2)} = -\frac{\gamma \Phi'' + \eta^2 B^2}{8\Phi^{*1/2}} \vec{q}^{\,2} + \frac{1}{2}\Phi^{*1/2}\vec{q}'^{\,2}$$

$$q^{(1)} = \vec{q}_0\, g_p(z) + \vec{q}_0{}' h_p(z) = s\vec{q}_0 + t\vec{q}_1 \qquad s = g - \frac{g_1}{h_1}h$$

$$t = \frac{h}{h_1}$$

$$S^{(2)} = \sqrt{2me}\int_{z_0}^{z_1} n^{(2)}(q^{(1)}, q^{(1)}{}', z)\, dz =$$

$$= \sqrt{2me}\int_{z_0}^{z_1}\left\{-\frac{\gamma\Phi''+\eta^2B^2}{8\Phi^{*1/2}}(s\vec{q}_0 + t\vec{q}_1)^2 + \frac{1}{2}\Phi^{*1/2}(s'\vec{q}_0 + t'\vec{q}_0{}')^2\right\} dz$$

$$= \sqrt{2me}\left[\int_{z_0}^{z_1}\left(-\frac{\gamma\Phi''+\eta^2B^2}{8\Phi^{*1/2}}s^2 + \frac{1}{2}\Phi^{*1/2}s'^2\right)dz\; \vec{q}_0^{\,2} +\right.$$

$$2\int_{z_0}^{z_1}\left(-\frac{\gamma\Phi''+\eta^2B^2}{8\Phi^{*1/2}}st + \frac{1}{2}\Phi^{*1/2}s't'\right)dz\; \vec{q}_0\vec{q}_1 +$$

$$+\int_{z_0}^{z_1}\left(-\frac{\gamma\Phi''+\eta^2B^2}{8\Phi^{*1/2}}t^2 + \frac{1}{2}\Phi^{*1/2}t'^2\right)dz\; \vec{q}_1^{\,2}$$

Paraxiální rov. $\quad \frac{d}{dz}(\Phi^{*1/2}u') + \frac{\gamma\Phi''+\eta^2B^2}{4\Phi^{*1/2}}u = 0$

$$\int_{z_0}^{z_1}\left\{-\frac{\gamma\Phi''+\eta^2B^2}{8\Phi^{*1/2}}u_1\cdot u_2 + \frac{1}{2}\Phi^{*1/2}u_1'u_2'\right\}dz = \int_{z_0}\left(\frac{d}{dz}\left(\frac{1}{2}\Phi^{*1/2}u_1'\right)u_2 + \frac{1}{2}\Phi^{*1/2}u_1'u_2'\right)dz$$

$$= \int_{z_0}^{z_1}\frac{d}{dz}\left(\frac{1}{2}\Phi^{*1/2}u_1'u_2\right)dz = \left[\frac{1}{2}\Phi^{*1/2}u_1'u_2\right]_{z_0}^{z_1}$$

$$= \sqrt{2me}\left\{\left(\frac{1}{2}\Phi_1^{*1/2}\overset{0}{s_1'}s_1 - \frac{1}{2}\Phi_0^{*1/2}s_0's_0\right)\vec{q}_0^{\,2} + 2\left(\frac{1}{2}\Phi_1^{*}t_1's_1 - \frac{1}{2}\Phi_0^{*1/2}\overset{0}{t_0'}s_0\right)\vec{q}_0\vec{q}_1\right.$$
$$\left. + \left(\frac{1}{2}\Phi_1^{*1/2}t_1't - \frac{1}{2}\Phi_0^{*1/2}t_0'\underset{0}{t_0}\right)\vec{q}_1^{\,2}\right\}$$

$$= \sqrt{2me}\left(\frac{1}{2}\Phi_0^{*1/2}s_0'\vec{q}_0^{\,2} + \Phi_0^{*1/2}t_0'\, q_0 q_1 + \frac{1}{2}\Phi_1^{*1/2}t_1\right)$$

pomocí traj. g_p a h_p:

$$S = \sqrt{2me}\left(\frac{1}{2}\frac{g_\alpha}{h_\alpha}\Phi_0^{*1/2}\vec{q}_0^{\,2} - \Phi_0^{*1/2}\vec{q}_0\vec{q}_1 + \frac{1}{2}\Phi_1^{*1/2}\frac{h_\alpha'}{h_\alpha}\vec{q}_1^{\,2}\right)$$

$$S = \frac{g_0}{2h_p}\left(\frac{g(z)}{g_0}h_p'(z)\,\vec{q}^{\,2} - 2\vec{q}\,\vec{q}_0 + g_p\,\vec{q}_0^{\,2}\right)$$

$g_0,\ g$ – kinetický moment

$g_p,\ h_p$ – paraxiální trajektorie

Užitím $g_0 = g(z)\,(g_p h_p' - h_p g_p')$ – zák. zach. Wr.

$$g(z)\frac{h_p'}{h_p} = g\frac{g_p'}{g_p} + g_0/g_p h_p$$

$$S(\vec{q}_0, \vec{q}) = \frac{g\,g_p'}{2g_p}\vec{q}^{\,2} + \frac{g_0\,g_p}{2h_p}\left(\vec{q}_0 - \frac{\vec{q}}{g_p}\right)^2$$

$$j = \frac{\hbar}{2mi}\left(\psi^* \nabla \psi - \psi \nabla \psi^*\right) + \frac{e|\psi|^2}{m}\vec{A} =$$

$$\psi = F e^{\frac{i}{\hbar}S} \qquad \nabla\psi = \nabla F e^{\frac{i}{\hbar}S} + \frac{i}{\hbar}F \nabla S\, e^{\frac{i}{\hbar}S}$$

$$\psi^*\nabla\psi - \psi\nabla\psi^* = F e^{-\frac{i}{\hbar}S}\left(\nabla F e^{\frac{i}{\hbar}S} + \frac{i}{\hbar} F \nabla S\, e^{\frac{i}{\hbar}S}\right) -$$

$$- F e^{\frac{i}{\hbar}S}\left(\nabla F e^{-\frac{i}{\hbar}S} - \frac{i}{\hbar} F \nabla S\, e^{-\frac{i}{\hbar}S}\right) =$$

$$= \frac{2i}{\hbar} F^2 \nabla S$$

$$j = \frac{\cancel{\hbar}}{\cancel{2}mi}\left(\frac{\cancel{2}i}{\cancel{\hbar}} F^2 \vec{\nabla} S\right) + \frac{e\,\cancel{|\psi|^2}\,F^2}{m}\vec{A} =$$

$$= \frac{F^2}{m}\underbrace{\left(\nabla S + e\vec{A}\right)}_{\vec{g}} = \frac{F^2}{m}\vec{g}$$

$$0 = \nabla j \Rightarrow 0 = \int_{dV} \nabla j = \oint_S j\, d\vec{S} = 0$$

↳ rovnice kontinuity