A SHORT COURSE ON THE PRINCIPLES OF PLASMA DISCHARGES AND MATERIALS PROCESSING

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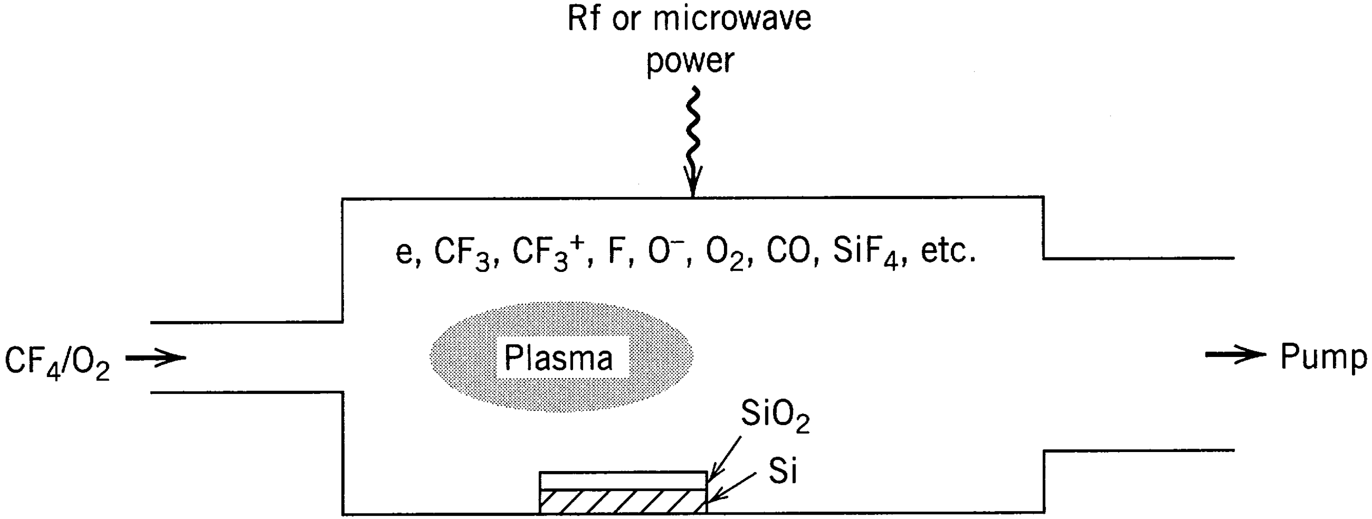

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THE NANOELECTRONICS REVOLUTION

- Transistors/chip doubling every $1\frac{1}{2}$–2 years since 1959
- Billion-fold increase in performance for the same cost over the last 40 years
- CMOS transistors with 4 nm (16 atoms) gate length

EQUIVALENT AUTOMOTIVE ADVANCE

- 60 billion miles/hr (90 × speed of light!)
- 20 billion miles/gal
- 1 cm long × 3 mm wide

DAY 1 (ETCHING EMPHASIS)

- **8:00 AM – 8:30 AM: Registration**
- **8:30 AM – 10:00 AM**
  - Introduction to Plasma Discharges and Processing
  - Summary of Plasma Fundamentals (Undriven)
- **10:00 AM – 10:30 AM: Coffee Break**
- **10:30 AM – 12 Noon**
  - Summary of Plasma Fundamentals (Driven)
  - Summary of Discharge Fundamentals
  - Analysis of Discharge Equilibrium
- **12:00 Noon – 1:30 PM: Lunch**
- **1:30 PM – 3:00 PM**
  - Capacitive RF Discharges: Symmetric Homogeneous Model
  - Capacitive RF Discharges: Self-Consistent Sheath Results
  - Capacitive RF Discharges: Simulation and Experimental Results
  - Capacitive RF Discharges: Example Equilibrium Calculations
- **3:00 PM – 3:30 PM: Coffee Break**
- **3:30 AM – 5:00 PM**
  - Capacitive RF Discharges: Asymmetric Systems
  - Inductive RF Discharges: Transformer Model and Matching
  - Inductive RF Discharges: Power Balance

DAY 2 (ETCHING EMPHASIS)

- **8:30 AM – 9:30 AM**
  - Capacitive RF Sheaths: Ion Transit Time Effects
  - Capacitive RF Sheaths: Ion Energy Distribution (IED)
- **9:30 AM – 10:00 AM: Coffee Break**
- **10:00 AM – 12 Noon**
  - Chemical Fundamentals: Atoms and Molecules
  - Chemical Fundamentals: Gas Phase Kinetics
  - Chemical Fundamentals: Adsorption and Desorption
  - Chemistry in Discharges: Neutral Free Radicals
- **12:00 Noon – 1:30 PM: Lunch**
- **1:30 PM – 3:00 PM**
  - Chemistry in Discharges: Negative Ions
  - Chemistry in Discharges: Example of Oxygen
  - Chemistry in Discharges: Time-Varying Global Models
  - Chemistry in Discharges: Etching Processes
- **3:00 PM – 3:30 PM: Coffee Break**
- **3:30 AM – 5:00 PM**
  - Plasma-Induced Charging Damage OR Pulsed Discharges
  - Dual Frequency Capacitive Discharges

DAY 1 (DEPOSITION EMPHASIS)

- **8:00 AM – 8:30 AM: Registration**
- **8:30 AM – 10:00 AM**
  - Introduction to Plasma Discharges and Processing
  - Summary of Plasma Fundamentals (Undriven)
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  - Capacitive RF Discharges: Simulation and Experimental Results
  - Capacitive RF Discharges: Example Equilibrium Calculations
- **3:00 PM – 3:30 PM: Coffee Break**
- **3:30 AM – 5:00 PM**
  - Capacitive RF Discharges: Asymmetric Systems
  - High Pressure Discharges
  - High Pressure Capacitive Discharges

DAY 2 (DEPOSITION EMPHASIS)

- **8:30 AM – 9:30 AM**
  - Alpha-To-Gamma Transition
- **9:30 AM – 10:00 AM: Coffee Break**
- **10:00 AM – 12 Noon**
  - Chemical Fundamentals: Atoms and Molecules
  - Chemical Fundamentals: Gas Phase Kinetics
  - Chemical Fundamentals: Adsorption and Desorption
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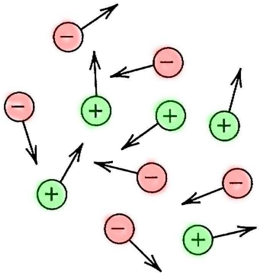
  - Chemistry in Discharges: Deposition Kinetics
- **3:00 PM – 3:30 PM: Coffee Break**
- **3:30 AM – 5:00 PM**
  - Pulsed Discharges
  - Dual Frequency Capacitive Discharges

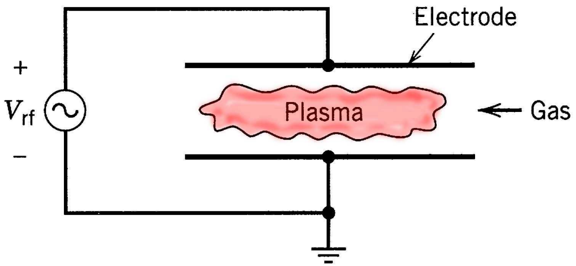
INTRODUCTION TO PLASMA DISCHARGES AND PROCESSING

PLASMAS AND DISCHARGES

- Plasmas
  A collection of freely moving charged particles which is, on the average, electrically neutral
- Discharges
  Are driven by voltage or current sources
  Charged particle collisions with neutral particles are important
  There are boundaries at which surface losses are important
  The electrons are not in thermal equilibrium with the ions



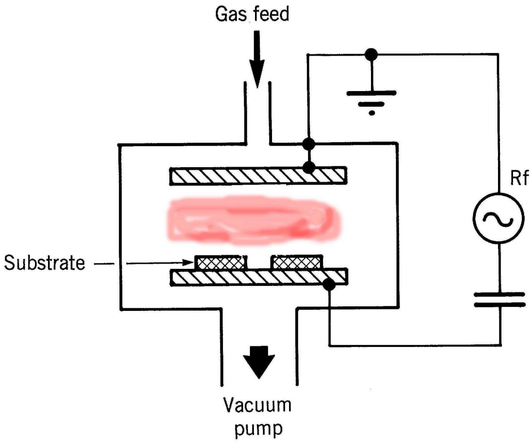
(a)



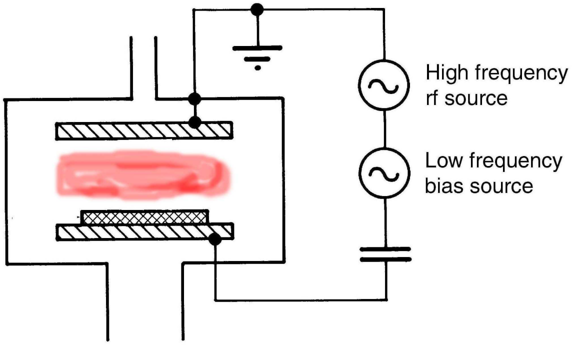
(b)

- Device sizes ∼ 30 cm – 1 m
- Frequencies from DC to rf (13.56 MHz) to microwaves (2.45 GHz)

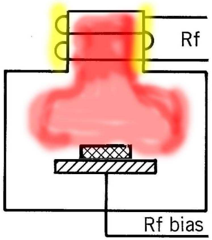
EVOLUTION OF ETCHING DISCHARGES



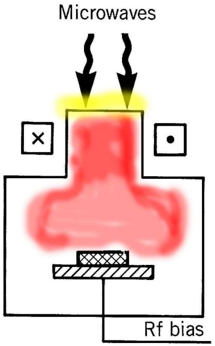
Capacitive
(Anisotropic etch)



Dual frequency
capacitive discharge

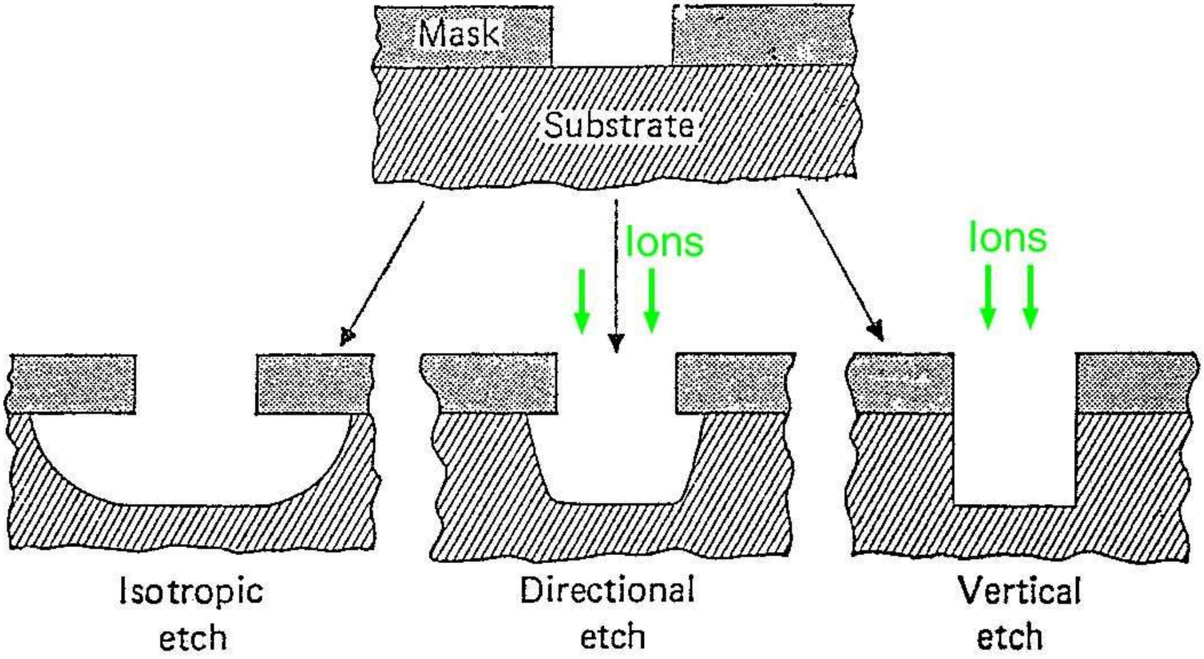


Inductive



ECR

ANISOTROPIC ETCHING



Wet Etching
Plasma Etching

Ion Enhanced Plasma Etching

ISOTROPIC PLASMA ETCHING

1. Start with inert molecular gas CF_4
2. Make discharge to create reactive species

$$\mathrm{CF_4 \longrightarrow CF_3 + F}$$

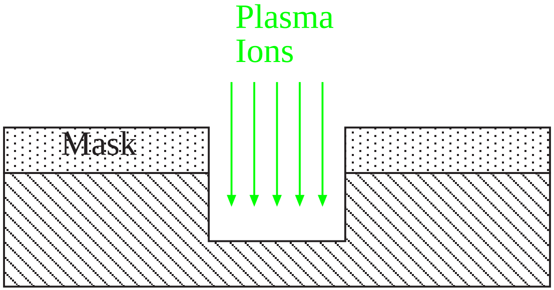
3. Species reacts with material, yielding volatile product

$$\mathrm{Si + 4F \longrightarrow SiF_4} \uparrow$$

4. Pump away product
5. CF_4 does not react with Si; SiF_4 is volatile

ANISOTROPIC PLASMA ETCHING

6. Energetic ions bombard trench bottom, but not sidewalls
   (a) Increase etching reaction rate at trench bottom
   (b) Clear passivating films from trench bottom



UNITS AND CONSTANTS

- SI units: meters (m), kilograms (kg), seconds (s), coulombs (C)
  $e = 1.6 \times 10^{-19}$ C, electron charge $= -e$
- Energy unit is joule (J)
  Often use electron-volt

$$1 \text{ eV} = 1.6 \times 10^{-19} \text{ J}$$

- Temperature unit is kelvin (K)
  Often use equivalent voltage of the temperature

$$\boxed{\mathrm{T}_e(\text{volts}) = \frac{kT_e(\text{kelvins})}{e}}$$

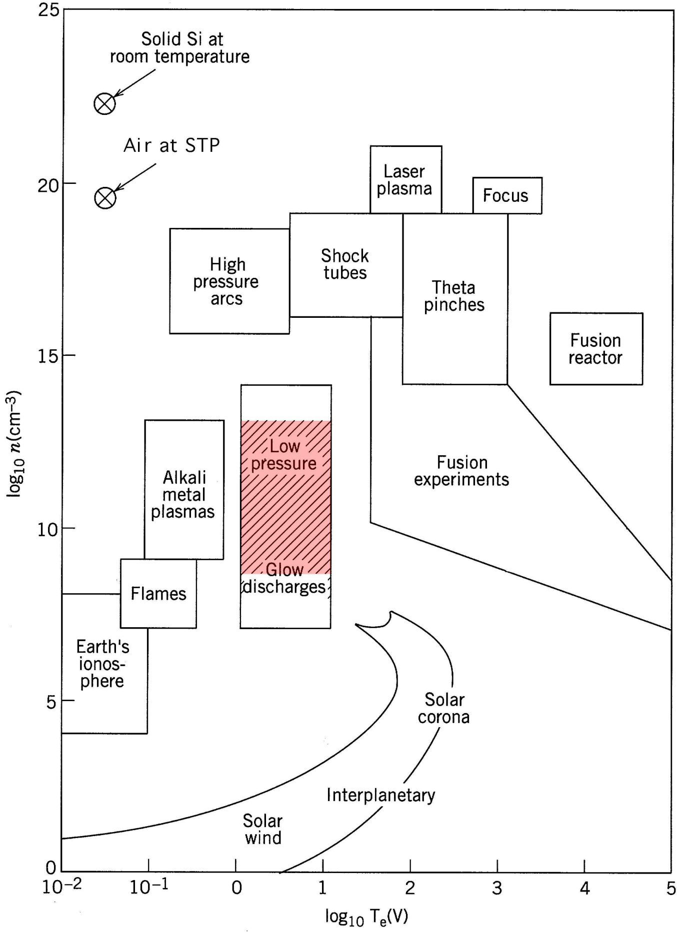
where k = Boltzmann's constant $= 1.38 \times 10^{-23}$ J/K

$$\boxed{1 \text{ V} \Longleftrightarrow 11,600 \text{ K}}$$

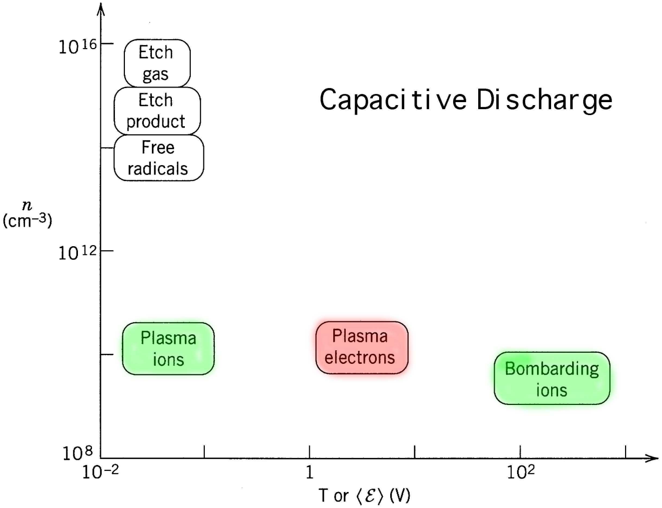
- Pressure unit is pascal (Pa); $1 \text{ Pa} = 1 \text{ N/m}^2$
  Atmospheric pressure $\equiv 1 \text{ bar} \approx 10^5 \text{ Pa} \approx 760 \text{ Torr}$

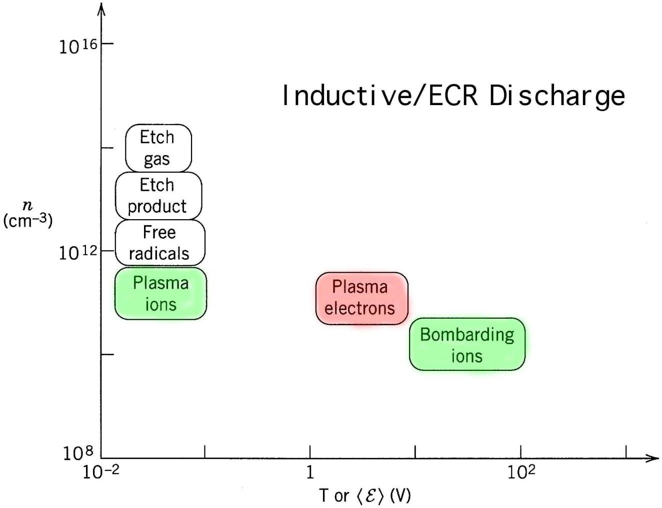
$$\boxed{1 \text{ Pa} \Longleftrightarrow 7.5 \text{ mTorr}}$$

PLASMA DENSITY VERSUS TEMPERATURE



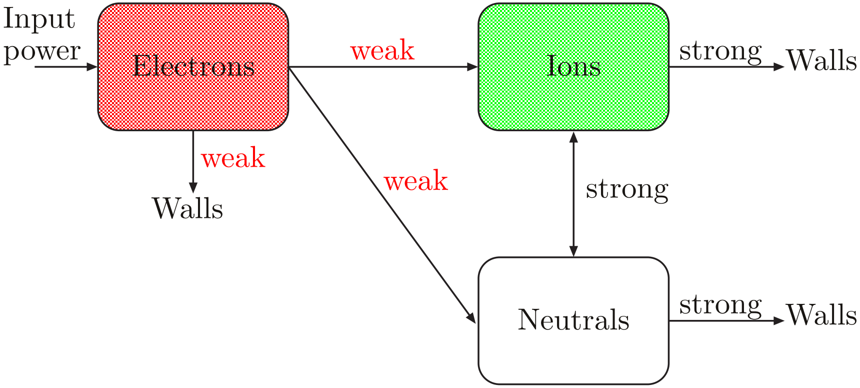
RELATIVE DENSITIES AND ENERGIES





NON-EQUILIBRIUM

- Energy coupling between electrons and heavy particles is weak



Input power

Electrons

weak

Ions

strong

Walls

weak

Walls

weak

strong

Neutrals

strong

Walls

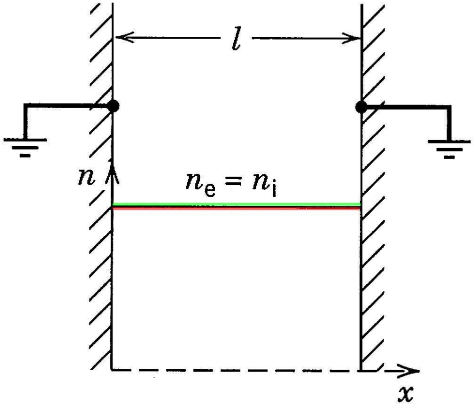
- Electrons are *not* in thermal equilibrium with ions or neutrals

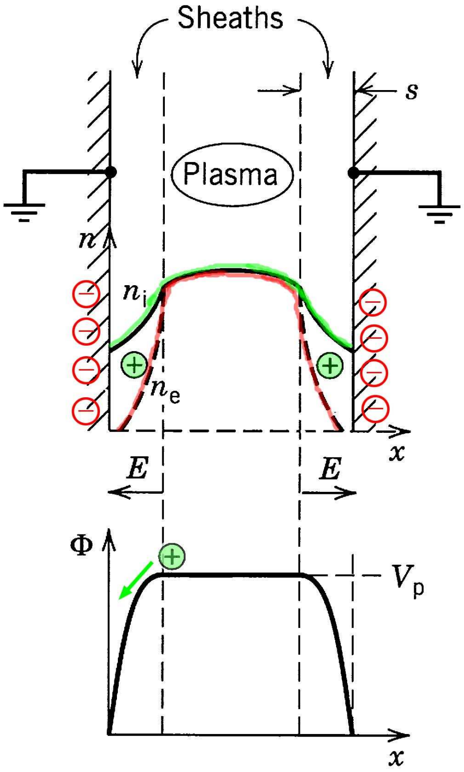
$$\boxed{\begin{array}{c} T_e \gg T_i \quad \text{in plasma bulk} \\[1em] \text{Bombarding ion } \mathcal{E}_i \gg T_e \quad \text{at wafer surface} \end{array}}$$

- “High temperature processing at low temperatures”
  1. Wafer can be near room temperature
  2. Electrons produce free radicals $\Longrightarrow$ chemistry
  3. Electrons produce electron-ion pairs $\Longrightarrow$ ion bombardment

ELEMENTARY DISCHARGE BEHAVIOR

- Uniform density of electrons and ions n_e and n_i at time $t = 0$
- Low mass warm electrons quickly drain to the wall, forming sheaths



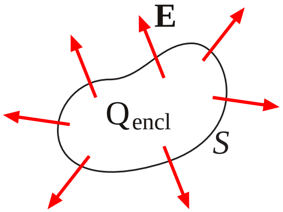


- Bulk plasma is quasi-neutral
  $\Longrightarrow n_e \approx n_i$

- Ions accelerated to walls; ion bombarding energy $\mathcal{E}_i$ = plasma-wall potential V_p

SUMMARY OF PLASMA FUNDAMENTALS

POISSON'S EQUATION



- An electric field can be generated by charges

$$\nabla\cdot\mathbf{E}=\frac{\rho}{\epsilon_0}\qquad\text{or}\qquad\oiint_S\bar{\mathbf{E}}\cdot d\mathbf{A}=\frac{Q_{\text{encl}}}{\epsilon_0}$$

- For slow time variations (dc, rf, but not microwaves)

$$\mathbf{E}\approx-\nabla\Phi$$

  $\mathbf{E}$ = electric field (V/m), ρ = charge density (C/m^3),
  Φ = potential (V), $\epsilon_0 = 8.85\times10^{-12}$ F/m

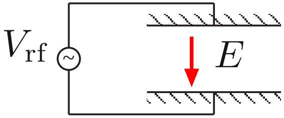
- In 1D planar geometry

$$\boxed{\frac{dE_x}{dx}=\frac{\rho}{\epsilon_0},\qquad\frac{d\Phi}{dx}=-E_x}$$

  Combining these yields Poisson's equation

$$\boxed{\frac{d^2\Phi}{dx^2}=-\frac{\rho}{\epsilon_0}}$$

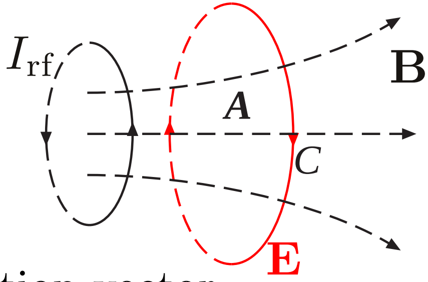
- This field powers a capacitive discharge or the wafer bias power of an inductive or ECR discharge



FARADAY'S LAW

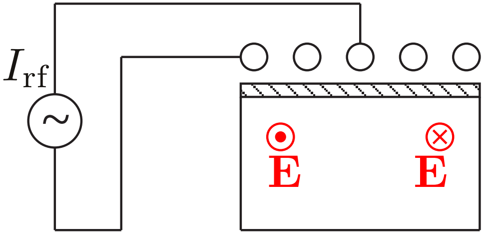
- An electric field can be generated by a time-varying magnetic field

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t} \qquad \text{or} \qquad \oint_{\mathcal{C}} \mathbf{E} \cdot d\mathbf{l} = -\frac{\partial}{\partial t} \iint_{\mathcal{A}} \mathbf{B} \cdot d\mathbf{A}$$



$\mathbf{B}$ = magnetic induction vector

- This field powers the coil of an inductive discharge (top power)



AMPERE'S LAW

- Both conduction currents and displacement currents generate magnetic fields **H**

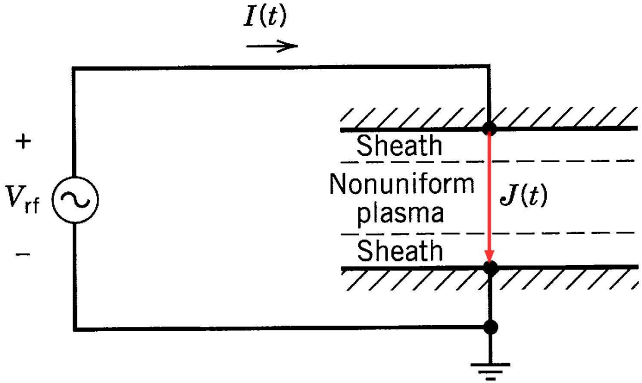
$$\nabla \times \mathbf{H} = \mathbf{J}_c + \epsilon_0 \frac{\partial \mathbf{E}}{\partial t} = \mathbf{J} \qquad [\mathrm{A/m^2}]$$

  $\mathbf{J}_c$ = conduction current density (physical motion of charges)
  $\epsilon_0 \partial \mathbf{E}/\partial t$ = displacement current density (flows in vacuum)
  $\mathbf{J}$ = total current density

- Note the vector identity $\nabla \cdot (\nabla \times \mathbf{H}) = 0 \;\Rightarrow\; \nabla \cdot \mathbf{J} = 0$

- In 1D

$$\boxed{\frac{\partial J(x,\,t)}{\partial x} = 0}$$

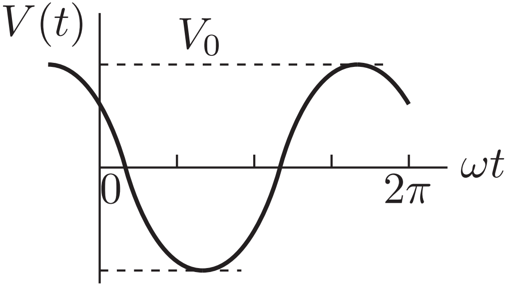


$$\boxed{\text{Total current } J \text{ is independent of } x}$$

REVIEW OF PHASORS

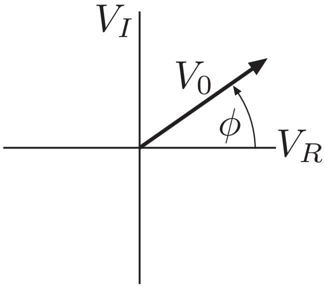
- Physical voltage (or current), a real sinusoidal function of time

$$V(t) = V_0 \cos(\omega t + \phi)$$



- Phasor voltage (or current), a complex number, independent of time

$$\widetilde{V} = V_0\, \mathrm{e}^{j\phi} = V_R + jV_I$$



- Note that

$$\boxed{V(t) = \mathrm{Re}\left(\widetilde{V}\, \mathrm{e}^{j\omega t}\right)}$$

- Hence

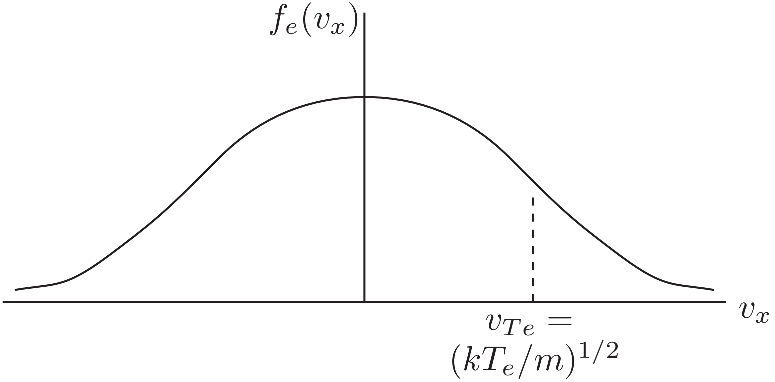
$$V(t) \Longleftrightarrow \widetilde{V} \qquad \text{(given } \omega\text{)}$$

THERMAL EQUILIBRIUM PROPERTIES

- Electrons generally near thermal equilibrium
  Ions generally *not* in thermal equilibrium
- Maxwellian distribution of electrons

$$f_e(v) = n_e \left(\frac{m}{2\pi k T_e}\right)^{3/2} \exp\left(-\frac{mv^2}{2kT_e}\right)$$

where $v^2 = v_x^2 + v_y^2 + v_z^2$



- Pressure $p = nkT$
  For neutral gas at room temperature (300 K)

$$\boxed{n_g(\text{cm}^{-3}) \approx 3.3 \times 10^{16}\, p(\text{Torr})}$$

AVERAGES OVER MAXWELLIAN DISTRIBUTION

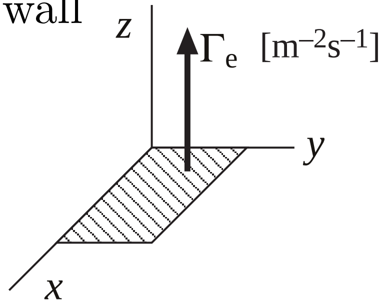
- Average energy

$$\langle \tfrac{1}{2}mv^2\rangle = \frac{1}{n_e}\int d^3v\, \tfrac{1}{2}mv^2 f_e(v) = \tfrac{3}{2}kT_e$$

- Average speed

$$\boxed{\bar{v}_e = \left(\frac{8kT_e}{\pi m}\right)^{1/2}} \qquad \left(= \frac{1}{n_e}\int d^3v\, v\, f_e(v)\right)$$

- Average electron flux lost to a wall



$$\boxed{\Gamma_e = \frac{1}{4}n_e\bar{v}_e} \quad \left(= \int_{-\infty}^{\infty} dv_x \int_{-\infty}^{\infty} dv_y \int_0^{\infty} dv_z v_z f_e(v)\right)$$

- Average kinetic energy lost per electron lost to a wall

$$\boxed{\mathcal{E}_e = 2\,\mathrm{T}_e}$$

FORCES ON PARTICLES

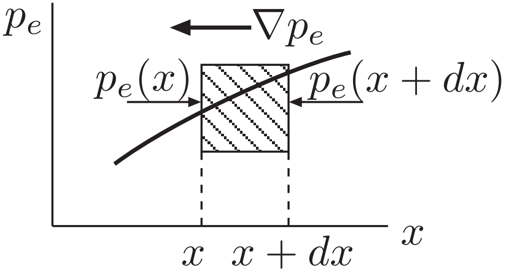
- For a unit volume of electrons (or ions)

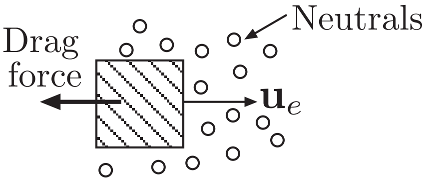
$$\boxed{mn_e\frac{d\mathbf{u}_e}{dt} = qn_e\mathbf{E} - \nabla p_e - mn_e\nu_m\mathbf{u}_e}$$

mass × acceleration = electric field force +

+ pressure gradient force + friction (gas drag) force

- m = electron mass
  n_e = electron density
  $\mathbf{u}_e$ = electron flow velocity
  $q = -e$ for electrons ($+e$ for ions)
  $\mathbf{E}$ = electric field
  $p_e = n_e kT_e$ = electron pressure
  ν_m = collision frequency of electrons with neutrals [p. 36]





BOLTZMANN FACTOR FOR ELECTRONS

- If electric field and pressure gradient forces almost balance

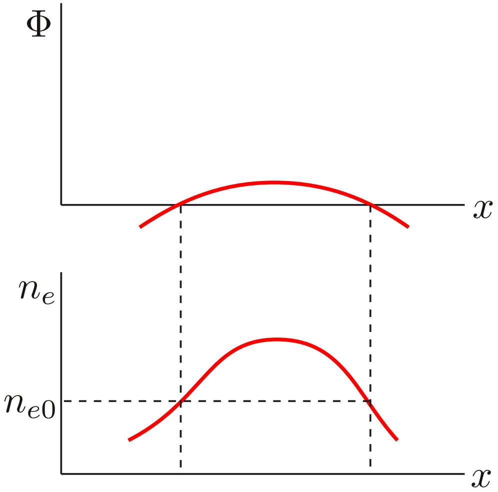
$$0 \approx -en_e\mathbf{E} - \nabla p_e$$

- Let $\mathbf{E} = -\nabla\Phi$ and $p_e = n_e kT_e$

$$\nabla\Phi = \frac{kT_e}{e}\frac{\nabla n_e}{n_e}$$

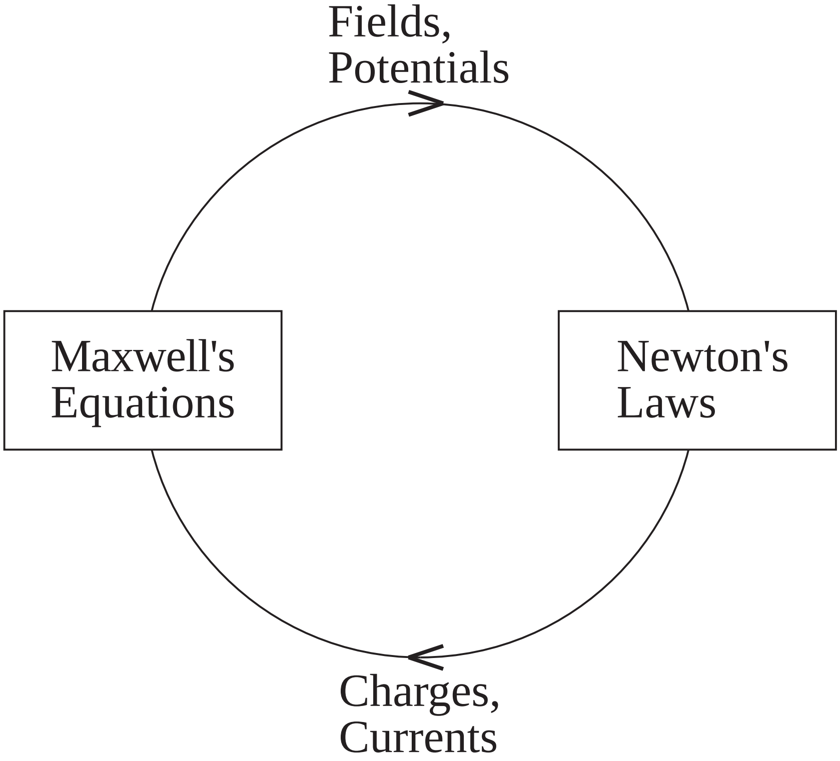
- Put $kT_e/e = \mathrm{T}_e$ (volts) and integrate to obtain

$$\boxed{n_e(\mathbf{r}) = n_{e0}\,\mathrm{e}^{\Phi(\mathbf{r})/\mathrm{T}_e}}$$



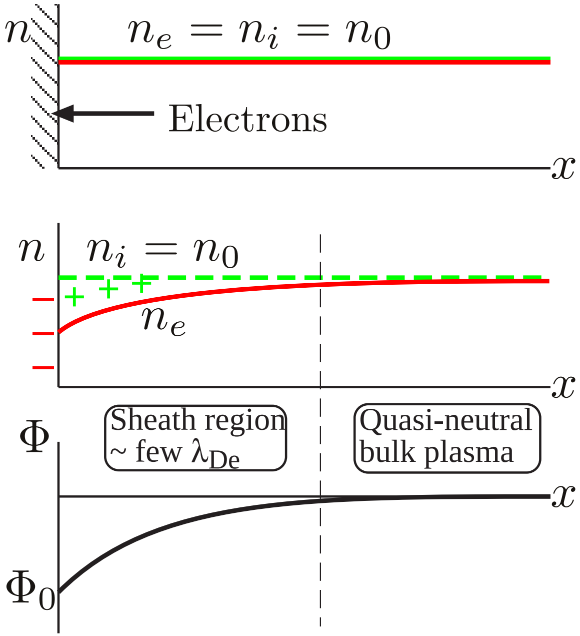
UNDERSTANDING PLASMA BEHAVIOR

- The field equations and the force equations are coupled



DEBYE LENGTH λ_{De}

- The characteristic length scale of a plasma
- Low voltage sheaths ∼ few Debye lengths thick
- Let's consider how a sheath forms near a wall
  Electrons leave plasma before ions and charge wall negative



Assume electrons in thermal equilibrium and stationary ions

DEBYE LENGTH λ_{De} (CONT'D)

- Newton's laws [p. 18]

$$n_e(x) = n_0\, \mathrm{e}^{\Phi/\mathrm{T}_e}, \qquad n_i = n_0$$

- Use in Poisson's equation [p. 12]

$$\frac{d^2\Phi}{dx^2} = -\frac{en_0}{\epsilon_0}\left(1 - \mathrm{e}^{\Phi/\mathrm{T}_e}\right)$$

- Linearize $\mathrm{e}^{\Phi/\mathrm{T}_e} \approx 1 + \Phi/\mathrm{T}_e$

$$\frac{d^2\Phi}{dx^2} = \frac{en_0}{\epsilon_0 \mathrm{T}_e}\Phi$$

- Solution is

$$\Phi(x) = \Phi_0\, \mathrm{e}^{-x/\lambda_{De}}, \qquad \boxed{\lambda_{De} = \left(\frac{\epsilon_0 \mathrm{T}_e}{en_0}\right)^{1/2}}$$

- In practical units

$$\lambda_{De}(\mathrm{cm}) = 740\sqrt{\mathrm{T}_e/n_0}, \qquad \mathrm{T}_e \text{ in volts, } n_0 \text{ in cm}^{-3}$$

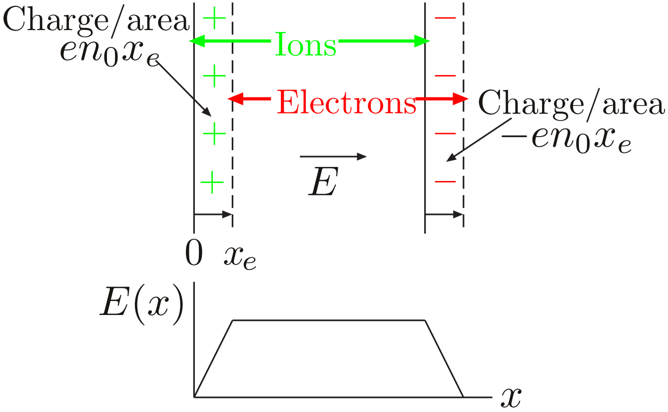
- Example

  At $\mathrm{T}_e = 1$ V and $n_0 = 10^{10}$ cm^{-3}, $\lambda_{De} = 7.4 \times 10^{-3}$ cm

  $\Longrightarrow$ Sheath is $\sim$ 0.15 mm thick (Very thin!)

ELECTRON PLASMA FREQUENCY $\omega_{\mathbf{pe}}$

- The fundamental timescale for a plasma
- Consider a plasma slab (no walls). Displace all electrons to the right a small distance x_{e0}, and release them



- Maxwell's equations (parallel plate capacitor) [p. 12]

$$E = \frac{en_0x_e(t)}{\epsilon_0}$$

ELECTRON PLASMA FREQUENCY $\omega_{\mathbf{pe}}$ (CONT'D)

- Newton's laws (electron motion) [p. 18]

$$m\frac{d^2x_e(t)}{dt^2} = -eE = -\frac{e^2n_0}{\epsilon_0}x_e(t)$$

- Solution is electron plasma oscillations

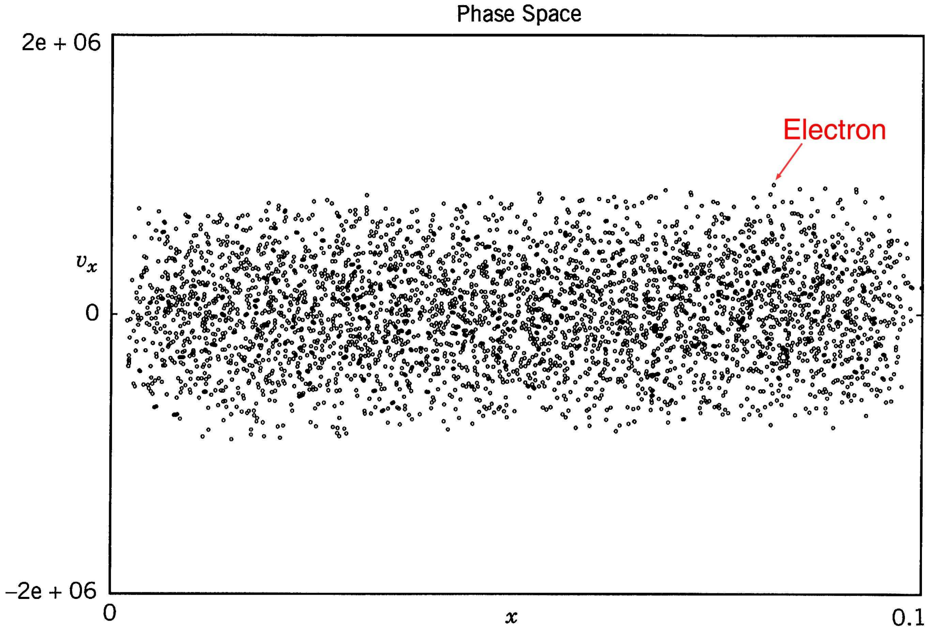
$$x_e(t) = x_{e0}\cos\omega_{pe}t, \qquad \boxed{\omega_{pe} = \left(\frac{e^2n_0}{\epsilon_0 m}\right)^{1/2}}$$

- Practical formula is $f_{pe}(\mathrm{Hz}) = 9000\sqrt{n_0}, \quad n_0$ in cm^{-3}

$\Longrightarrow$ microwave frequencies ($\gtrsim$ 1 GHz) for typical plasmas

1D SIMULATION OF SHEATH FORMATION

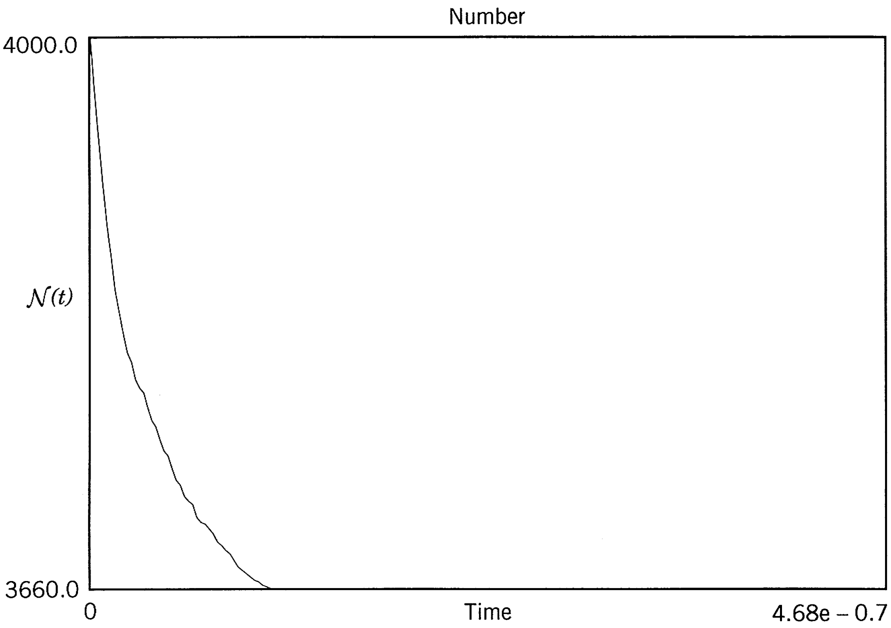
- Particle-in-cell (PIC) simulations with uniform fixed ion density; 4000 electron sheets; solve Newton's laws + Maxwell's equations
- $T_e = 1$ V (random), $n_e = n_i = 10^{13}$ m^{-3} (low), $l = 0.1$ m
- Electron v_x–x phase space at $t = 0.77$ μs



- Note absence of electron sheets near the walls

1D SIMULATION OF SHEATH FORMATION (CONT'D)

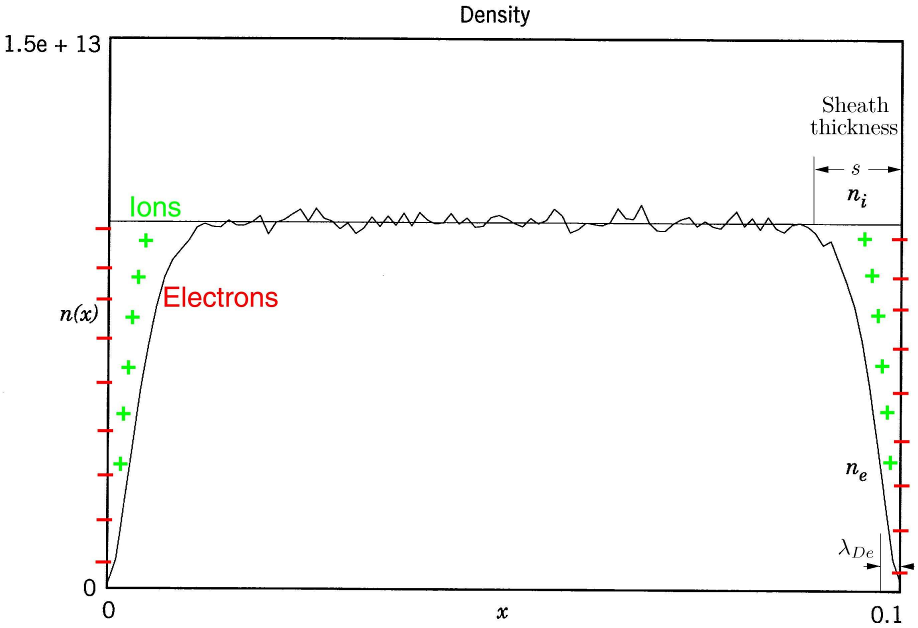
- Electron number $\mathcal{N}$ versus t



- Note 340 electron sheets lost to walls to form sheaths

1D SIMULATION OF SHEATH FORMATION (CONT'D)

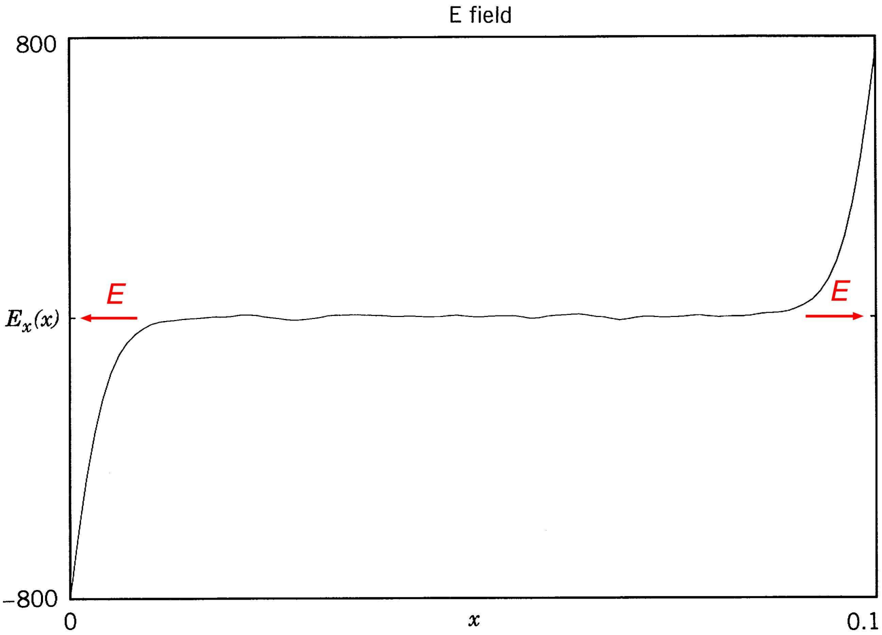
- Electron density $n_e(x)$ at $t = 0.77\ \mu$s



- Note sheath width is a few Debye lengths

1D SIMULATION OF SHEATH FORMATION (CONT'D)

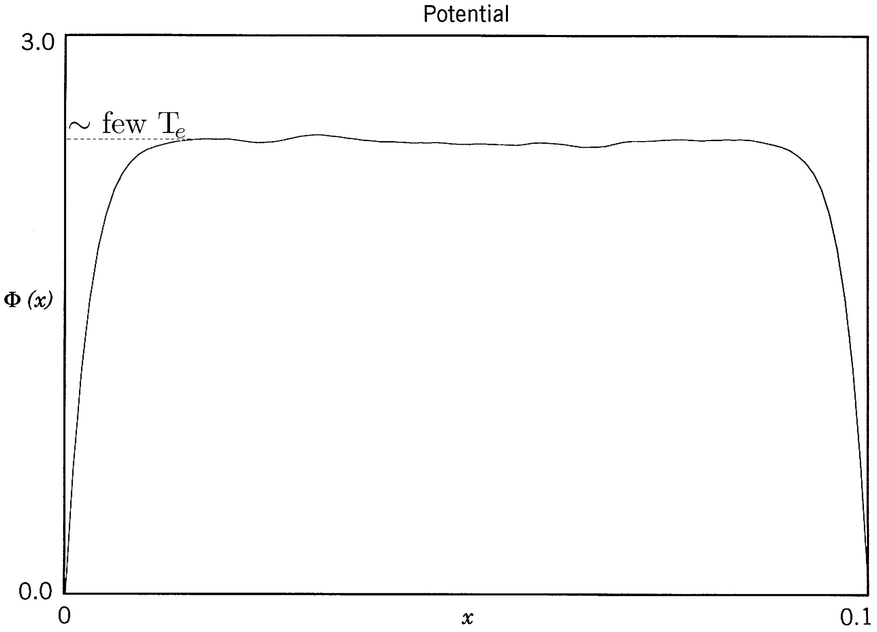
- Electric field $E(x)$ at $t = 0.77\ \mu$s



- Note electric field retards electrons, accelerates ion into walls

1D SIMULATION OF SHEATH FORMATION (CONT'D)

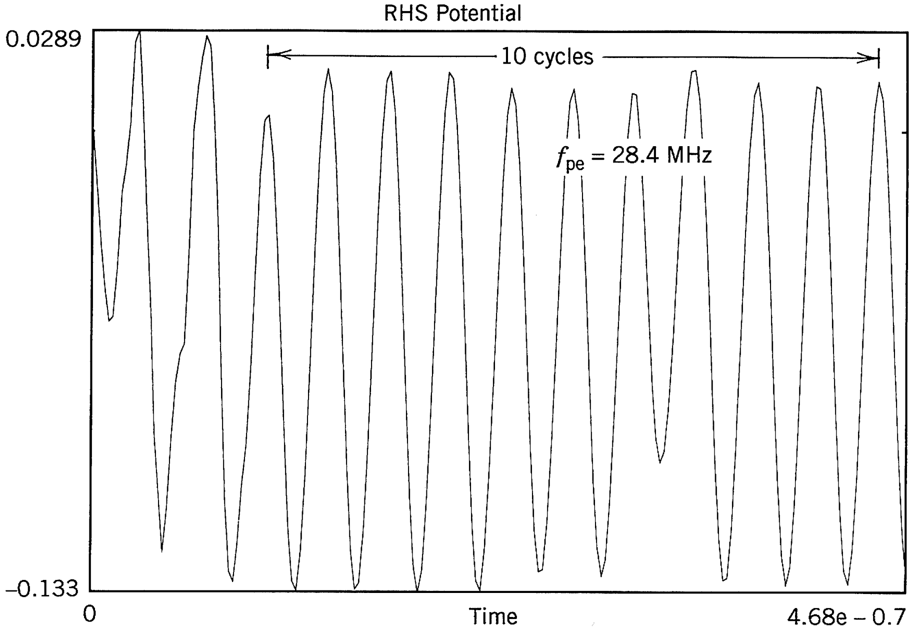
- Potential $\Phi(x)$ at $t = 0.77\ \mu$s



- Note plasma potential builds up to a few T_e with respect to wall

1D SIMULATION OF SHEATH FORMATION (CONT'D)

- Right hand potential $\Phi(x = l)$ versus t



- Due to asymmetric electron initial conditions, a small oscillation of the right hand potential is excited at the plasma frequency

PLASMA DIELECTRIC CONSTANT ϵ_p

- RF discharges are driven at a frequency ω

$$E(t) = \mathrm{Re}\,(\widetilde{E}\,\mathrm{e}^{j\omega t}), \quad \text{etc. [p. 15]}$$

- Define ϵ_p from the total current in Maxwell's equations [p. 14]

$$\nabla \times \tilde{H} = \underbrace{\tilde{J}_c + j\omega\epsilon_0\widetilde{E}}_{\text{Total current } \tilde{J}} \equiv j\omega\epsilon_p\widetilde{E}$$

- Conduction current $\tilde{J}_c = -en_e\tilde{u}_e$ is due to electrons
- Newton's law (electric field and neutral drag) is [p. 18]

$$j\omega m\tilde{u}_e = -e\widetilde{E} - m\nu_m\tilde{u}_e$$

- Solve for $\tilde{u}_e$ and evaluate $\tilde{J}_c$ to obtain

$$\boxed{\epsilon_p \equiv \epsilon_0\kappa_p = \epsilon_0\left[1 - \frac{\omega_{pe}^2}{\omega(\omega - j\nu_m)}\right]}$$

with $\omega_{pe} = (e^2n_e/\epsilon_0 m)^{1/2}$ the electron plasma frequency [p. 23]

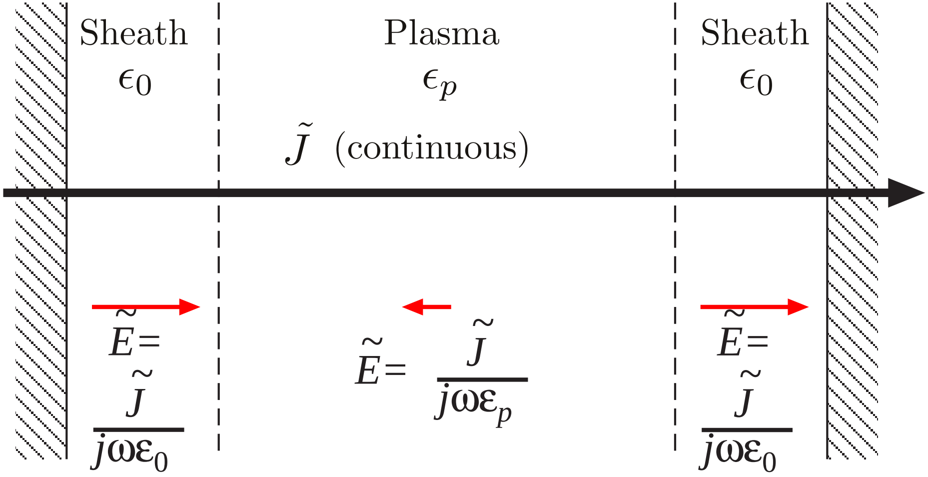
- For $\omega \gg \nu_m$, ϵ_p is mainly real (nearly lossless dielectric)
  For $\nu_m \gg \omega$, ϵ_p is mainly imaginary (very lossy dielectric)

RF FIELDS IN LOW PRESSURE DISCHARGES

- Consider mainly lossless plasma ($\omega \gg \nu_m$)

$$\boxed{\epsilon_p = \epsilon_0 \left(1 - \frac{\omega_{pe}^2}{\omega^2}\right)}$$

- For rf discharges, $\omega_{pe} \gg \omega \implies \epsilon_p$ is negative ($\epsilon_p = -1000\,\epsilon_0$)
- RF current density $\tilde{J}$ is continuous across the discharge [p. 14]



- Electric field in plasma is 1000 × smaller than in sheaths!
- Although field in plasma is small, it sustains the plasma!

PLASMA CONDUCTIVITY $\sigma_{\mathbf{p}}$

- It is useful to introduce rf plasma conductivity $\tilde{J}_c \equiv \sigma_p \widetilde{E}$
- Find $\tilde{J}_c$ to be a linear function of $\widetilde{E}$ [p. 31]

$$\sigma_p = \frac{e^2 n_e}{m(\nu_m + j\omega)}$$

- DC plasma conductivity ($\omega \ll \nu_m$)

$$\boxed{\sigma_{\mathrm{dc}} = \frac{e^2 n_e}{m\nu_m}}$$

- The plasma dielectric constant and conductivity are related by

$$j\omega\epsilon_p = \sigma_p + j\omega\epsilon_0$$

- RF current flowing through the plasma heats electrons (just like a resistor)

OHMIC HEATING POWER

- Time average power absorbed/volume

$$\boxed{p_d = \langle \mathbf{J}(t) \cdot \mathbf{E}(t) \rangle = \frac{1}{2}\,\mathrm{Re}\,(\tilde{J} \cdot \widetilde{E}^*) \qquad [\mathrm{W/m^3}]}$$

Here $\widetilde{E}^*$ = complex conjugate of $\widetilde{E}$

- Since $\tilde{J}$ is the same everywhere in the discharge [p. 32], put $\widetilde{E} = \tilde{J}/j\omega\epsilon_p$ to find p_d in terms of $\tilde{J}$ alone

- For discharges with $\omega \ll \omega_{pe}$ (all rf discharges)

$$\boxed{p_d = \frac{1}{2}|\tilde{J}|^2 \frac{1}{\sigma_{\mathrm{dc}}} \qquad [\mathrm{W/m^3}]}$$

SUMMARY OF DISCHARGE FUNDAMENTALS

ELECTRON COLLISIONS WITH ARGON

- Maxwellian electrons collide with Ar atoms (density n_g)

$$\frac{\text{\# collisions of a particular kind}}{\text{s-m}^3} = \nu n_e = K n_g n_e$$

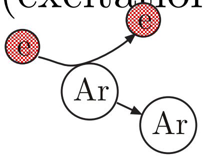
ν = collision frequency [s^{-1}], $K(T_e)$ = rate coefficient [m^3/s]

- Electron-Ar collision processes

$\text{e} + \text{Ar} \longrightarrow \text{Ar}^+ + 2\text{e}$ (ionization)

$\text{e} + \text{Ar} \longrightarrow \text{e} + \text{Ar}^* \longrightarrow \text{e} + \text{Ar} +$ photon (excitation)

$\text{e} + \text{Ar} \longrightarrow \text{e} + \text{Ar}$ (elastic scattering)

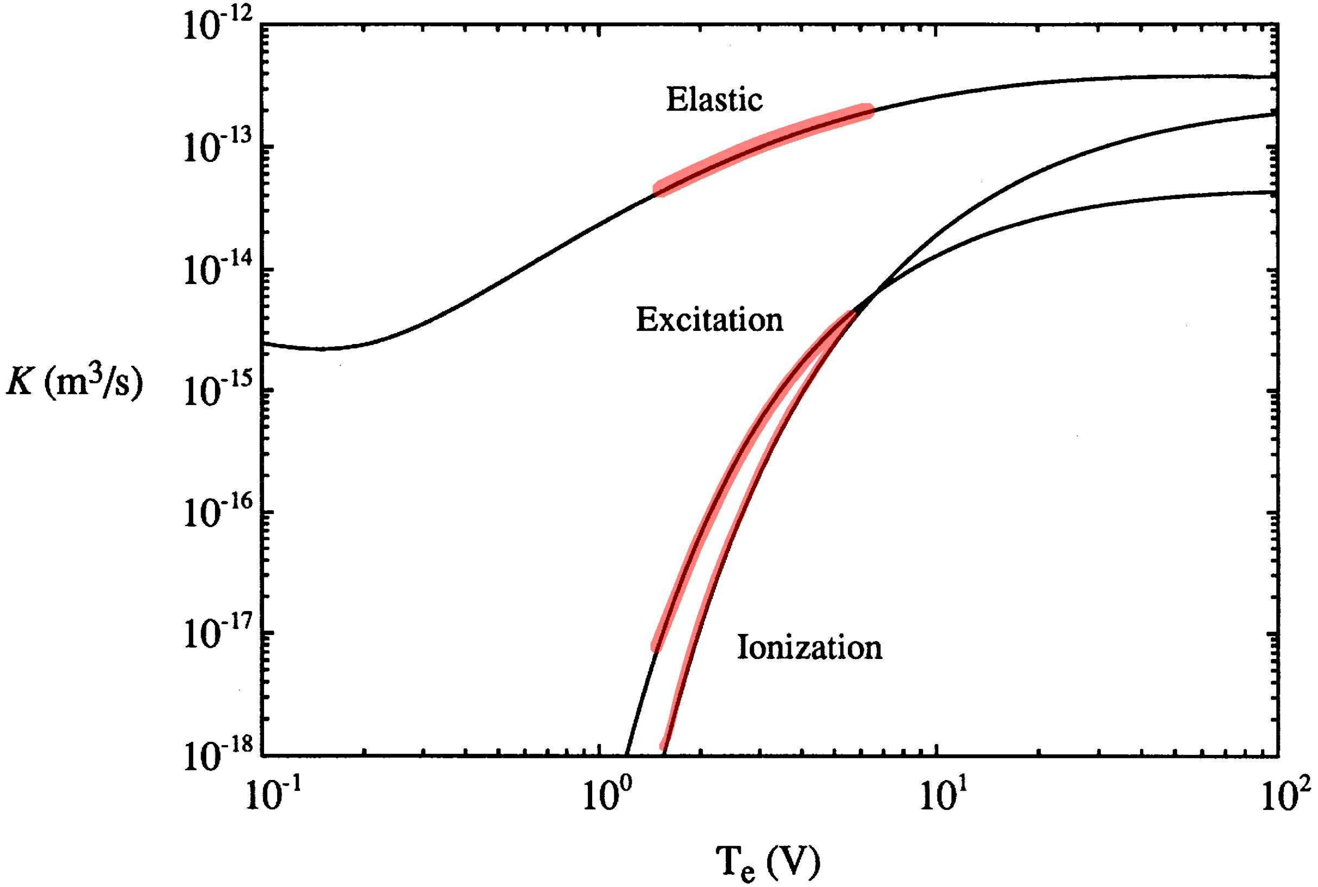


- Rate coefficient $K(T_e)$ is average of cross section $\sigma(v_R)$ [m^2] for process, over Maxwellian distribution

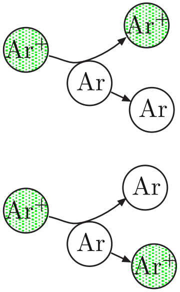
$$K(T_e) = \langle \sigma \, v_R \rangle_{\text{Maxwellian}}$$

v_R = relative velocity of colliding particles

ELECTRON-ARGON RATE COEFFICIENTS



ION COLLISIONS WITH ARGON



- Argon ions collide with Ar atoms

  $\mathrm{Ar^+ + Ar \longrightarrow Ar^+ + Ar}$ (elastic scattering)

  $\mathrm{Ar^+ + Ar \longrightarrow Ar + Ar^+}$ (charge transfer)

- Total cross section for room temperature ions $\sigma_i \approx 10^{-14}\ \mathrm{cm}^2$
- Ion-neutral mean free path (distance ion travels before colliding)

$$\boxed{\lambda_i = \frac{1}{n_g \sigma_i}}$$

- Practical formula

$$\boxed{\lambda_i(\mathrm{cm}) = \frac{1}{330\,p}, \qquad p \text{ in Torr}}$$

- Ion-neutral collision frequency

$$\nu_i = \frac{\bar{v}_i}{\lambda_i}$$

with $\bar{v}_i = (8kT_i/\pi M)^{1/2}$

THREE ENERGY LOSS PROCESSES

1. Collisional energy $\mathcal{E}_c$ lost per electron-ion pair created

$$K_{\rm iz}\mathcal{E}_c = K_{\rm iz}\mathcal{E}_{\rm iz} + K_{\rm ex}\mathcal{E}_{\rm ex} + K_{\rm el}(2m/M)(3{\rm T}_e/2)$$

$$\Longrightarrow \mathcal{E}_c({\rm T}_e) \qquad \text{(voltage units)}$$

   $\mathcal{E}_{\rm iz}$, $\mathcal{E}_{\rm ex}$, and $(3m/M){\rm T}_e$ are energies lost by an electron due to an ionization, excitation, and elastic scattering collision

2. Electron kinetic energy lost to walls [p. 17]

$$\mathcal{E}_e = 2\,{\rm T}_e$$

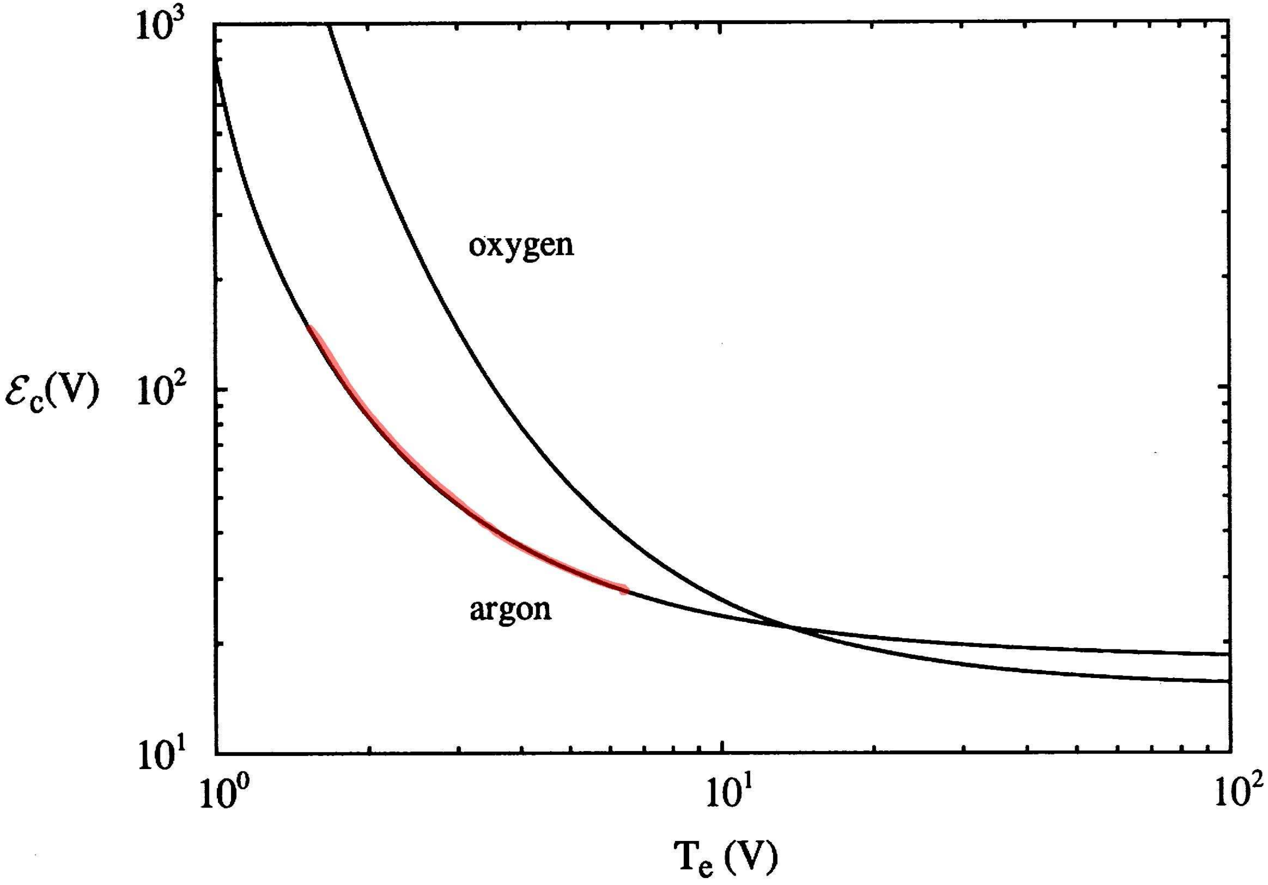
3. Ion kinetic energy lost to walls is mainly due to the dc potential $\bar{V}_s$ across the sheath

$$\mathcal{E}_i \approx \bar{V}_s$$

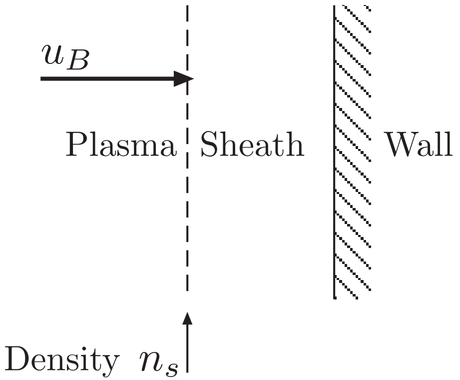
- Total energy lost per electron-ion pair lost to walls

$$\boxed{\mathcal{E}_T = \mathcal{E}_c + \mathcal{E}_e + \mathcal{E}_i}$$

COLLISIONAL ENERGY LOSSES



BOHM (ION LOSS) VELOCITY u_B



- Due to formation of a "presheath", ions arrive at the plasma-sheath edge with directed energy $kT_e/2$

$$\frac{1}{2}Mu_B^2 = \frac{kT_e}{2}$$

- Electron-ion pairs are lost at the Bohm velocity at the plasma-sheath edge (density n_s)

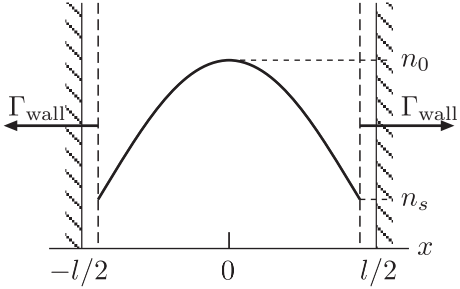
$$\boxed{\Gamma_{\text{wall}} = n_s u_B, \qquad u_B = \left(\frac{kT_e}{M}\right)^{1/2}}$$

AMBIPOLAR DIFFUSION AT HIGH PRESSURES

- Plasma bulk is quasi-neutral ($n_e \approx n_i = n$) and the electron and ion loss fluxes are equal ($\Gamma_e \approx \Gamma_i \approx \Gamma$)
- Fick's law

$$\Gamma = -D_a \nabla n$$

  with ambipolar diffusion coefficient $D_a = kT_e / M\nu_i$
- Density profile is sinusoidal



- Loss flux to the wall is

$$\Gamma_{\text{wall}} = n_s u_B \equiv h_l n_0 u_B$$

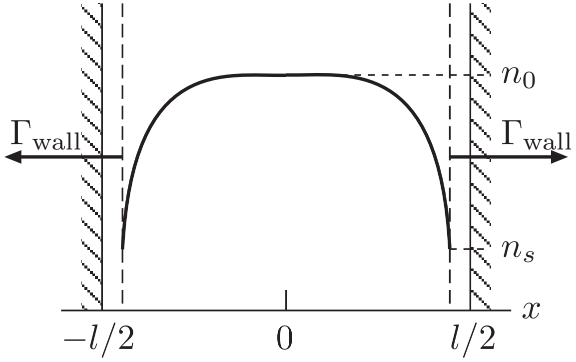
- From diffusion theory, edge-to-center density ratio is

$$h_l \equiv \frac{n_s}{n_0} = \frac{\pi}{l} \frac{u_B}{\nu_i}$$

- Applies for pressures > 100 mTorr in argon

AMBIPOLAR DIFFUSION AT LOW PRESSURES

- The diffusion coefficient is not constant
- Density profile is relatively flat in the center and falls sharply near the sheath edge

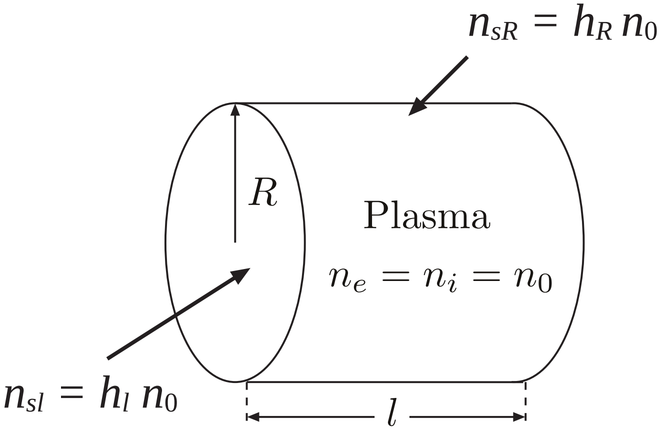


- The edge-to-center density ratio is

$$\boxed{h_l \equiv \frac{n_s}{n_0} \approx \frac{0.86}{(3 + l/2\lambda_i)^{1/2}}}$$

where λ_i = ion-neutral mean free path [p. 38]
- Applies for pressures < 100 mTorr in argon

AMBIPOLAR DIFFUSION IN LOW PRESSURE CYLINDRICAL DISCHARGE



- For a cylindrical plasma of length l and radius R, loss fluxes to axial and radial walls are

$$\boxed{\Gamma_{\text{axial}} = h_l n_0 u_B, \qquad \Gamma_{\text{radial}} = h_R n_0 u_B}$$

where the edge-to-center density ratios are

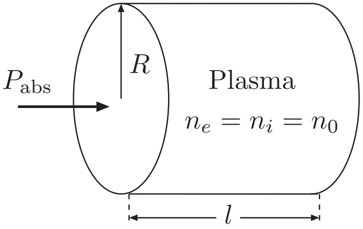
$$\boxed{h_l \approx \frac{0.86}{(3 + l/2\lambda_i)^{1/2}}, \qquad h_R \approx \frac{0.8}{(4 + R/\lambda_i)^{1/2}}}$$

- Applies for pressures < 100 mTorr in argon

ANALYSIS OF DISCHARGE EQUILIBRIUM

PARTICLE BALANCE AND $\mathbf{T_e}$

- Assume uniform cylindrical plasma absorbing power $P_{\rm abs}$



- Particle balance

$$\text{Production due to ionization} = \text{loss to the walls}$$

$$\boxed{K_{\rm iz} n_g \cancel{n_0} \pi R^2 l = (2\pi R^2 h_l \cancel{n_0} + 2\pi R l h_R \cancel{n_0}) u_B}$$

- Solve to obtain

$$\boxed{\frac{K_{\rm iz}(T_e)}{u_B(T_e)} = \frac{1}{n_g d_{\rm eff}}}$$

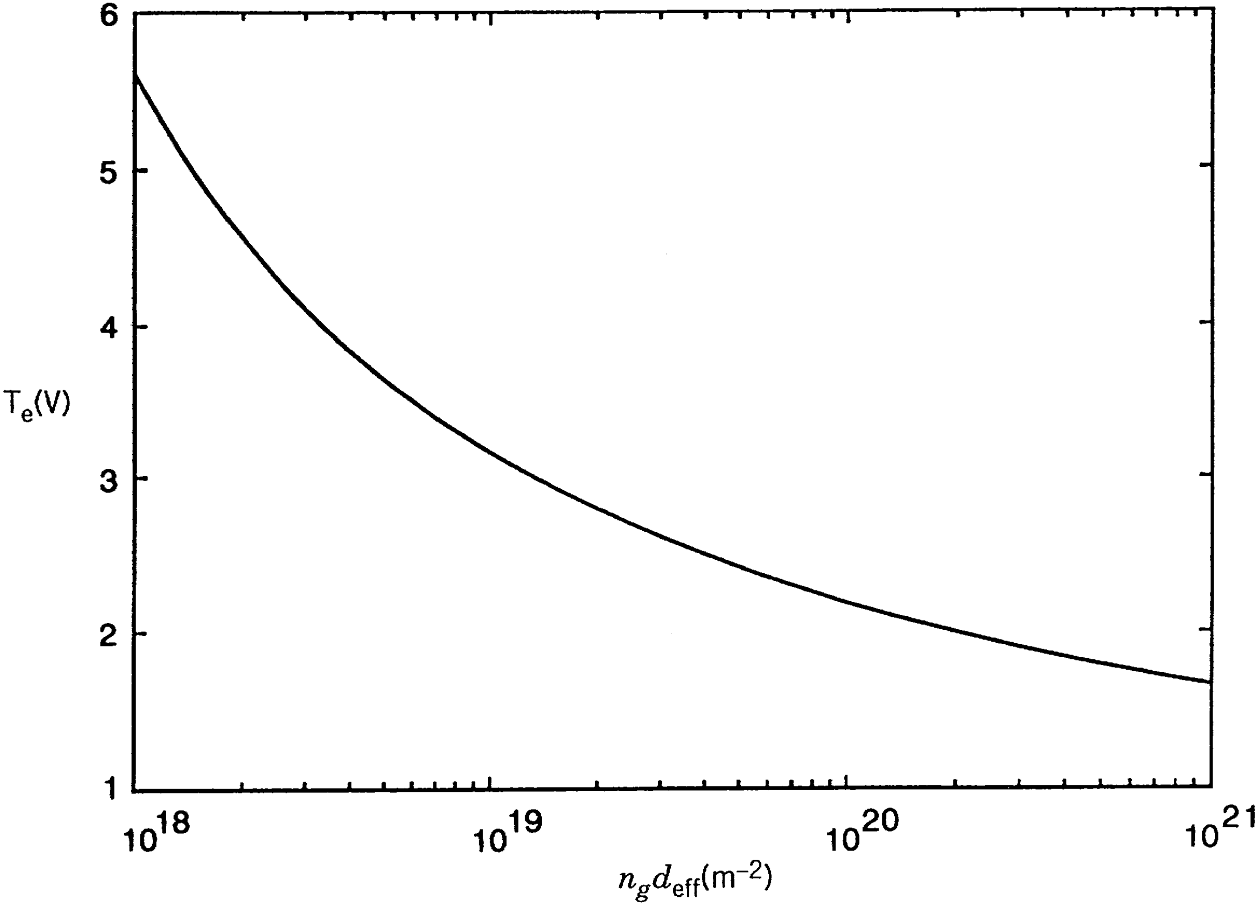
where

$$d_{\rm eff} = \frac{1}{2}\frac{Rl}{Rh_l + lh_R}$$

is an effective plasma size

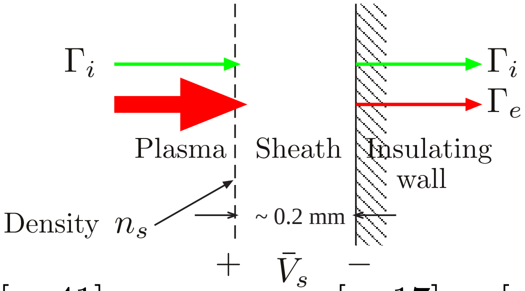
- Given n_g and $d_{\rm eff}$ $\Longrightarrow$ electron temperature T_e
- T_e varies over a narrow range of 2–5 volts

ELECTRON TEMPERATURE IN ARGON DISCHARGE



ION ENERGY FOR LOW VOLTAGE SHEATHS

- $\mathcal{E}_i$ = energy entering sheath + energy gained traversing sheath
- Ion energy entering sheath = $\mathrm{T}_e/2$ (voltage units) [p. 41]
- Sheath voltage determined from particle conservation



[p. 41] [p. 17] [p. 19]

$$\Gamma_i = n_s u_B, \qquad \Gamma_e = \underbrace{\frac{1}{4} n_s \bar{v}_e}_{\text{Random flux at sheath edge}} \mathrm{e}^{-\overline{V}_s/\mathrm{T}_e}$$

with $\bar{v}_e = (8e\mathrm{T}_e/\pi m)^{1/2}$

- The ion and electron fluxes at the wall must balance

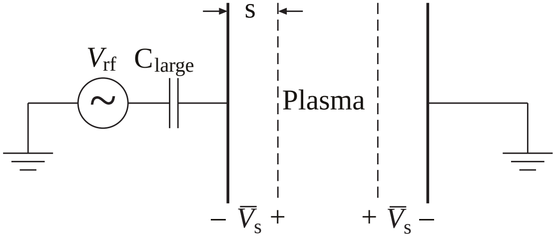
$$\boxed{\overline{V}_s = \frac{\mathrm{T}_e}{2} \ln\left(\frac{M}{2\pi m}\right)}$$

or $\overline{V}_s \approx 4.7\,\mathrm{T}_e$ for argon

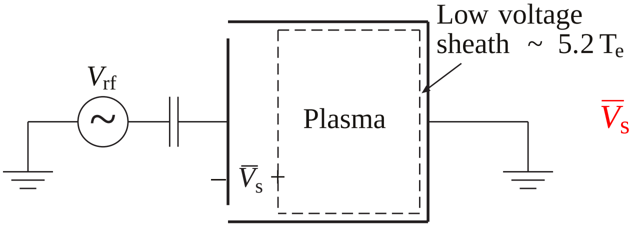
- Accounting for the initial ion energy, $\mathcal{E}_i \approx 5.2\,\mathrm{T}_e$

ION ENERGY FOR HIGH VOLTAGE SHEATHS

- Large ion bombarding energies can be gained near rf-driven electrodes embedded in the plasma



$\bar{V}_s \sim 0.4\ V_{rf}$



$\bar{V}_s \sim 0.8\ V_{rf}$

- The sheath thickness s (~ 0.5 cm) is given by the Child Law

$$\bar{J}_i = e n_s u_B = \frac{4}{9}\epsilon_0 \left(\frac{2e}{M}\right)^{1/2} \frac{\bar{V}_s^{3/2}}{s^2}$$

- Estimating ion energy is not simple as it depends on the type of discharge and the application of bias voltages

POWER BALANCE AND n_0

- Assume low voltage sheaths at all surfaces

$$\mathcal{E}_T(\mathrm{T}_e) = \overset{\text{[p. 40]}}{\underbrace{\mathcal{E}_c(\mathrm{T}_e)}_{\text{Collisional}}} + \overset{\text{[p. 17]}}{\underbrace{2\,\mathrm{T}_e}_{\text{Electron}}} + \overset{\text{[p. 48]}}{\underbrace{5.2\,\mathrm{T}_e}_{\text{Ion}}} \qquad \text{[V]}$$

- Power balance

$$\text{Power in} = \text{power out}$$

$$\boxed{P_{\text{abs}} = (h_l n_0 2\pi R^2 + h_R n_0 2\pi R l)\, u_B\, e\mathcal{E}_T} \qquad \text{[W]}$$

- Solve to obtain

$$\boxed{n_0 = \frac{P_{\text{abs}}}{A_{\text{eff}} u_B e\mathcal{E}_T}}$$

where

$$A_{\text{eff}} = 2\pi R^2 h_l + 2\pi R l h_R$$

is an effective area for particle loss

- Density n_0 is proportional to the absorbed power P_{abs}
- Density n_0 depends on pressure p through h_l, h_R, and T_e

PARTICLE AND POWER BALANCE

- Particle balance $\Longrightarrow$ electron temperature T_e (independent of plasma density)
- Power balance $\Longrightarrow$ plasma density n_0 (once electron temperature T_e is known)

EXAMPLE 1

- Let $R = 0.15$ m, $l = 0.3$ m, $n_g = 3.3 \times 10^{19}$ m^{-3} ($p = 1$ mTorr at 300 K), and $P_{\text{abs}} = 800$ W
- Assume low voltage sheaths at all surfaces

- Find $\lambda_i = 0.03$ m [p. 38]. Then $h_l \approx h_R \approx 0.3$ [p. 44] and $d_{\text{eff}} \approx 0.17$ m [p. 46]
- T_e versus $n_g d_{\text{eff}}$ figure gives $\mathrm{T}_e \approx 3.5$ V [p. 47]
- $\mathcal{E}_c$ versus T_e figure gives $\mathcal{E}_c \approx 42$ V [p. 40]. Adding $\mathcal{E}_e = 2\mathrm{T}_e \approx 7$ V and $\mathcal{E}_i \approx 5.2\mathrm{T}_e \approx 18$ V yields $\mathcal{E}_T = 67$ V [p. 39]
- Find $u_B \approx 2.9 \times 10^3$ m/s [p. 41] and find $A_{\text{eff}} \approx 0.13$ m^2 [p. 50]
- Power balance yields $n_0 \approx 2.0 \times 10^{17}$ m^{-3} [p. 50]

- Ion current density $J_{il} = e h_l n_0 u_B \approx 2.9$ mA/cm^2 [p. 46]
- Ion bombarding energy $\mathcal{E}_i \approx 18$ V [p. 48]

EXAMPLE 2

- Apply a strong dc magnetic field along the cylinder axis
  $\Longrightarrow$ particle loss to radial wall is inhibited
- Assume no radial losses, then $d_{\text{eff}} = l/2h_l \approx 0.5$ m
- From the T$_e$ versus $n_g d_{\text{eff}}$ figure, T$_e \approx 3.3$ V (was 3.5 V)
- From the $\mathcal{E}_c$ versus T$_e$ figure, $\mathcal{E}_c \approx 46$ V. Adding $\mathcal{E}_e = 2\text{T}_e \approx 6.6$ V and $\mathcal{E}_i \approx 5.2\text{T}_e \approx 17$ V yields $\mathcal{E}_T = 70$ V
- Find $u_B \approx 2.8 \times 10^3$ m/s and find $A_{\text{eff}} = 2\pi R^2 h_l \approx 0.043$ m^2
- Power balance yields $n_0 \approx 5.8 \times 10^{17}$ m^{-3} (was 2×10^{17} m^{-3})
- Ion current density $J_{il} = e h_l n_0 u_B \approx 7.8$ mA/cm^2
- Ion bombarding energy $\mathcal{E}_i \approx 17$ V

$\Longrightarrow$ Slight decrease in electron temperature T_e

$\Longrightarrow$ Significant increase in plasma density n_0

EXPLAIN WHY!

- What happens to T_e and n_0 if there is a sheath voltage $V_s = 500$ V at each end plate?

CAPACITIVE RF DISCHARGES

SYMMETRIC HOMOGENEOUS MODEL

BASIC PROPERTIES

- Simplicity of concept
- RF rather than microwave powered
- Inherent high sheath voltages
- No independent control of plasma density and ion energy

- Control parameters

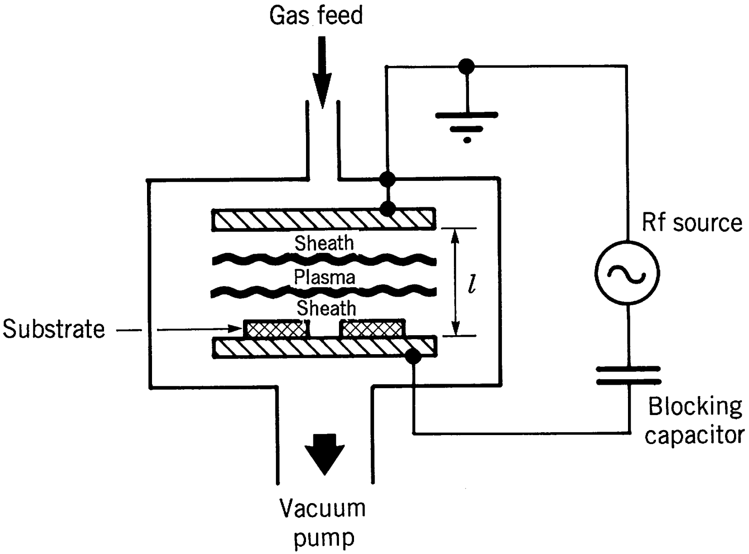
  RF current $\tilde{I}_{\text{rf}}$ (1–10 mA/cm^2)
  Driving frequency ω (2–13.56 MHz)
  Neutral gas density n_g (10^{14}–10^{16} cm^{-3})
  Electrode separation l (1–10 cm)

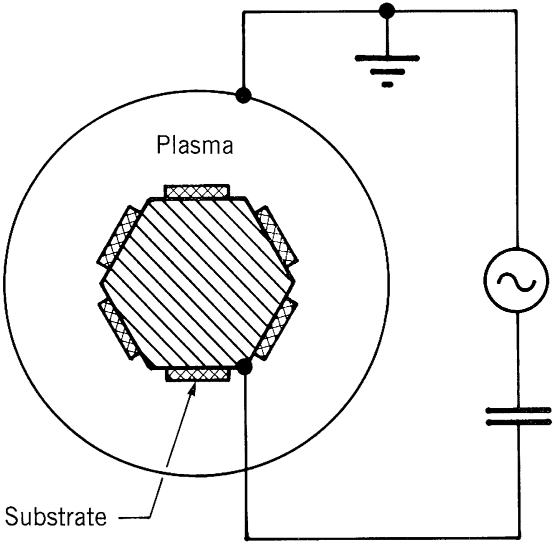
- Discharge parameters to find

  Plasma density n (10^9–10^{10} cm^{-3})
  Electron temperature T_e (2–4 V)
  Discharge voltage V_{rf} (100–1000 V)
  Discharge power P_{rf} (50–500 W)
  Ion bombarding energy $\mathcal{E}_i$ (50–500 V)

CONFIGURATIONS

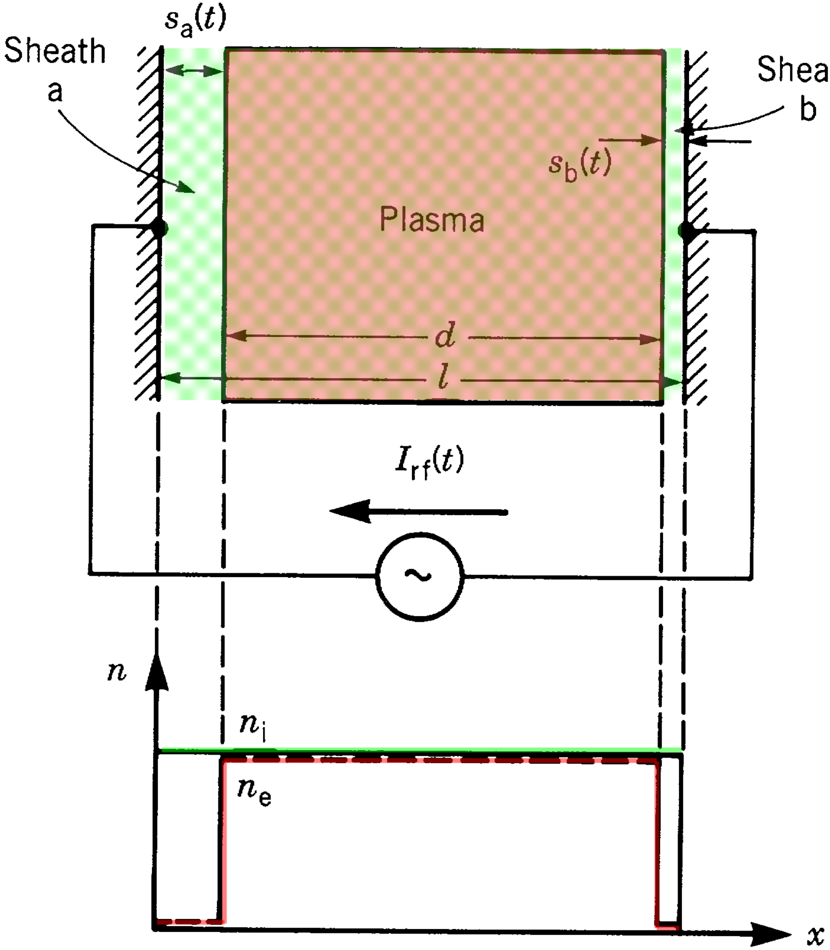
- Multi-wafer parallel plate and “hex” configurations (1980’s)





- Modern configurations are single wafer parallel plate, sometimes driven at multiple rf frequencies

CURRENT-DRIVEN HOMOGENEOUS MODEL



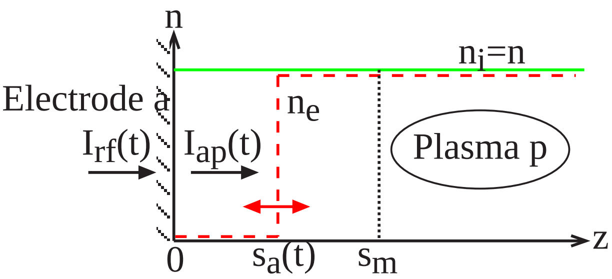
HOMOGENEOUS MODEL ASSUMPTIONS

- No transverse variations (along the electrodes)
- Electrons respond to instantaneous electric fields
- Ions respond to only time-average electric fields
- Electron density is zero in the sheath regions
- Ion density is constant in the plasma and sheath regions

$$n_i(z) = n_0$$

(We will correct this later)

ELECTRON SHEATH EDGE MOTION

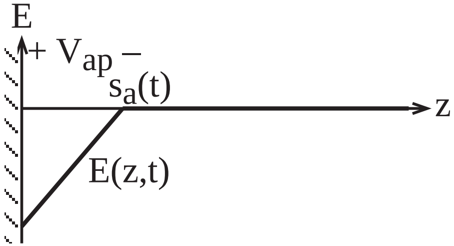


- The electric field is found by integrating the charge density in the sheath [p. 12]

$$\frac{dE}{dz} = \frac{en}{\epsilon_0}, \qquad z < s_a(t)$$

to obtain

$$E(z,t) = \frac{en}{\epsilon_0}\big(z - s_a(t)\big)$$



ELECTRON SHEATH EDGE MOTION (CONT'D)

- The displacement current in the sheath is [p. 14]

$$I_{ap} = \epsilon_0 A \frac{\partial E}{\partial t} = -enA \frac{ds_a}{dt}$$

- Let $I_{\mathrm{rf}}(t) = \tilde{I}_0 \cos \omega t$ and integrate to obtain

$$s_a(t) = \bar{s}_0 - \tilde{s}_0 \sin \omega t$$

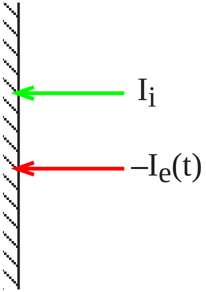
$$\boxed{\tilde{s}_0 = \frac{\tilde{I}_0}{en\omega A}}$$

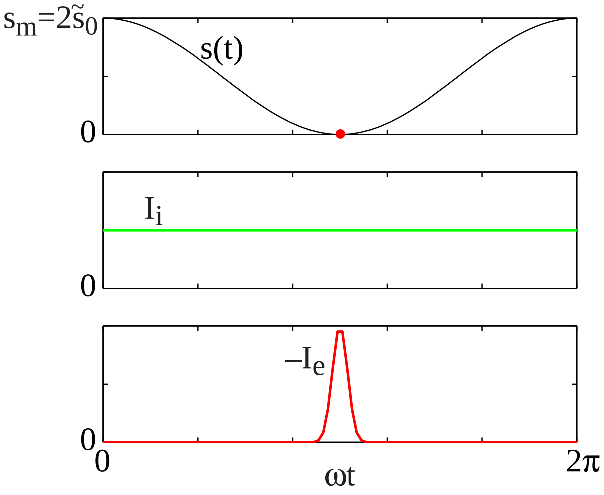
- The oscillation amplitude of the sheath motion is $\tilde{s}_0$, but what is the "constant of integration" $\bar{s}_0$?

CONDUCTION CURRENT

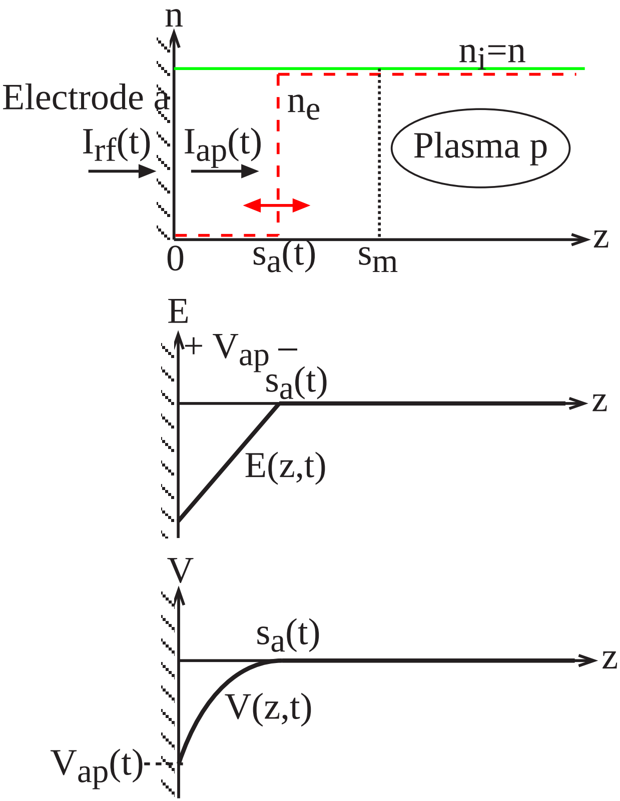
- Assume a steady loss of ions to electrode a
$$I_i = enu_B A$$
- The time-average total conduction current to the electrode is zero
- Hence electrons must be lost to the electrode
- The sheath thickness $s_a(t)$ must then collapse to zero at some time during the rf cycle $\Longrightarrow \bar{s}_0 = \tilde{s}_0$

$$\boxed{s_a(t) = \tilde{s}_0(1 - \sin\omega t)}$$





VOLTAGE ACROSS THE SHEATH



The voltage is found by integrating the electric field in the sheath

$$\frac{dV}{dz} = -E$$

VOLTAGE ACROSS THE SHEATH (CONT'D)

- Integrating the electric field in the sheath [p. 12]

$$\frac{dV}{dz} = -E$$

  we obtain

$$V_{ap}(t) = \int_0^{s_a(t)} E(z,t)\, dz = -\frac{en}{\epsilon_0}\frac{s_a^2(t)}{2}$$

- Using $s_a(t)$ [p. 61]

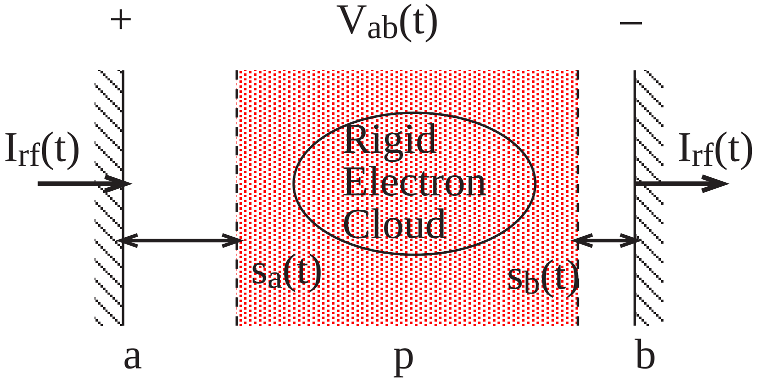
$$\boxed{V_{ap}(t) = -\frac{en}{2\epsilon_0}\tilde{s}_0^2\big(1 - \sin\omega t\big)^2}$$

- $V_{ap}(t)$ is a nonlinear function of $I_{\rm rf}$; there are second harmonics

VOLTAGE ACROSS BOTH SHEATHS

- By symmetry $s_b(t) = \tilde{s}_0(1 + \sin\omega t)$; since $s_a(t) = \tilde{s}_0(1 - \sin\omega t)$

$$\boxed{s_a(t) + s_b(t) = 2\,\tilde{s}_0 = \text{const}}$$



- There is a rigid bulk electron cloud oscillation

VOLTAGE ACROSS BOTH SHEATHS (CONT'D)

- Voltage across sheath b is

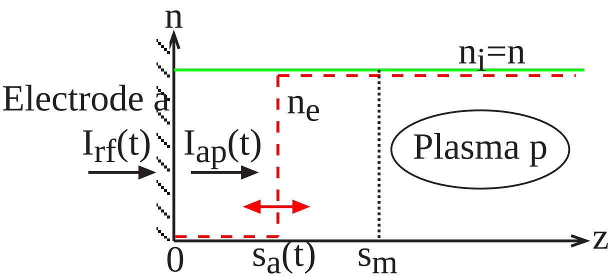
$$V_{bp}(t) = -\frac{en}{2\epsilon_0}\tilde{s}_0^2\bigl(1 + \sin\omega t\bigr)^2$$

- Voltage across the plasma is small because $I_{\mathrm{rf}} = j\omega\epsilon_p E A$ and ϵ_p is large $\Longrightarrow$ E across bulk plasma is small
- Discharge voltage is $V_{\mathrm{rf}} = V_{ap} + V_{pb}$

$$\boxed{V_{\mathrm{rf}}(t) = \frac{2en\tilde{s}_0^2}{\epsilon_0}\sin\omega t}$$

- Each sheath is nonlinear, but the combination of both sheaths is linear

DC VOLTAGE ACROSS ONE SHEATH



$$V_{pa}(t) = \frac{en}{2\epsilon_0}\tilde{s}_0^2\left(1 - 2\sin\omega t + \sin^2\omega t\right)$$

- Take time average

$$\overline{V}_s = \frac{3}{4}\frac{en}{\epsilon_0}\tilde{s}_0^2 = \mathcal{E}_i$$

- Compare to rf voltage across discharge [p. 65]

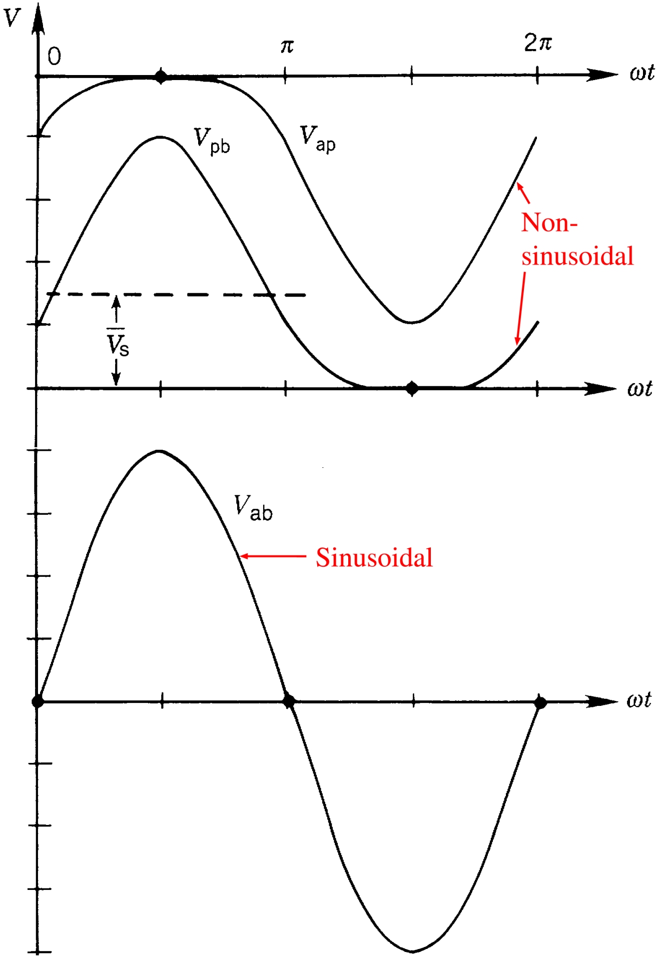
$$\Longrightarrow \overline{V}_s = \frac{3}{8}\widetilde{V}_{\rm rf}$$

- We can think of $\widetilde{V}_{\rm rf}$ as divided equally across the two sheaths

$$\boxed{\overline{V}_s = \frac{3}{4}\widetilde{V}_s \qquad \text{with} \qquad \widetilde{V}_s = \frac{1}{2}V_{\rm rf}}$$

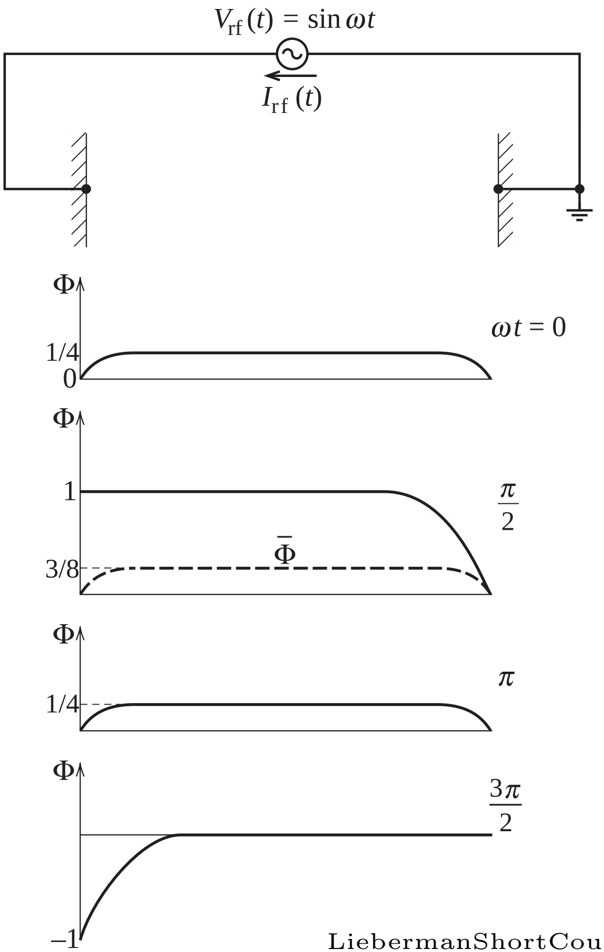
SHEATH VOLTAGES VERSUS TIME

Sheath voltages $V_{ap}(t)$, $V_{pb}(t)$, and their sum $V_{ab}(t) = V_{\mathrm{rf}}(t)$; the time average $\overline{V}_s$ of $V_{pb}(t)$ is also shown



SHEATH POTENTIAL VERSUS POSITION AT VARIOUS TIMES

Spatial variation of the total potential Φ (solid curves) for the homogeneous model at four different times during the rf cycle. The dashed curve shows the spatial variation of the time-average potential $\bar{\Phi} \equiv \overline{V}_s$



SHEATH CAPACITANCE

- Define total discharge capacitance by

$$I_{\rm rf}(t) = \tilde{I}_0 \cos\omega t, \qquad V_{\rm rf}(t) = \frac{2en\tilde{s}_0^2}{\epsilon_0}\sin\omega t \quad [p.\ 65]$$

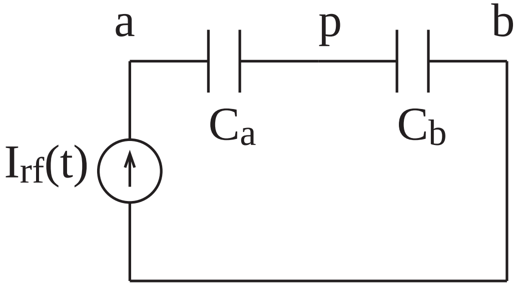
which yields

$$\boxed{I_{\rm rf} = C_s\frac{dV_{\rm rf}}{dt} \qquad \text{with} \qquad C_s = \frac{\epsilon_0 A}{2\tilde{s}_0}}$$

- We can think of each sheath as having a capacitance

$$\boxed{C_a = C_b = \frac{\epsilon_0 A}{\tilde{s}_0}}$$

- We now have a lossless discharge model

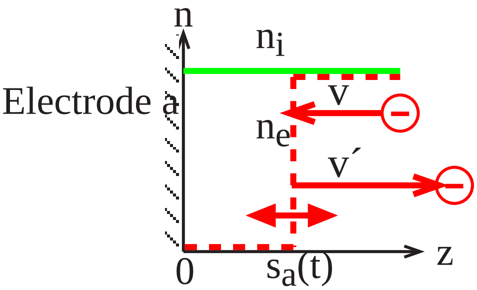


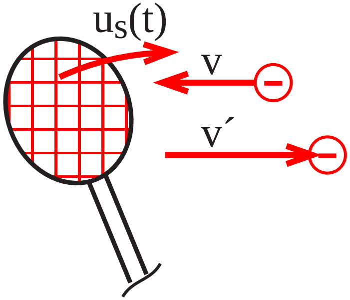
OHMIC AND STOCHASTIC HEATING

- Ohmic heating in the bulk plasma [p. 34]

$$P_\Omega = \frac{1}{2}|\tilde{J}_{\mathrm{rf}}|^2 \frac{m\nu_m d}{e^2 n} A \qquad [\text{watts}]$$

- Stochastic heating by oscillating sheaths





$$v' = -v + 2\,u_s(t)$$

with

$$u_s(t) = \frac{ds_a}{dt} = \tilde{u}_0 \cos\omega t \qquad \text{with} \qquad \tilde{u}_0 = \omega\tilde{s}_0$$

- Average energy transferred is

$$\Delta\mathcal{E}_e = \left[\frac{1}{2}m\big(-v + 2\,u_s(t)\big)^2 - \frac{1}{2}mv^2\right] = m\tilde{u}_0^2$$

- $\Delta\mathcal{E}_e$ is positive, so the oscillating sheath heats electrons

STOCHASTIC HEATING POWER

- For a Maxwellian distribution, the electron flux incident on a sheath is [p. 17]

  $$\Gamma_e = \frac{1}{4} n \bar{v}_e$$

  with

  $$\bar{v}_e = (8e\mathrm{T}_e/\pi m)^{1/2}$$

  the mean electron speed
- The time-average stochastic heating power is found to be

$$\boxed{P_{sa} = \Gamma_e A \cdot 2\,\Delta\mathcal{E}_e = \frac{1}{2} m n \bar{v}_e \omega^2 \tilde{s}_0^2 A \qquad [\mathrm{watts}]}$$

- This is a powerful electron heating mechanism in a capacitive discharge

HEATING POWERS VERSUS DRIVING VOLTAGE

- For stochastic heating [p. 71]

$$P_{sa} = \frac{1}{2} m n \bar{v}_e \omega^2 \tilde{s}_0^2 A$$

Using $\widetilde{V}_{\rm rf} = 2en\tilde{s}_0^2/\epsilon_0$ [p. 65] we obtain for the two sheaths

$$\boxed{P_s = P_{sa} + P_{sb} = \frac{m}{2e}\epsilon_0 \bar{v}_e \omega^2 \widetilde{V}_{\rm rf} A}$$

- For bulk ohmic heating [p. 70]

$$P_\Omega = \frac{1}{2}|\tilde{J}_{\rm rf}|^2 \frac{m\nu_m d}{e^2 n} A$$

Using $\tilde{J}_{\rm rf} = en\omega\tilde{s}_0$ and $\widetilde{V}_{\rm rf}$ given above

$$\boxed{P_\Omega = \frac{m}{4e}\epsilon_0 \nu_m d\, \omega^2 \widetilde{V}_{\rm rf} A}$$

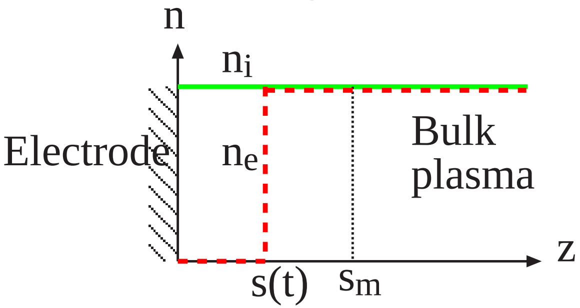
Ohmic and stochastic heating powers depend on $\widetilde{V}_{\rm rf}$

CAPACITIVE RF DISCHARGES

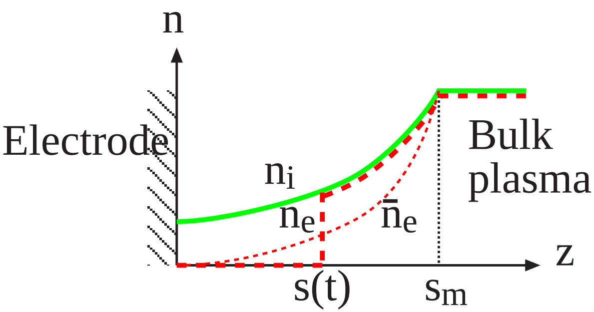
SELF-CONSISTENT SHEATH RESULTS

HOMOGENEOUS AND CHILD LAW SHEATHS

Homogeneous Model

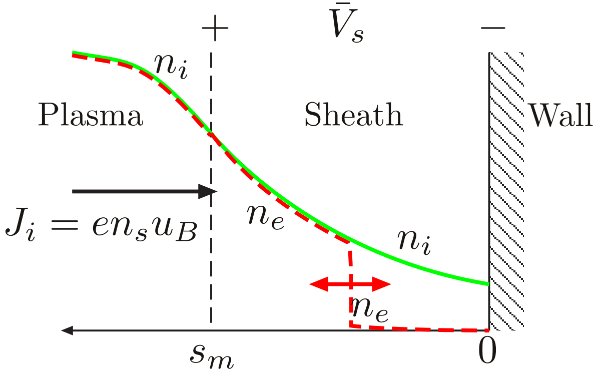


Self-Consistent Child Law



- Child law ion density decreases and sheath width increases compared to homogeneous model

COLLISIONLESS CHILD LAW SHEATH



- Larger sheath width $\Longrightarrow$ larger sheath oscillation velocity $u_s \propto \omega s_m$

$$\boxed{\text{Stochastic heating is larger than for homogeneous model}}$$

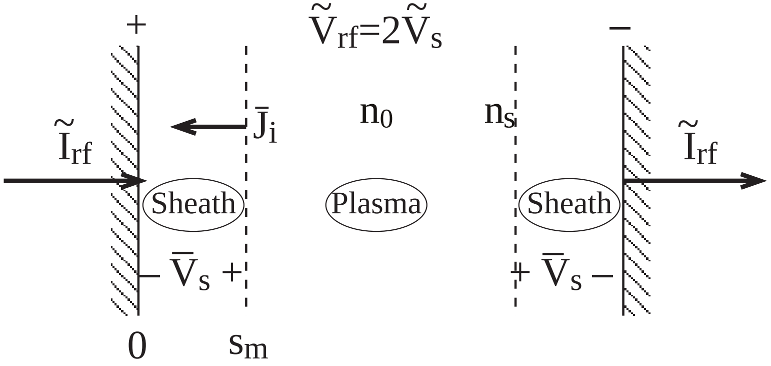
- Collisionless ion motion $\Longrightarrow$ Child law relating s_m to n_s and $\overline{V}_s$

$$\boxed{\bar{J}_i = en_s u_B = 0.82\,\epsilon_0 \left(\frac{2e}{M}\right)^{1/2} \frac{\overline{V}_s^{3/2}}{s_m^2}}$$

- RF voltage $\widetilde{V}_s$ across sheath $\Longrightarrow$ dc voltage $\overline{V}_s$

$$\boxed{\overline{V}_s \approx 0.83\,\widetilde{V}_s}$$

SUMMARY — SELF-CONSISTENT MODEL



$$\mathcal{E}_i = \overline{V}_s = 0.83\,\tilde{V}_s$$

$$J_i = en_s u_B = 0.82\,\epsilon_0 \left(\frac{2e}{M}\right)^{1/2} \frac{\overline{V}_s^{3/2}}{s_m^2}$$

$$\tilde{I}_{\mathrm{rf}} = 1.23\,j\omega \frac{\epsilon_0 A}{s_m}\tilde{V}_s$$

$$P_s = 1.12\,\frac{m}{2e}\omega^2 \epsilon_0 \bar{v}_e \tilde{V}_s A$$

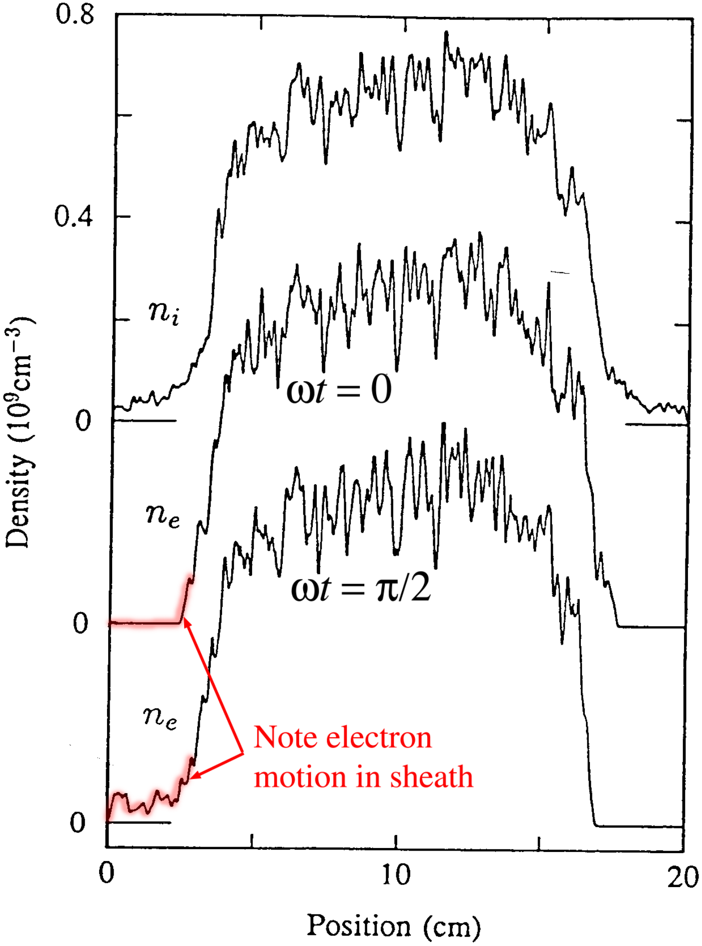
$$P_\Omega = 1.73 \frac{n_s}{n_0}\frac{m}{2e}\omega^2 \epsilon_0 \nu_m d\,(\mathrm{T}_e \tilde{V}_s)^{1/2} A$$

CAPACITIVE RF DISCHARGES

SIMULATION AND EXPERIMENTAL RESULTS

PIC SIMULATION OF DENSITIES

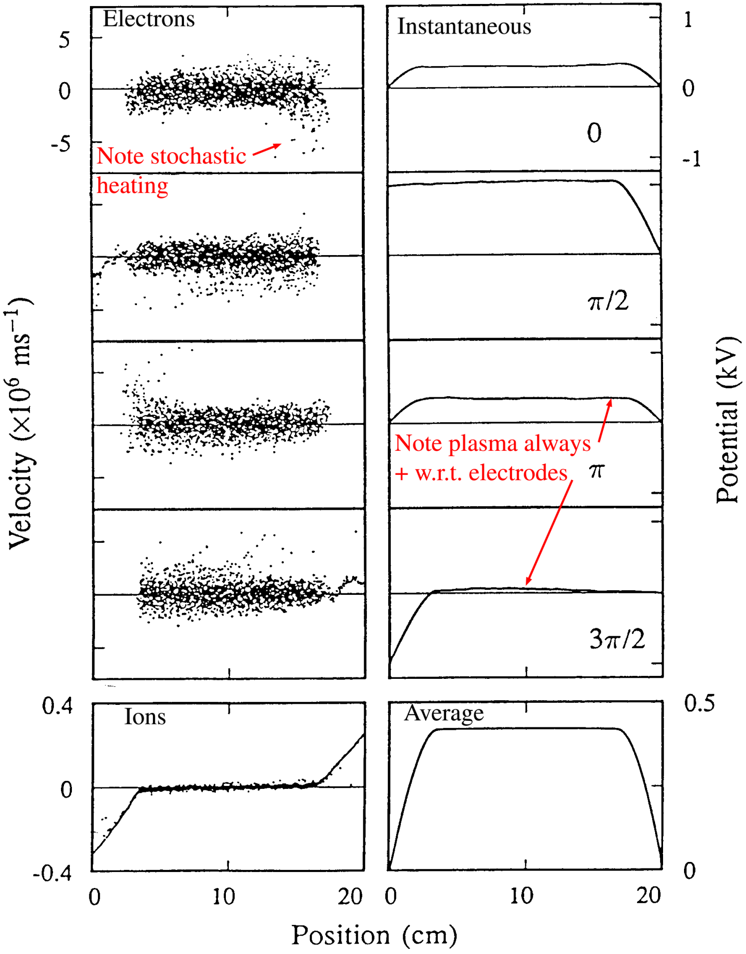
Symmetric rf discharge with right hand electrode grounded, $V_{\rm rf} = 1$ kV at 10 MHz, 20 mTorr hydrogen gas (Thesis of D. Vender, Australian National University, $\sim$ 1990)



Note electron motion in sheath

PHASE SPACE AND POTENTIALS VERSUS POSITION

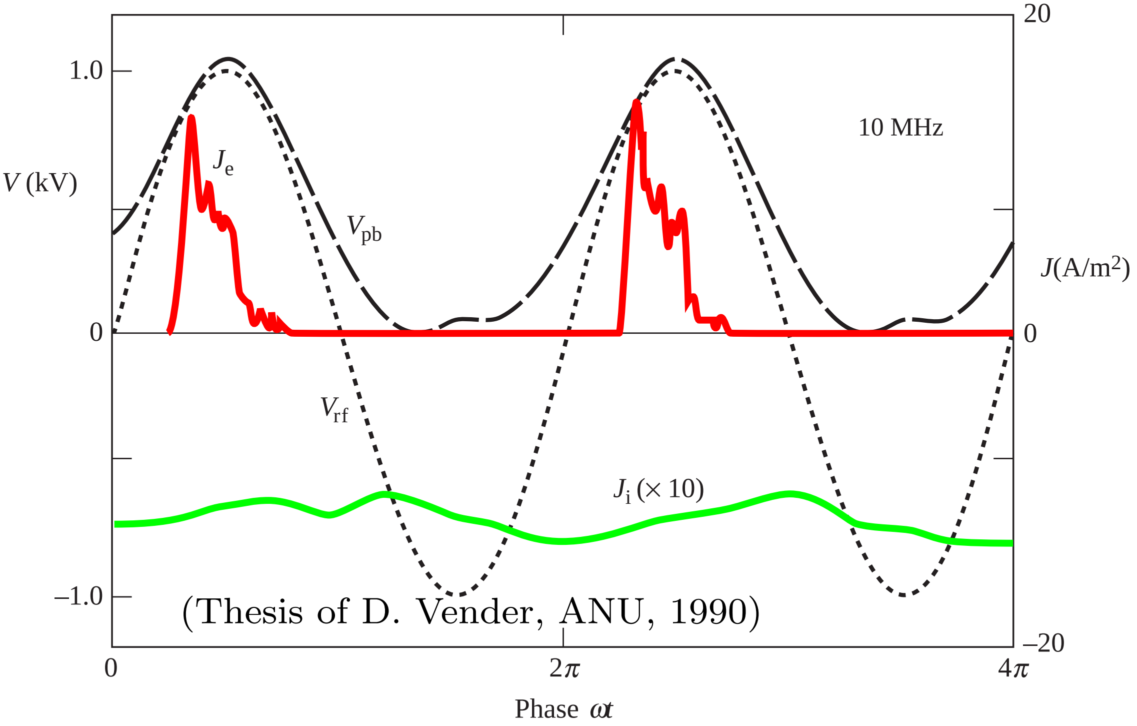
Symmetric rf discharge with right hand electrode grounded, $V_{\mathrm{rf}} = 1$ kV at 10 MHz, 20 mTorr hydrogen gas; left panels show electron and ion phase space; right panels show potentials (see [p. 68] for comparison to theory) (Thesis of D. Vender, Australian National University, $\sim$ 1990)



Note stochastic heating

Note plasma always + w.r.t. electrodes

TIME VARIATION OF VOLTAGES AND CURRENTS

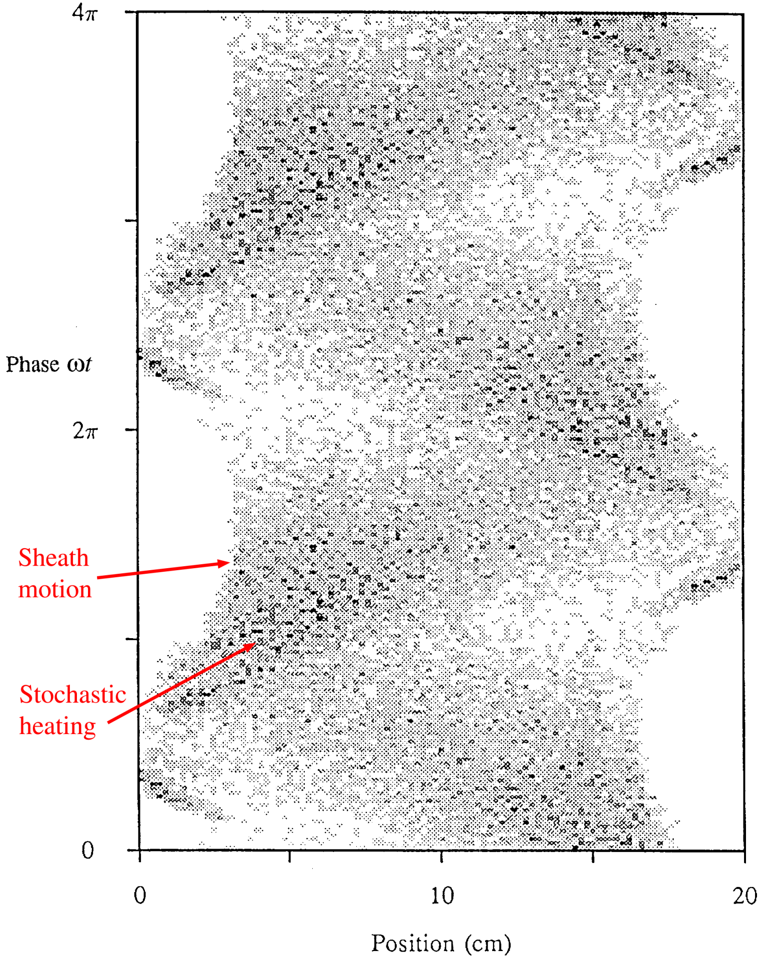


(Thesis of D. Vender, ANU, 1990)

- Note plasma always positive with respect to both electrodes
- Note steady ion current [p. 61]
- Note pulsed electron current when $V_{pa} = V_{pb} - V_{\rm rf} \to 0$ [p. 61]

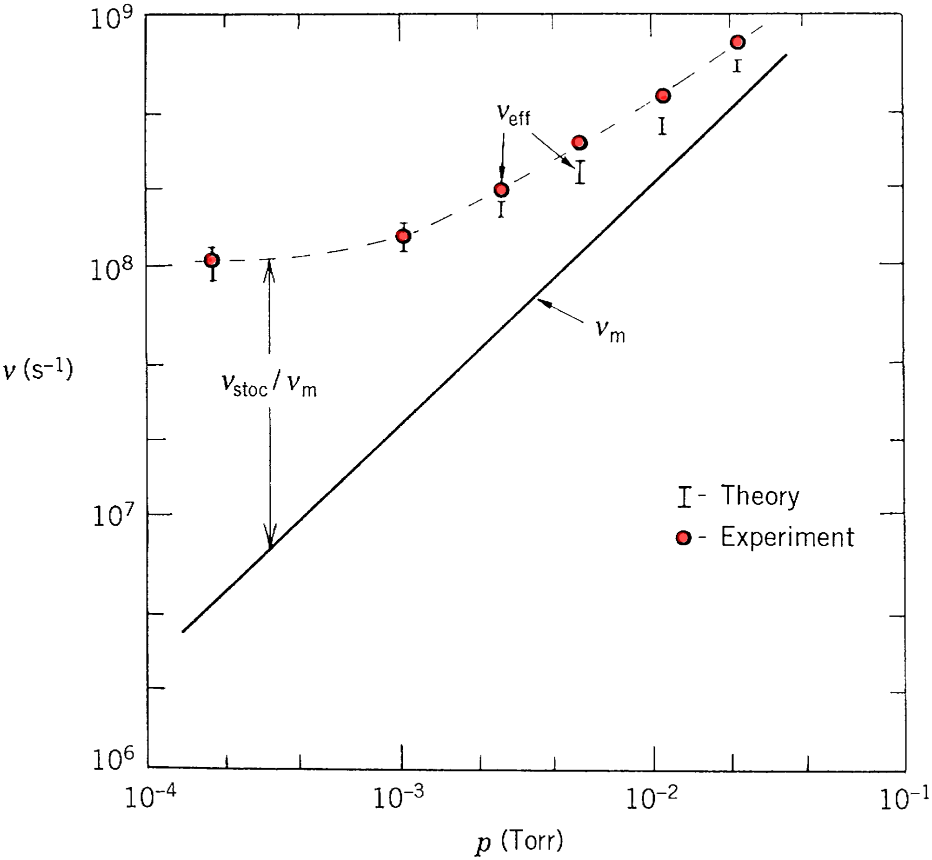
SPACE-TIME DISTRIBUTION OF IONIZING COLLISIONS

The darkness of each square is proportional to the number of ionizing collisions within that square of time and position intervals; symmetric rf discharge with right hand electrode grounded, $V_{\mathrm{rf}} = 1$ kV at 10 MHz, 20 mTorr hydrogen gas (Thesis of D. Vender, Australian National University, ∼ 1990)



MEASUREMENTS OF STOCHASTIC HEATING

Effective collision frequency ν_{eff} versus pressure p, for a mercury discharge driven at 40.8 MHz; the solid line shows the collision frequency due to ohmic dissipation alone (Popov and Godyak, 1985)



CAPACITIVE RF DISCHARGES

EXAMPLE EQUILIBRIUM CALCULATIONS

POWER BALANCE

- **Electron power balance**

$$\boxed{P_\Omega + 2P_s = en_s u_B\, 2A\, \underbrace{(\mathcal{E}_c + 2\mathrm{T}_e)}^{\text{El. col + kin}}}$$

where

$$P_\Omega = 1.73\, h_l \frac{m}{2e}\omega^2\epsilon_0(\nu_m d)(\mathrm{T}_e\widetilde{V}_s)^{1/2}A$$

$$P_s = 0.56\,\frac{m}{2e}\omega^2\epsilon_0\bar{v}_e\widetilde{V}_s A$$

Specify $\widetilde{V}_s \Longrightarrow n_s$

- **Total power balance**

$$\boxed{P_{\mathrm{abs}} = en_s u_B\, 2A\, (\overbrace{\mathcal{E}_c + 2\mathrm{T}_e}^{\text{El. col.+kin.}} + \overbrace{0.83\,\widetilde{V}_s}^{\text{Ion kin.}})}$$

- **Eliminate $\mathbf{n_s}$ from electron and total power balance**

$$\boxed{P_{\mathrm{abs}} \approx (P_\Omega + 2P_s)\left(1 + \frac{0.83\widetilde{V}_s}{\mathcal{E}_c + 2\mathrm{T}_e}\right)}$$

Specify $P_{\mathrm{abs}} \Longrightarrow \widetilde{V}_s$
In this case, electron or total power balance $\Longrightarrow n_s$

EXAMPLE 1

- Let $p = 3$ mTorr argon at 300 K, $l = 10$ cm, $A = 1000$ cm^2, $f = 13.56$ MHz ($\omega = 8.52 \times 10^7$ s^{-1}), and $V_{\text{rf}} = 500$ V
- Start with estimate $s_m \approx 1$ cm
- Ion mean free path $\lambda_i = 1/n_g\sigma_i \approx 1.0$ cm [p. 38]
- With bulk plasma thickness $d = l - 2s_m = 8$ cm, $\lambda_i/d \approx 0.125$
- $h_l = n_s/n_0 \approx 0.325$ [p. 43] and $d_{\text{eff}} = d/2h_l = 12.3$ cm [p. 46]
- With $n_g d_{\text{eff}} \approx 1.23 \times 10^{19}$ m^{-2}, the T_e versus $n_g d_{\text{eff}}$ figure [p. 47] yields $\mathrm{T}_e \approx 3.1$ V
- Bohm velocity $u_B \approx 2.7 \times 10^3$ m/s [p. 41]
- $\mathcal{E}_c$ versus T_e figure [p. 40] yields $\mathcal{E}_c \approx 47$ V and $\mathcal{E}_c + 2\mathrm{T}_e \approx 53$ V.
- Use the K_{el} versus T_e figure [p. 37] to find $\nu_m \approx K_{\text{el}} n_g \approx 1.0 \times 10^7$ s^{-1}

EXAMPLE 1 (CONT'D)

- Evaluate ohmic and stochastic electron heating [p. 76 or 84]

$$P_\Omega \approx 0.0145\,\widetilde{V}_s^{1/2} \qquad [\mathrm{W}]$$

$$P_s \approx 0.0121\,\widetilde{V}_s \qquad [\mathrm{W}]$$

- Use $\widetilde{V}_s \approx V_{\mathrm{rf}}/2 = 250$ V in above to find $P_\Omega \approx 0.229$ W and $P_s \approx 3.03$ W
- Electron power balance [p. 84] yields $n_s \approx 1.37 \times 10^{15}\ \mathrm{m}^{-3}$
- Since $h_l \approx 0.325$, $n_0 \approx 4.23 \times 10^{15}\ \mathrm{m}^{-3}$
- Using $\mathcal{E}_i \approx 0.83\,\widetilde{V}_s$ [p. 76] yields $\mathcal{E}_i \approx 208$ V
- $J_i = en_s u_B \approx 0.59\ \mathrm{A/m^2}$ [p. 76]
- The Child law [p. 76] gives $s_m \approx 0.90$ cm
- $J_{\mathrm{rf}} \approx 1.23\,\omega\epsilon_0 \widetilde{V}_s / s_m \approx 25.8\ \mathrm{A/m^2}$ [p. 76]
- Total power balance [p. 84] gives $P_{\mathrm{abs}} \approx 30.8$ W
- s_m reasonably close to the initial estimate
  $\Longrightarrow$ iteration over d is not useful

EXAMPLE 2

- Let $p = 3$ mTorr argon at 300 K, $l = 10$ cm, $A = 1000$ cm^2, $f = 13.56$ MHz and $P_{\text{abs}} = 200$ W
- As before, $h_l \approx 0.325$, $\mathrm{T}_e \approx 3.1$ V, $u_B \approx 2.7 \times 10^3$ m/s, and $\mathcal{E}_c + 2\mathrm{T}_e \approx 53$ V.
- Because n_g and T_e are the same, P_Ω and P_s are the same functions of $\widetilde{V}_s$ as in EXAMPLE 1
- Using $P_{\text{abs}} = 200$ W, we obtain the equation for the rf sheath voltage $\widetilde{V}_s$ [p. 84]

$$200 = \left(0.0145\widetilde{V}_s^{1/2} + 0.0242\widetilde{V}_s\right)\left(1 + \frac{0.83\widetilde{V}_s}{53}\right)$$

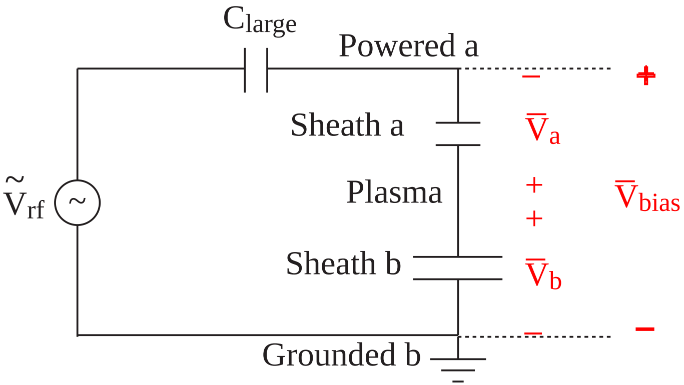
- A numerical solution gives $\widetilde{V}_s = 687$ V
- Then $V_{\text{rf}} \approx 2\widetilde{V}_s \approx 1374$ V and $\mathcal{E}_i = 0.83\,\widetilde{V}_s \approx 570$ V
- Use this in total power balance [p. 84] to find $n_s \approx 3.72 \times 10^{15}$ m^{-3} and $n_0 \approx 1.14 \times 10^{16}$ m^{-3}
- We then find $\bar{J}_i \approx 1.6$ A/m^2, $s_m \approx 1.16$ cm, and $J_{\text{rf}} \approx 54.9$ A/m^2

CAPACITIVE RF DISCHARGES

ASYMMETRIC SYSTEMS

ASYMMETRIC RF DISCHARGE

- Powered electrode area A_a smaller than grounded area A_b



$\overline{V}_a$ = dc sheath voltage from plasma to powered electrode a
$\overline{V}_b$ = dc sheath voltage from plasma to grounded electrode b
$\tilde{V}_a$ = rf voltage amplitude across sheath a
$\tilde{V}_b$ = rf voltage amplitude across sheath b

$$\boxed{\overline{V}_a = 0.83\,\tilde{V}_a, \qquad \overline{V}_b = 0.83\,\tilde{V}_b, \qquad \tilde{V}_{\text{rf}} = \tilde{V}_a + \tilde{V}_b}$$

- A negative DC bias voltage $V_{\text{bias}} = \overline{V}_b - \overline{V}_a$ appears

DEPENDENCE OF VOLTAGES ON AREAS

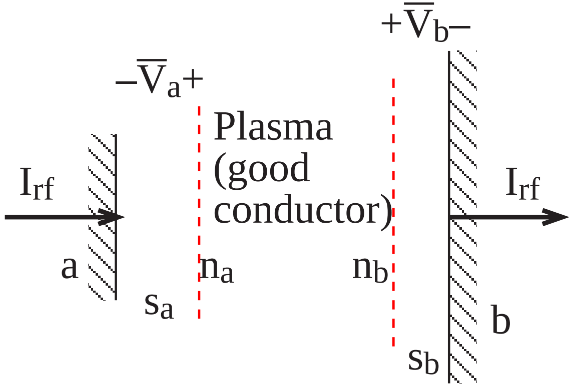
- Given $\widetilde{V}_{\text{rf}}$, A_a and A_b, what are $\overline{V}_a$, $\overline{V}_b$ and $\overline{V}_{\text{bias}}$?
- Voltage ratio $\overline{V}_a/\overline{V}_b$ depends on area ratio A_b/A_a

$$\boxed{\frac{\overline{V}_a}{\overline{V}_b} = \left(\frac{A_b}{A_a}\right)^q}$$

$q =$ area ratio scaling exponent

- Experiments show $q \sim 1$–2.5
- Collisionless Child law gives $q = 4$

COLLISIONLESS CHILD LAW ANALYSIS



- Electrodes and plasma are good conductors, so $\overline{V}_a$ is the same everywhere along electrode a
- Capacitive sheath [p. 76]

$$I_{\text{rf}} \propto \frac{\overline{V}_a A_a}{s_a}$$

- Child law [p. 76]

$$n_a \propto \frac{\overline{V}_a^{3/2}}{s_a^2}$$

- Eliminating s_a

$$I_{\text{rf}} \propto \overline{V}_a^{1/4} n_a^{1/2} A_a$$

COLLISIONLESS CHILD LAW ANALYSIS (CONT'D)

- Similarly

$$I_{\mathrm{rf}} \propto \overline{V}_b^{1/4} n_b^{1/2} A_b$$

- Set currents at sheaths a and b equal and solve for $\overline{V}_a/\overline{V}_b$

$$\frac{\overline{V}_a}{\overline{V}_b} = \left(\frac{n_b}{n_a}\right)^2 \left(\frac{A_b}{A_a}\right)^4$$

- Simplest assumption for plasma is equal densities at the sheath edges a and b

$$\boxed{\frac{\overline{V}_a}{\overline{V}_b} = \left(\frac{A_b}{A_a}\right)^4}$$

COLLISIONAL CHILD LAW SHEATH SCALING

- For pressures above 3–10 mTorr, ions suffer collisions with neutrals in the sheaths
- The Child law is modified to the scaling

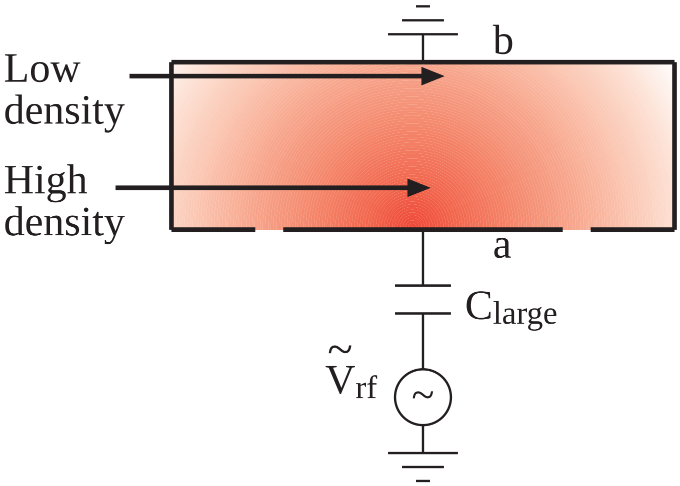
$$n_a \propto \frac{\overline{V}_a^{3/2}}{s_a^{5/2}}$$

- This leads to

$$\boxed{\frac{\overline{V}_a}{\overline{V}_b} = \left(\frac{A_b}{A_a}\right)^{2.5}}$$

- The weaker scaling $q = 2.5$ is more in agreement with experiments

VARIATION OF DENSITIES



- Plasma density near powered electrode is usually larger than near grounded electrode
- This leads to additional modifications in scaling

INDUCTIVE DISCHARGES

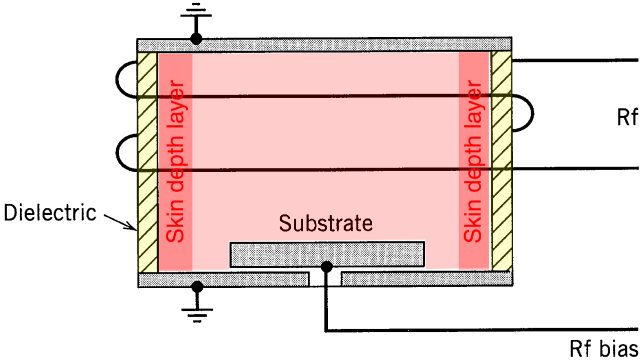
TRANSFORMER MODEL AND MATCHING

MOTIVATION

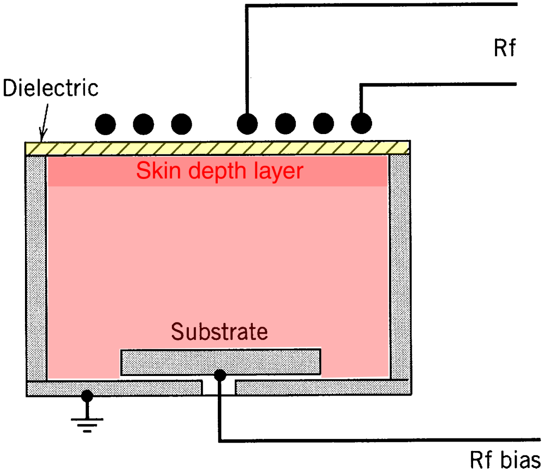
- High density (compared to capacitive discharge)
- Independent control of plasma density and ion energy
- Simplicity of concept
- RF rather than microwave powered
- No source magnetic fields

CYLINDRICAL AND PLANAR CONFIGURATIONS

- Cylindrical coil

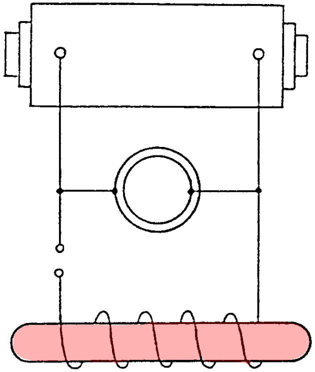


- Planar coil

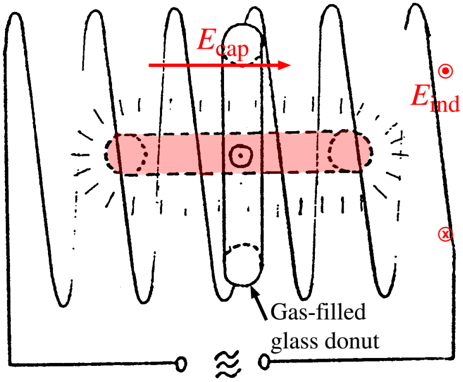


EARLY HISTORY

- First inductive discharge by Hittorf (1884)

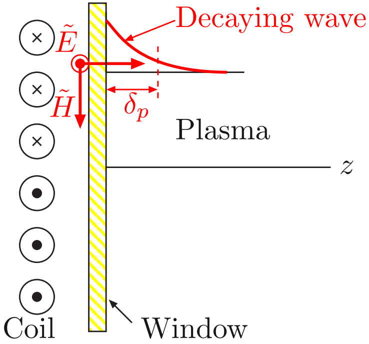


- Arrangement to test discharge mechanism by Lehmann (1892)



HIGH DENSITY REGIME

- Inductive coil launches decaying wave into plasma



- Wave decays exponentially into plasma

$$\widetilde{E} = \widetilde{E}_0\, \mathrm{e}^{-z/\delta_p}, \qquad \delta_p = \frac{c}{\omega}\frac{1}{\mathrm{Im}(\kappa_p^{1/2})}$$

where κ_p = plasma dielectric constant [p. 31]

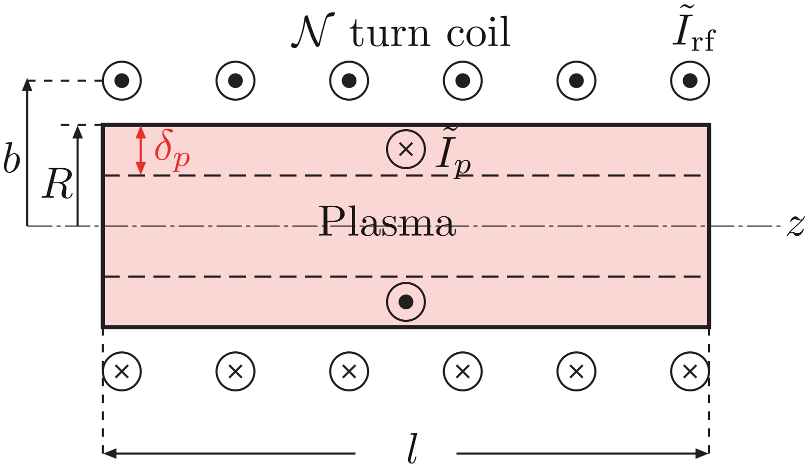
$$\kappa_p = 1 - \frac{\omega_{pe}^2}{\omega(\omega - j\nu_m)}$$

For typical high density, low pressure ($\nu_m \ll \omega$) discharge

$$\boxed{\delta_p \approx \frac{c}{\omega_{pe}} = \left(\frac{m}{e^2\mu_0 n_e}\right)^{1/2} \sim \ 1\text{–}2 \text{ cm}}$$

TRANSFORMER MODEL

- For simplicity consider a long cylindrical discharge



- Current $\tilde{I}_{\rm rf}$ in $\mathcal{N}$ turn coil induces current $\tilde{I}_p$ in 1-turn plasma skin

$$\boxed{\Longrightarrow \text{A transformer}}$$

PLASMA RESISTANCE AND INDUCTANCE

- Plasma resistance R_p

$$R_p = \frac{1}{\sigma_{\rm dc}} \frac{\text{circumference of plasma loop}}{\text{average cross sectional area of loop}}$$

where

$$\sigma_{\rm dc} = \frac{e^2 n_{es}}{m\nu_m} \qquad \text{[p. 33]}$$

with n_{es} = density at plasma-sheath edge

$$\boxed{\Longrightarrow R_p = \frac{\pi R}{\sigma_{\rm dc} l \delta_p}}$$

- Plasma inductance L_p

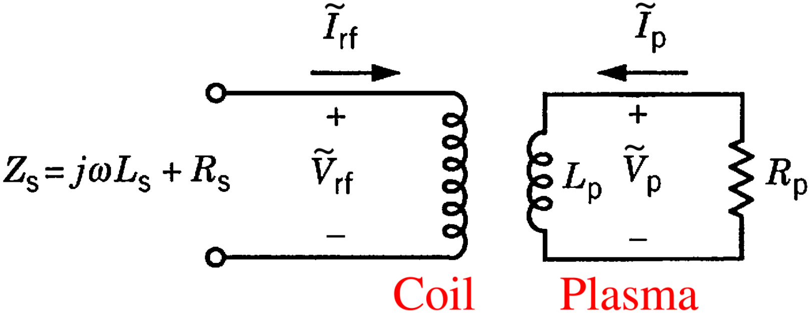
$$L_p = \frac{\text{magnetic flux produced by plasma current}}{\text{plasma current}}$$

- Using magnetic flux $= \pi R^2 \mu_0 \tilde{I}_p / l$

$$\boxed{\Longrightarrow L_p = \frac{\mu_0 \pi R^2}{l}}$$

COUPLING OF COIL TO PLASMA

- Model the source as a transformer



$$\tilde{V}_{\rm rf} = j\omega L_{11}\tilde{I}_{\rm rf} + j\omega L_{12}\tilde{I}_p$$

$$\tilde{V}_p = j\omega L_{21}\tilde{I}_{\rm rf} + j\omega L_{22}\tilde{I}_p$$

- Transformer inductances

$$L_{11} = \frac{\text{magnetic flux linking coil}}{\text{coil current}} = \frac{\mu_0\pi b^2\mathcal{N}^2}{l}$$

$$L_{12} = L_{21} = \frac{\text{magnetic flux linking plasma}}{\text{coil current}} = \frac{\mu_0\pi R^2\mathcal{N}}{l}$$

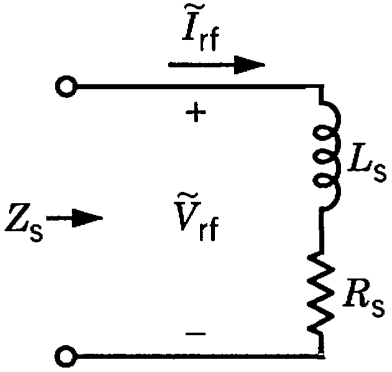
$$L_{22} = L_p = \frac{\mu_0\pi R^2}{l}$$

SOURCE CURRENT AND VOLTAGE

- Put $\widetilde{V}_p = -\tilde{I}_p R_p$ in transformer equations and solve for the impedance $Z_s = \widetilde{V}_{\rm rf}/\tilde{I}_{\rm rf}$ seen at coil terminals

$$Z_s = j\omega L_{11} + \frac{\omega^2 L_{12}^2}{R_p + j\omega L_p} \equiv R_s + j\omega L_s$$

- Equivalent circuit at coil terminals



$$R_s = \mathcal{N}^2 \frac{\pi R}{\sigma_{\rm dc} l \delta_p}$$

$$L_s = \frac{\mu_0 \pi R^2 \mathcal{N}^2}{l}\left(\frac{b^2}{R^2} - 1\right)$$

- Power balance $\Longrightarrow \tilde{I}_{\rm rf}$

$$\boxed{P_{\rm abs} = \frac{1}{2}\tilde{I}_{\rm rf}^2 R_s}$$

- From source impedance $\Longrightarrow V_{\rm rf}$

$$\boxed{\widetilde{V}_{\rm rf} = \tilde{I}_{\rm rf} Z_s}$$

EXAMPLE

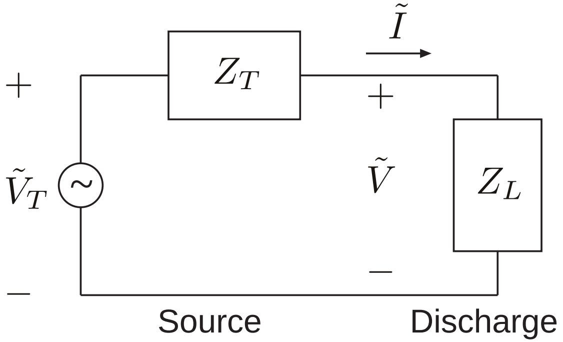
- Assume plasma radius $R = 10$ cm, coil radius $b = 15$ cm, length $l = 20$ cm, $\mathcal{N} = 3$ turns, gas density $n_g = 6.6\times 10^{14}$ cm^{-3} (20 mTorr argon at 300 K), $\omega = 85 \times 10^6$ s^{-1} (13.56 MHz), absorbed power $P_{\text{abs}} = 600$ W, and low voltage sheaths

- At 20 mTorr, $\lambda_i \approx 0.15$ cm, $h_l \approx h_R \approx 0.1$, $d_{\text{eff}} \approx 34$ cm [p. 38, 44, 46]
- Particle balance (T$_e$ versus $n_g d_{\text{eff}}$ figure [p. 47]) yields T$_e \approx 2.1$ V
- Collisional energy losses ($\mathcal{E}_c$ versus T$_e$ figure [p. 40]) are $\mathcal{E}_c \approx 110$ V. Adding $\mathcal{E}_e + \mathcal{E}_i = 7.2\,T_e$ yields total energy losses $\mathcal{E}_T \approx 126$ V [p. 39]
- $u_B \approx 2.3 \times 10^5$ cm/s [p. 41] and $A_{\text{eff}} \approx 185$ cm^2 [p. 50]
- Power balance yields $n_e \approx 7.1 \times 10^{11}$ cm^{-3} and $n_{se} \approx 7.4 \times 10^{10}$ cm^{-3} [p. 50]

- Use n_{se} to find skin depth $\delta_p \approx 2.0$ cm [p. 99]; estimate $\nu_m = K_{\text{el}} n_g$ (K_{el} versus T$_e$ figure [p. 37]) to find $\nu_m \approx 3.4 \times 10^7$ s^{-1}
- Use ν_m and n_{se} to find $\sigma_{\text{dc}} \approx 61$ Ω^{-1}-m^{-1} [p. 33]
- Evaluate impedance elements $R_s \approx 23.5$ Ω and $L_s \approx 2.2$ μH; $|Z_s| \approx \omega L_s \approx 190$ Ω [p. 103]
- Power balance yields $\tilde{I}_{\text{rf}} \approx 7.1$A; from source impedance $|Z_s| = 190$ Ω, $\widetilde{V}_{\text{rf}} \approx 1360$ V [p. 103]

MATCHING DISCHARGE TO POWER SOURCE

- Consider an rf power source connected to a discharge



- Source impedance $Z_T = R_T + jX_T$ is given
  Discharge impedance $Z_L = R_L + jX_L$
- Time-average power delivered to discharge $P_{\text{abs}} = \frac{1}{2}\text{Re}\,(\widetilde{V}\tilde{I}^*)$
- For fixed source $\widetilde{V}_T$ and Z_T, maximize power delivered to discharge

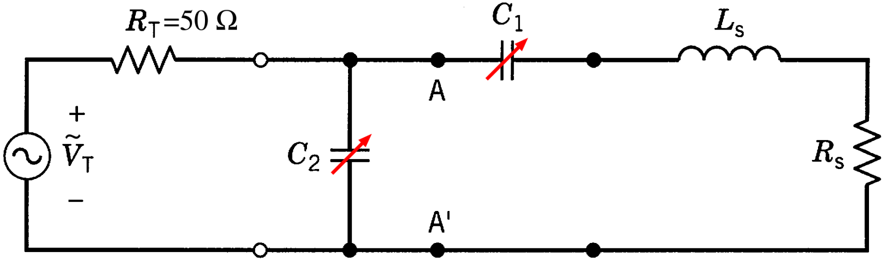
$$\boxed{\begin{aligned} X_L &= -X_T \\ R_L &= R_T \end{aligned}}$$

- Maximum time-average power delivered to discharge

$$P_{\text{abs}} = \frac{1}{8}\frac{|\widetilde{V}_T|^2}{R_T}$$

MATCHING NETWORK

- Insert lossless matching network between power source and coil



Power source Matching network Discharge coil

- Continue EXAMPLE [p. 104] with $R_s = 23.5\ \Omega$ and $\omega L_s = 190\ \Omega$; assume $R_T = 50\ \Omega$ (corresponds to a conductance $1/R_T = 1/50$ S)
- Choose C_1 such that the conductance seen looking to the right at terminals AA' is 1/50 S

$$\boxed{\Longrightarrow C_1 = 71\ \text{pF}}$$

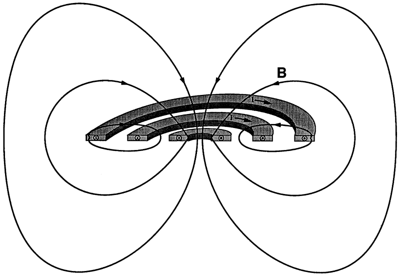
- Choose C_2 to cancel the reactive part of the impedance seen at AA'

$$\boxed{\Longrightarrow C_2 = 249\ \text{pF}}$$

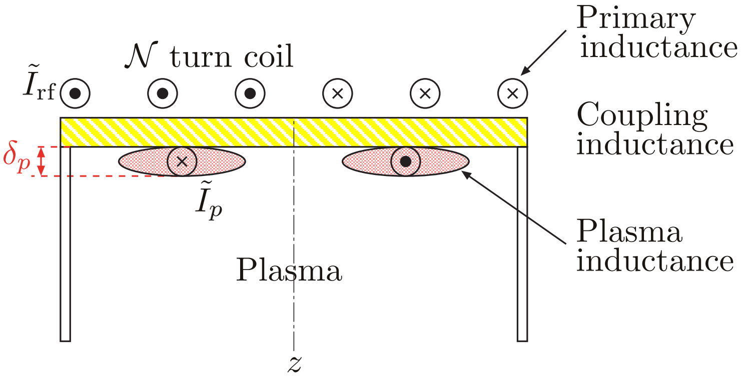
- For $P_{\text{abs}} = 600$ W, the 50 Ω source supplies $\tilde{I}_{\text{rf}} = 4.9$ A
- Voltage at source terminals (AA') = $\tilde{I}_{\text{rf}} R_T = 245$ V

PLANAR COIL DISCHARGE

- Magnetic field produced by planar coil



- RF power is deposited in a ring-shaped plasma volume



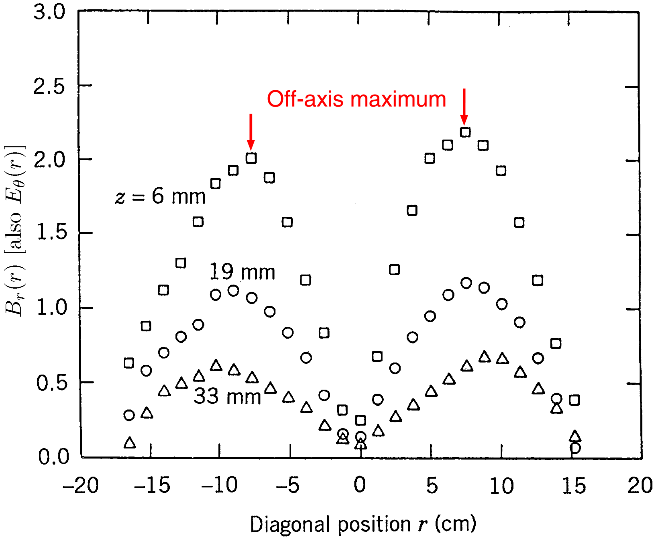
- As for a cylindrical discharge, there is a primary (L_{11}), coupling ($L_{12} = L_{21}$) and secondary ($L_p = L_{22}$) inductance

PLANAR COIL FIELDS

- A ring-shaped plasma forms because

$$\text{Induced electric field} = \begin{cases} 0, & \text{on axis} \\ \max, & \text{at } r \approx \frac{1}{2} R_{\text{wall}} \\ 0, & \text{at } r = R_{\text{wall}} \end{cases}$$

- Measured radial variation of B_r (and E_θ) at three distances below the window (5 mTorr argon, 500 W, Hopwood et al, 1993)



INDUCTIVE DISCHARGES

POWER BALANCE

RESISTANCE AT HIGH AND LOW DENSITIES

- Plasma resistance seen by the coil [p. 103]

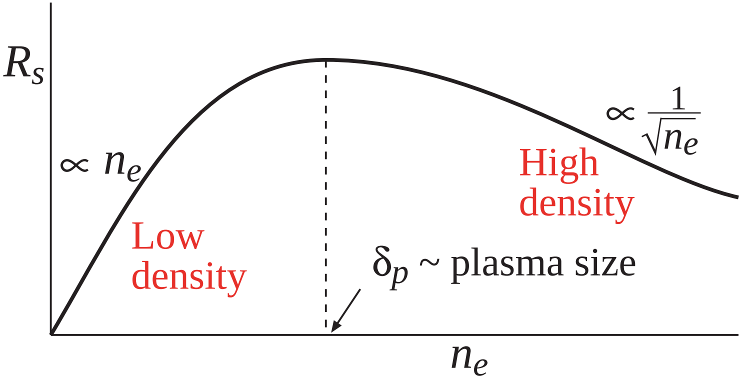
$$R_s = R_p \frac{\omega^2 L_{12}^2}{R_p^2 + \omega^2 L_p^2}$$

- High density (normal inductive operation) [p. 103]

$$R_s \propto R_p \propto \frac{1}{\sigma_{\mathrm{dc}} \delta_p} \propto \frac{1}{\sqrt{n_e}}$$

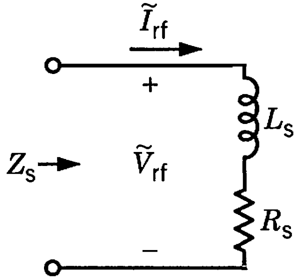
- Low density (skin depth > plasma size)

$$R_s \propto \text{ number of electrons in the heating volume} \propto n_e$$

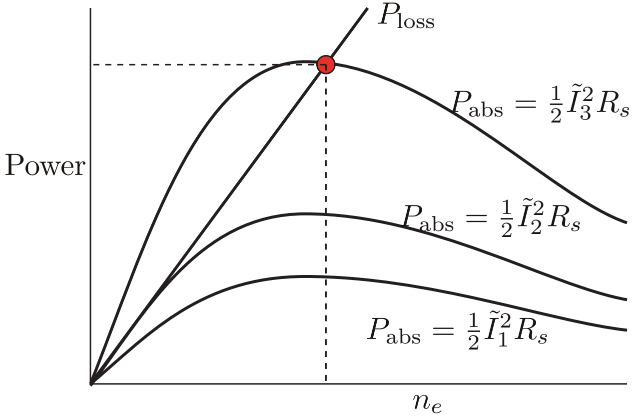


POWER BALANCE WITHOUT MATCHING

- Drive discharge with rf current



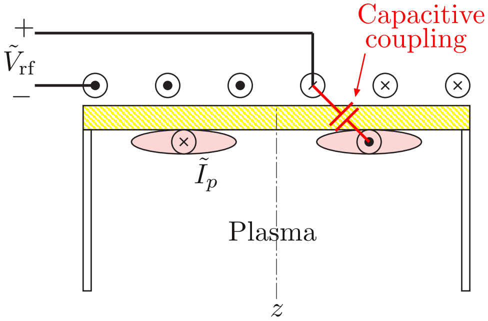
- Power absorbed by discharge is $P_{\text{abs}} = \frac{1}{2}|\tilde{I}_{\text{rf}}|^2 R_s(n_e)$ [p. 110]
  Power lost by discharge $P_{\text{loss}} \propto n_e$ [p. 50]
- Intersection (red dot) gives operating point; let $\tilde{I}_1 < \tilde{I}_2 < \tilde{I}_3$



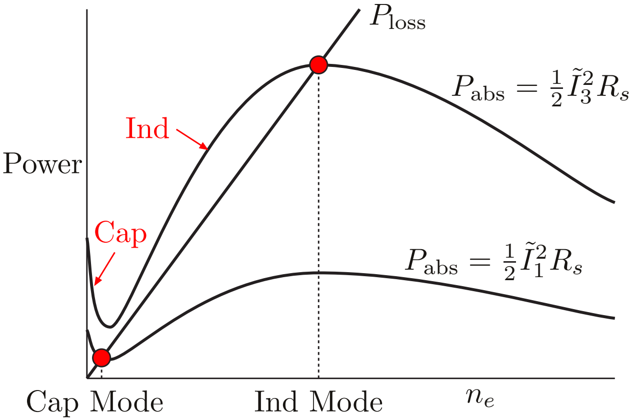
- Inductive operation impossible for $\tilde{I}_{\text{rf}} \leq \tilde{I}_2$

CAPACITIVE COUPLING OF COIL TO PLASMA

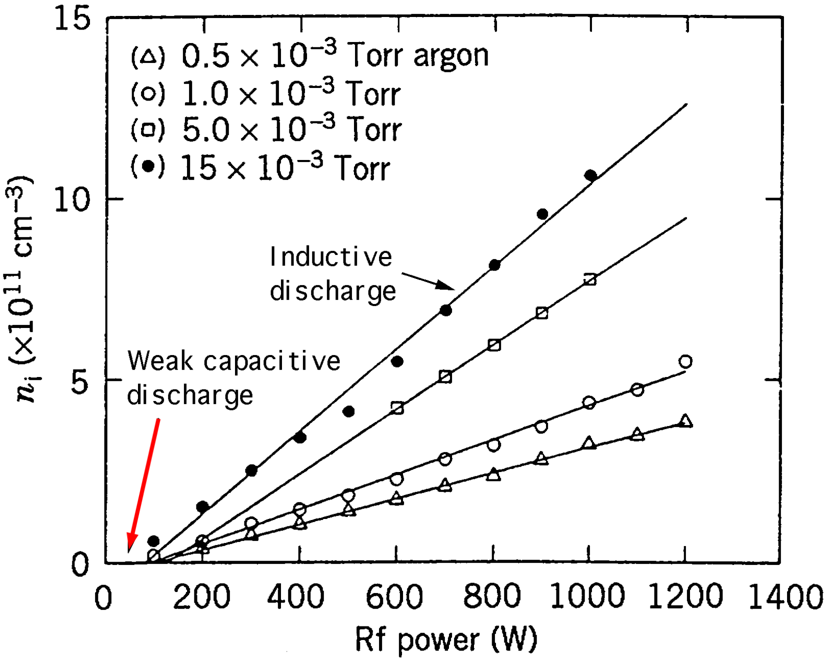
- For $\tilde{I}_{\text{rf}}$ below the minimum current $\tilde{I}_2$, there is only a weak capacitive coupling of the coil to the plasma



- A small capacitive power is absorbed
  $\Longrightarrow$ low density capacitive discharge



MEASURMENTS OF ARGON ION DENSITY



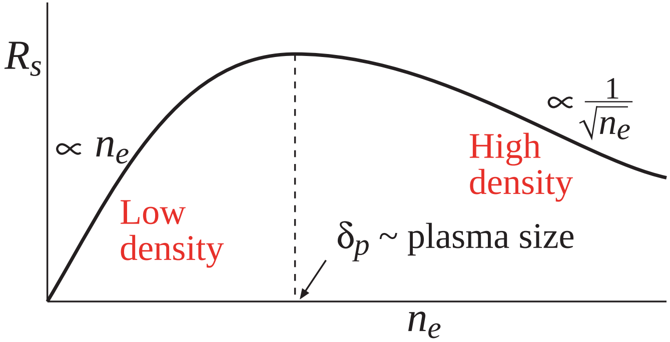
- Above 100 W, discharge is inductive and $n_e \propto P_{\rm abs}$
- Below 100 W, a weak capacitive discharge is present

SOURCE EFFICIENCY

- The source coil has some winding resistance R_{coil}
- R_{coil} is in series with the plasma resistance R_s
- Power transfer efficiency is

$$\eta = \frac{R_s}{R_s + R_{\text{coil}}}$$

- High efficiency $\Longrightarrow$ maximum R_s



- Power transfer efficiency decreases at low and high densities
- Poor power transfer at low or high densities is analogous to poor power transfer in an ordinary transformer with an open or shorted secondary winding