Limita funkce

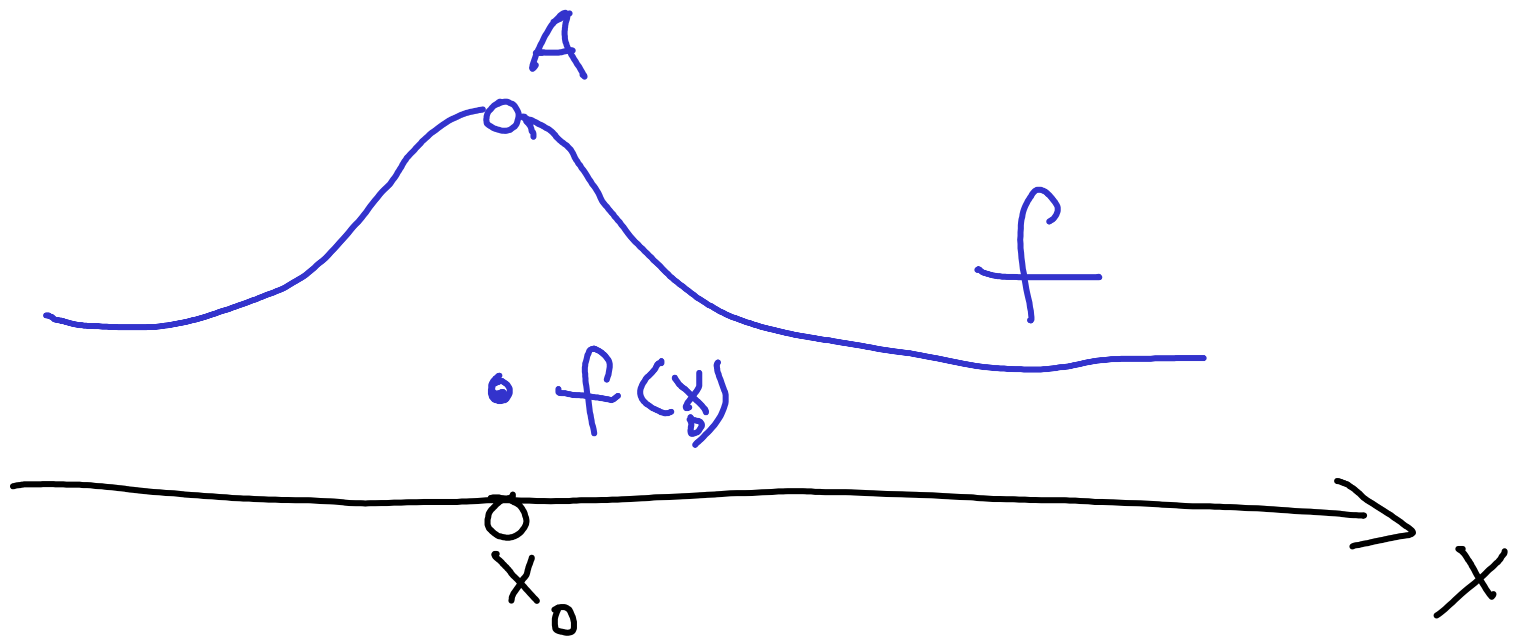
$f: (x_0-\Delta, x_0+\Delta) \setminus \{x_0\} \longrightarrow \mathbb{R}$

$$\lim_{x \to x_0} f(x) = L \in \mathbb{R}$$

$\forall \varepsilon > 0$ $\exists \delta > 0$ $\forall x \in (x_0-\Delta, x_0+\Delta) \setminus \{x_0\}$ $|f(x) - L| < \varepsilon$

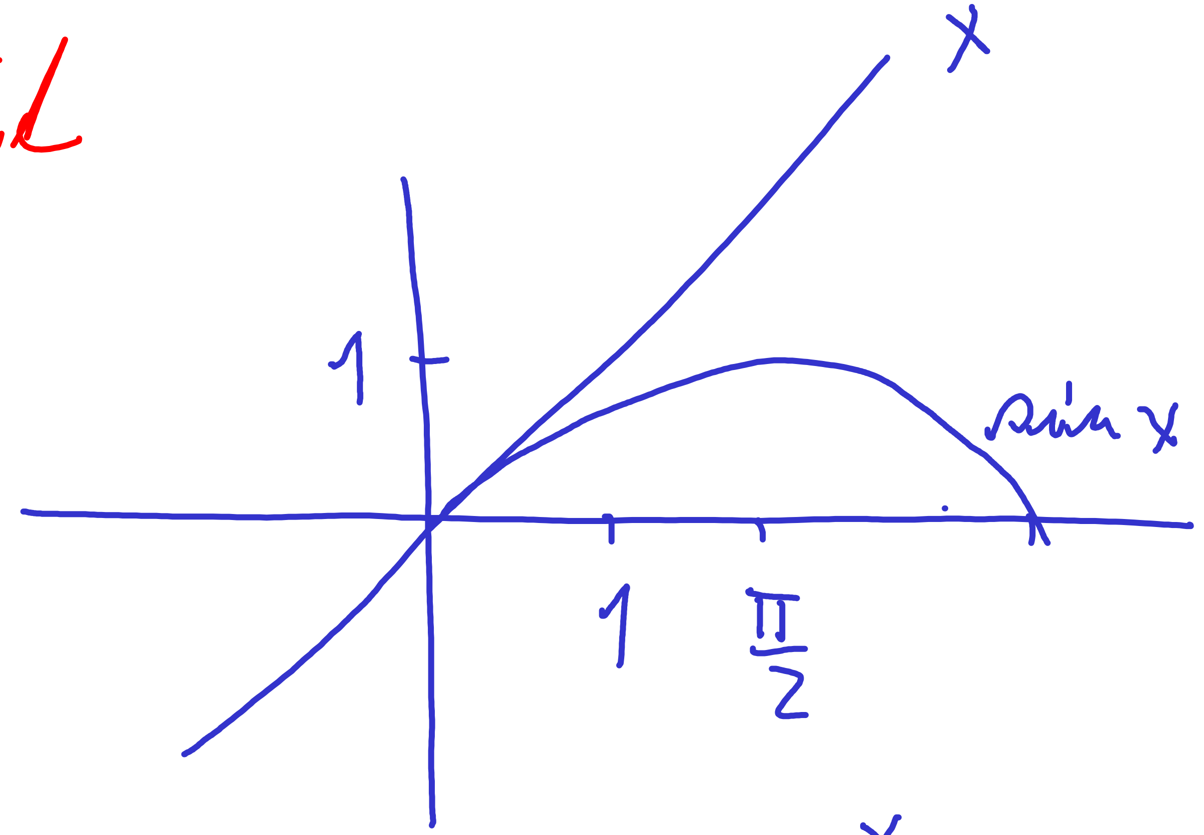
pro každé   existuje   pro všechna x: $0 < |x - x_0| < \delta$

$$\lim_{x \to x_0} f(x) = A \neq f(x_0)$$



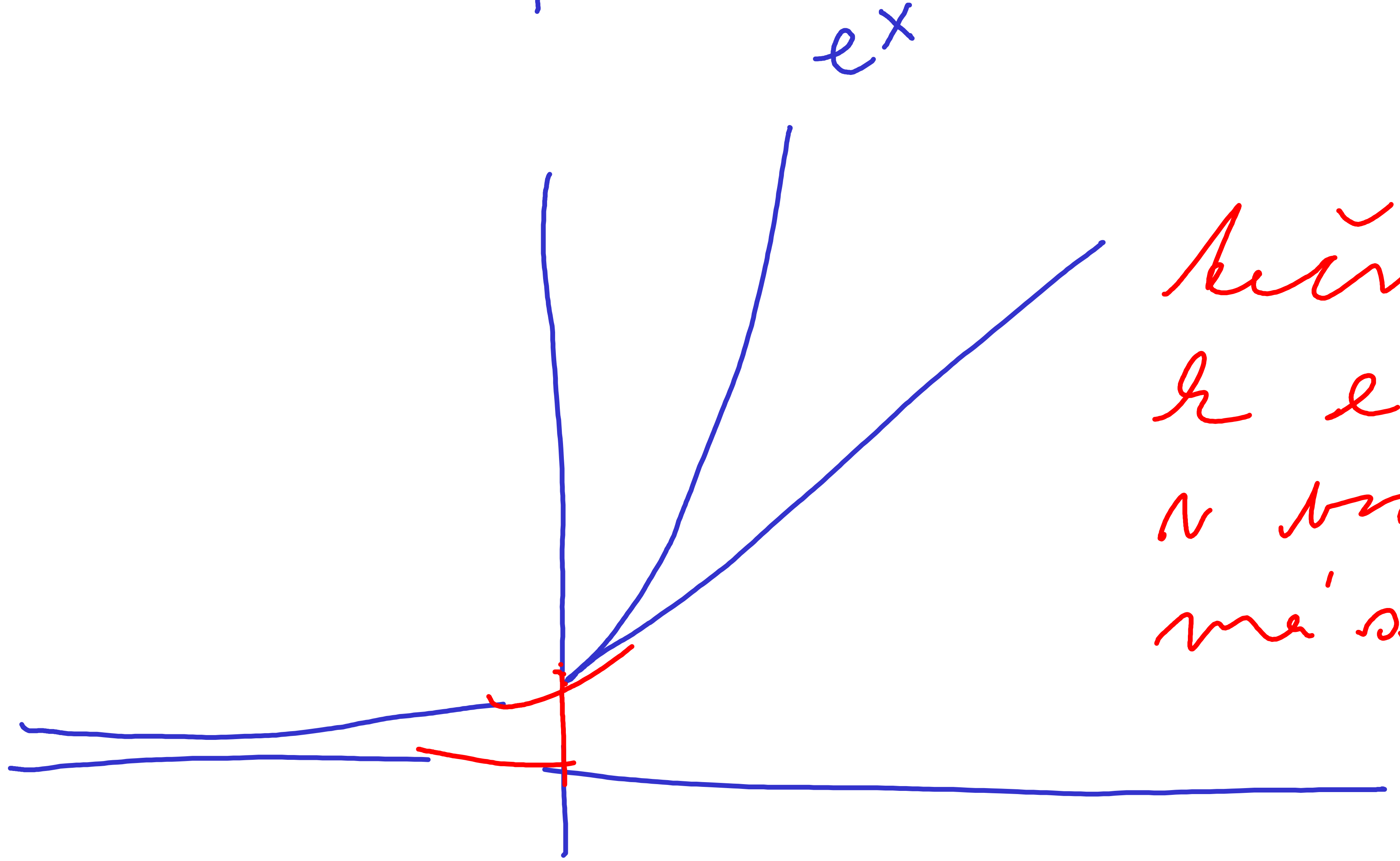
Seznam důležitých základních limit

① $$\lim_{x \to 0} \frac{\sin x}{x} = 1$$



② $$\lim_{x \to 0} \frac{e^x - 1}{x} = 1$$

③ $$\lim_{x \to 1} \frac{\ln x}{x - 1} = 1$$



tečna k e^x v bodě $(0,1)$ má směrnici 1

(4) $\lim\limits_{x\to\infty} \dfrac{e^x}{x^n} = \infty$ pro všechna přirozená čísla n

(5) $\lim\limits_{x\to\infty} \dfrac{\ln x}{x^\alpha} = 0$ pro všechna reálná $\alpha > 0$

Věta o limitě složené funkce

Nechť je g spojitá v bodě a a nechť $g(a) = b$ a současně $g(x) \neq b$ pro $x \in (a-\Delta, a+\Delta) \setminus \{a\}$.
Nechť $\lim\limits_{y\to b} f(y) = L$.
Potom $\lim\limits_{x\to a} f(g(x)) = L$.

Příklad

$$\lim_{x \to 0} \frac{\sin 5x}{x} = \lim_{x \to 0} f(g(x)) = \lim_{y \to 0} f(y) = 5$$

$$f(y) = \frac{\sin y}{\frac{y}{5}} = 5\,\frac{\sin y}{y} \qquad \lim_{y \to 0} f(y) = 5$$

$g(x) = 5x$ spojité $\quad g(x) = 0$ pouze pro $x = 0$. $\quad \lim_{x \to 0} g(x) = 0$

Příklady: cvičení 6 úloha 5, goniometrické rovnice

(a) Řešte v $\mathbb{R}$: $\sin 2x = \sin x$

$$\sin 2x = 2 \sin x \cos x = \sin x$$

$$\sin x \,(2\cos x - 1) = 0$$

(1) $\sin x = 0$

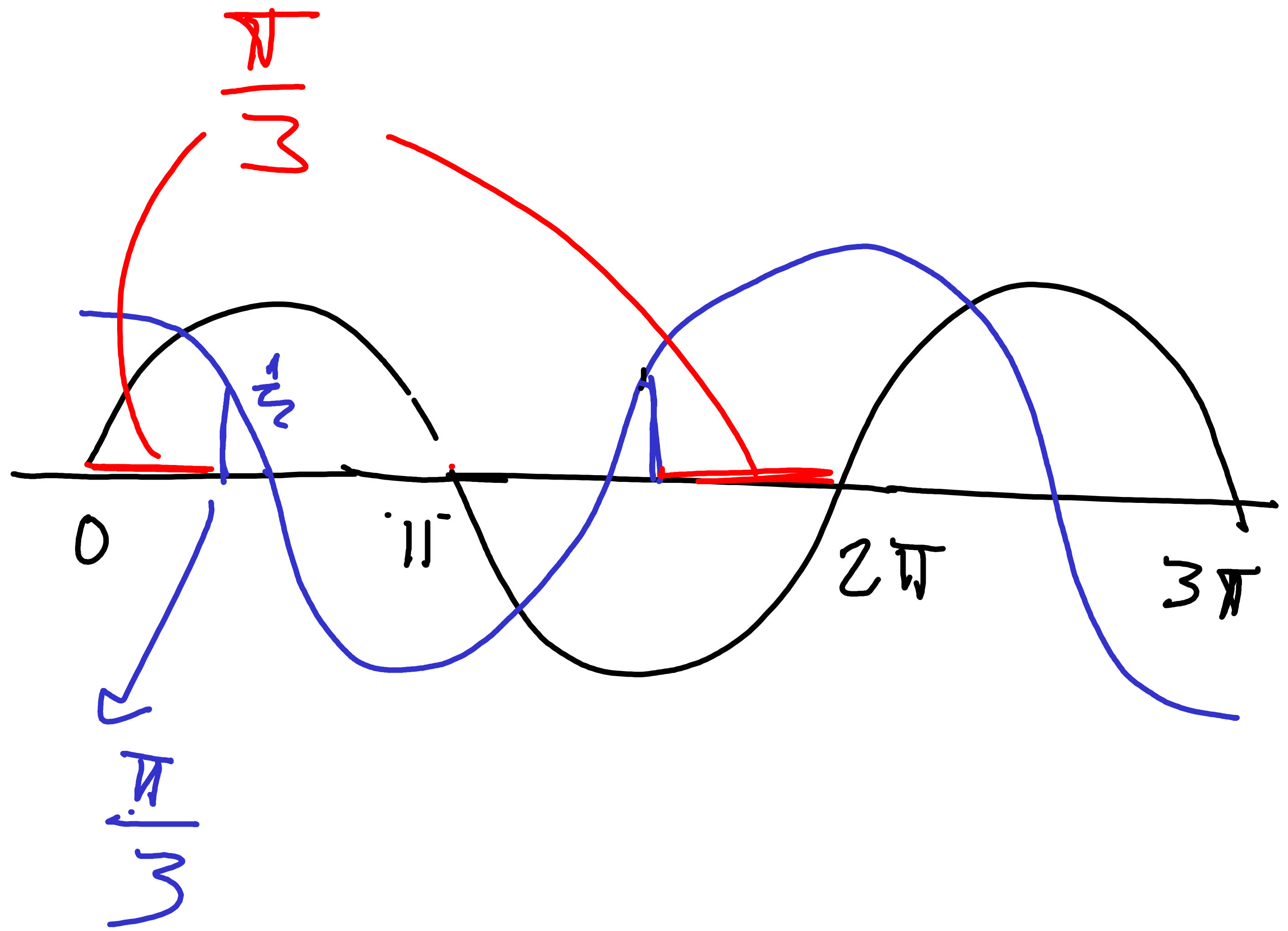
$x = k\pi, \ k \in \mathbb{Z}$

(2) $2\cos x - 1 = 0$

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3} + 2k\pi$

$x = \frac{5}{3}\pi + 2k\pi$



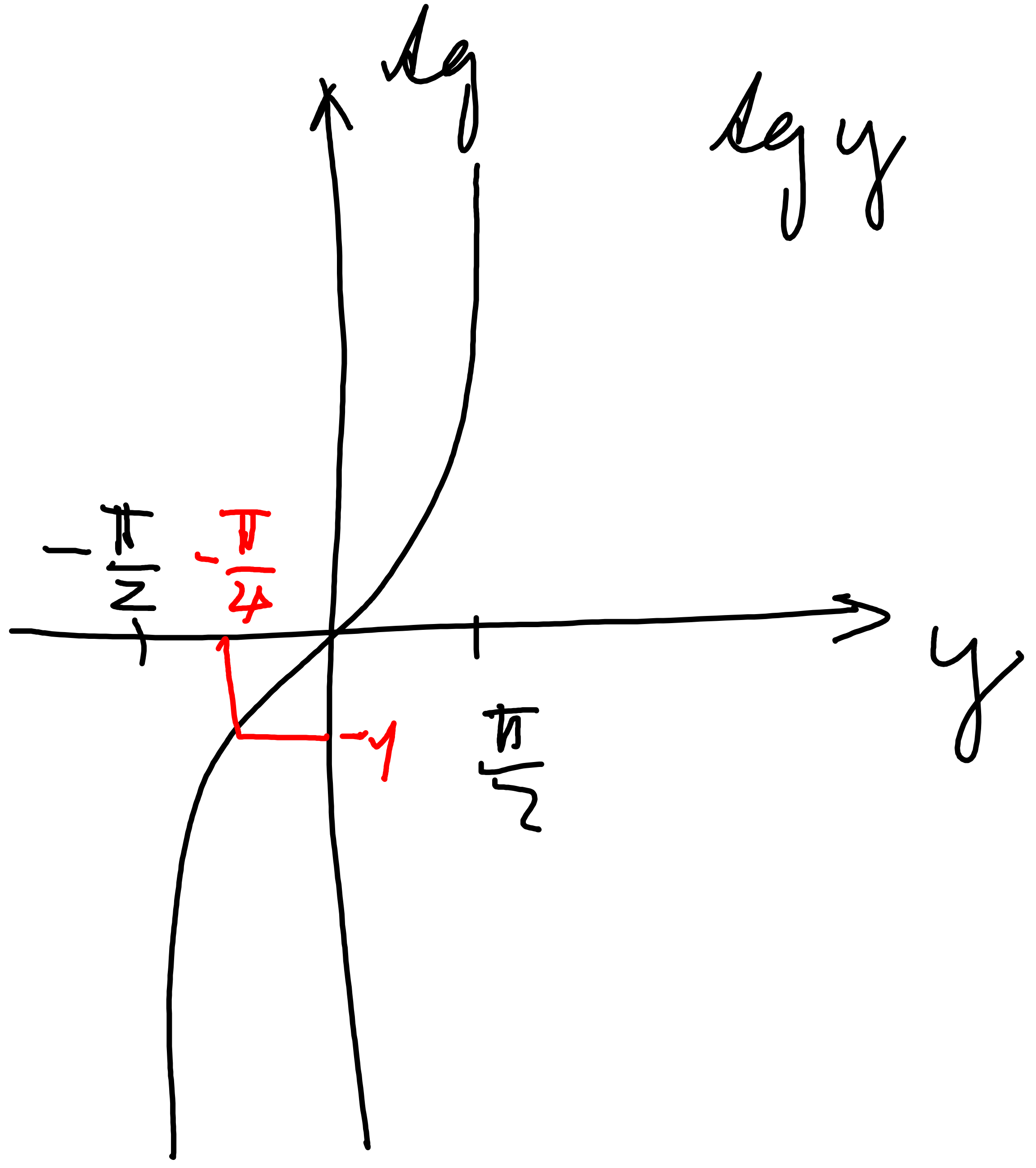
(b) $\cos 3x + \sin 3x = 0$ předp. $\cos 3x \neq 0$

$$1 + \frac{\sin 3x}{\cos 3x} = 0 \qquad / : \cos 3x$$

$$\operatorname{tg} 3x = -1$$

$$3x = -\frac{\pi}{4} + k\pi$$

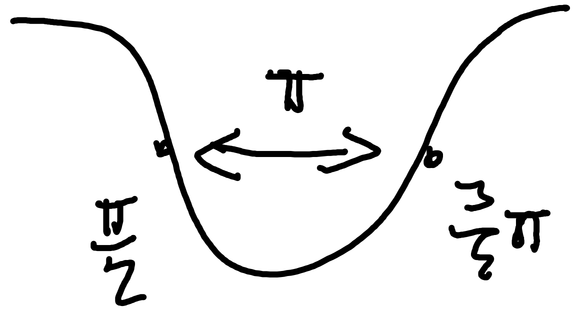
$$\boxed{x = -\frac{\pi}{12} + k\frac{\pi}{3}}$$



$\cos 3x = 0 \Rightarrow \sin 3x \neq 0$ x nemůže být řešení

$(\sin 3x)^2 + (\cos 3x)^2 = 1$

(d) $\cos 3x + \underbrace{\sin 2x - \sin 4x} = 0$



$$\sin\alpha - \sin\beta = 2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}$$

$$\cos 3x + 2\cos\frac{2x+4x}{2}\sin\frac{2x-4x}{2} = 0$$

$$\cos 3x + 2\cos 3x \sin(-x) = 0$$

$$-\sin x = \sin(-x)$$

$$\cos 3x - 2\cos 3x \cdot \sin x = 0$$

$$\cos 3x(1 - 2\sin x) = 0$$

(1) $\cos 3x = 0$

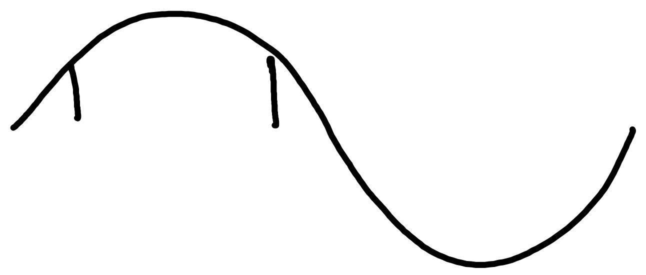
$$3x = \frac{\pi}{2} + k\pi$$

$$x = \frac{\pi}{6} + k\frac{\pi}{3}$$

liché násobky $\frac{\pi}{6}$

(2) $1 - 2\sin x = 0$

$$\sin x = \frac{1}{2}$$



$$x = \frac{\pi}{6} + 2k\pi$$

$$x = \frac{5}{6}\pi + 2k\pi$$

Řešení v $[0, 2\pi]$:

$\frac{\pi}{6}\pi, \frac{5}{6}\pi, \frac{\pi}{2}, \frac{7}{6}\pi, \frac{3}{2}\pi, \frac{11}{6}\pi$

(e) $2\sin^2 x + 7\cos x - 5 = 0$

$2(1-\cos^2 x) + 7\cos x - 5 = 0$

$2 - 2\cos^2 x + 7\cos x - 5 = 0$

$y = \cos x$

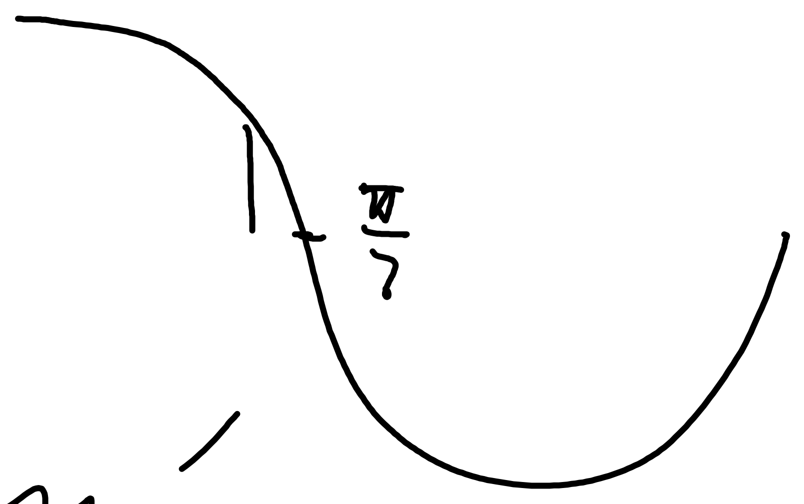
$2y^2 - 7y + 3 = 0$

$D = 49 - 4\cdot 3\cdot 2 = 25$

$y_{1,2} = \dfrac{7\pm 5}{4} = \begin{matrix} 3 \\ \frac{1}{2} \end{matrix}$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$



$\cos x = 3$ nemá řešení

$\cos x = \frac{1}{2}$

$x = \frac{\pi}{3} + 2k\pi$

$x = \frac{5}{3}\pi + 2k\pi$

(c)

$$1 \cdot \cos x + \sqrt{3} \sin x = 1 \quad \Big| :2$$

$$(1)^2 + (\sqrt{3})^2 = 1 + 3 = 4$$

Promień wynosi 2

$$\left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$$

$$\underbrace{\frac{1}{2}}\cos x + \underbrace{\frac{\sqrt{3}}{2}}\sin x = \frac{1}{2}$$

$$\sin\frac{\pi}{6}\cos x + \cos\frac{\pi}{6}\sin x = \frac{1}{2}$$

$$\sin\alpha\cos\beta + \cos\alpha\sin\beta = \sin(\alpha+\beta)$$

$$\sin\left(\frac{\pi}{6} + x\right) = \frac{1}{2}$$

$$\left(\frac{\pi}{6} + x\right) = \frac{\pi}{6} + 2k\pi \qquad x = 2k\pi$$

$$= \frac{5}{6}\pi + 2k\pi \qquad x = \frac{2}{3}\pi + 2k\pi$$