$$x = B \cdot (x)_B = \sum_{j=1}^{n} x_j v_j$$

$$(x)_B = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$\varphi(x) = \sum_{j=1}^{n} x_j \varphi(v_j) = \left(\varphi(v_1), \ldots, \varphi(v_n)\right) \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$$

$$\left(\varphi(x)\right)_A = \sum_{j=1}^{n} x_j \left(\varphi(v_j)\right)_A = (\varphi)_{A,B} \cdot (x)_B$$

$$(\varphi(x))_\alpha = A(x)_\beta$$
$$(\varphi(x))_\alpha = B(x)_\beta \overset{?}{\Longrightarrow} \underline{A = B}$$

$$(\varphi(v_1))_\alpha = A(v_1)_\beta = A\begin{pmatrix}1\\0\\\vdots\\0\end{pmatrix} = c_1(A)$$
$$\| $$
$$c_1(B)$$

✓

$$\psi : W \to V, \quad \varphi : V \to U$$

$$(\varphi \circ \psi)_{\alpha,\gamma} = \big(\varphi(\psi(w_1)), \dots, \varphi(\psi(w_r))\big)$$

$$\gamma = (w_1, \dots, w_r), \quad r = \dim W$$

$$\big(\varphi(\psi(w_1)) \dots\big)\begin{pmatrix} x_1 \\ \vdots \\ x_r \end{pmatrix} = \varphi(\psi(x))$$

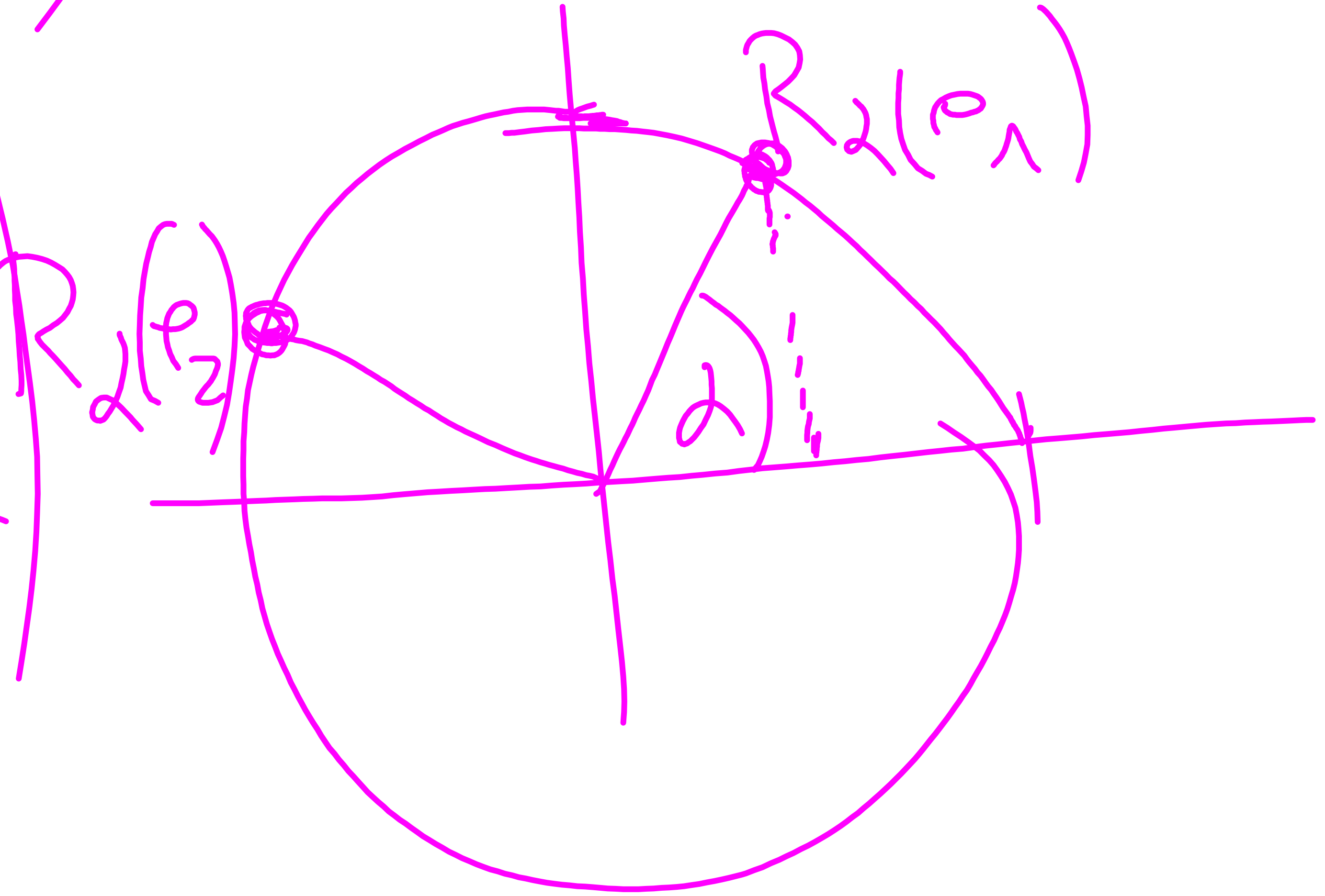
$$x \in W \qquad (x)_\gamma = \begin{pmatrix} x_1 \\ \vdots \\ x_r \end{pmatrix}$$

$$(\varphi)_{\alpha,\beta} \cdot (\psi)_{\beta,\gamma} \begin{pmatrix} x_1 \\ \vdots \\ x_r \end{pmatrix} = (\varphi)_{\alpha,\beta} (\psi(x))_\beta = (\varphi(\psi(x)))_\alpha$$

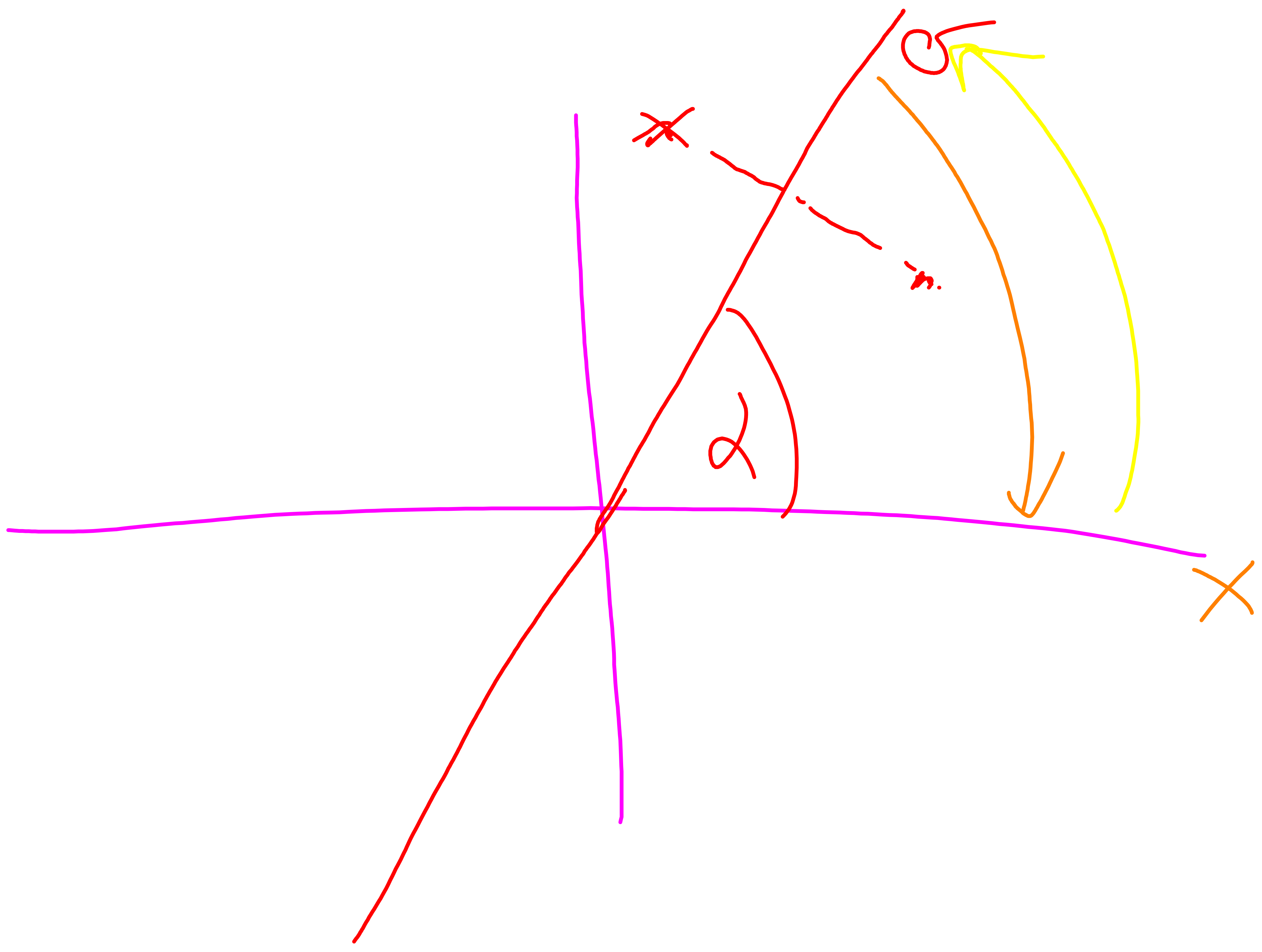
$$e_1 = \begin{pmatrix}1\\0\end{pmatrix}, \quad e_2 = \begin{pmatrix}0\\1\end{pmatrix}$$

$$R_\alpha\begin{pmatrix}1\\0\end{pmatrix} \qquad R_\alpha\begin{pmatrix}0\\1\end{pmatrix}$$

$$R_\alpha(e_2) = \begin{pmatrix}-\sin\alpha\\ \cos\alpha\end{pmatrix}$$



$$\begin{pmatrix}\cos\alpha\\ \sin\alpha\end{pmatrix} = R_\alpha(e_1)$$



$$S_0 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$S_\alpha = R_\alpha \circ S_0 \circ R_{-\alpha}$$

$\mathcal{L}(V, U) \subseteq U^V$ $\quad (\varphi+\psi)(x) = \varphi(x) + \psi(x)$

$\varphi, \psi \Rightarrow \varphi + \psi \in \mathcal{L}(V, U)$

$\boxed{\varphi \in \mathcal{L}(V, U) \Rightarrow c\varphi \in \mathcal{L}(V, U)}$

$x, y \in V, \quad a, b \in \mathbb{K}$

$(\varphi+\psi)(ax+by) = \varphi(ax+by) + \psi(ax+by)$

$= a\varphi(x) + b\varphi(y) + a\psi(x) + b\psi(y) = a(\varphi+\psi)(x) + b(\varphi+\psi)(y)$

3,2

$$\begin{tikzcd} \varphi \in \mathcal{L}(V, W) \arrow[d, mapsto] & \varphi_A \\ (\varphi)_{\alpha,\beta} & A \arrow[u, mapsto] \end{tikzcd}$$

$$(\varphi+\psi)_{\alpha,\beta} \stackrel{?}{=} (\varphi)_{\alpha,\beta} + (\psi)_{\alpha,\beta}$$

$$\parallel$$

$$\big(((\varphi+\psi)(v_1))_{\alpha}, \dots, ((\varphi+\psi)(v_n))_{\alpha}\big) =$$

$$= \big((\varphi(v_1))_{\alpha} + (\psi(v_1))_{\alpha}, \dots, (\varphi(v_n))_{\alpha} + (\psi(v_n))_{\alpha}\big) =$$

$$= (\varphi)_{\alpha,\beta} + (\psi)_{\alpha,\beta}$$

$$V \qquad\qquad V^{*} \qquad\qquad V^{**}$$

$$\begin{array}{ccc} V & & V^{**} \\ \cup & & \cup \\ x & \longmapsto & \hat{x} \end{array}$$

$$\underset{\in K}{\hat{x}(\varphi)} = \underset{\in K}{\varphi(x)}$$

$$\hat{x}(\varphi+\psi) = \varphi(x)+\psi(x) = \hat{x}(\varphi)+\hat{x}(\psi)$$

$\alpha(x) = \hat{x}$ $\qquad V \cong V^* \cong V^{**}$

$\alpha(x) = 0 \Longrightarrow x = 0$

$\hat{x}(\varphi) = 0 \quad \forall \varphi \in V^* \qquad x \neq 0$

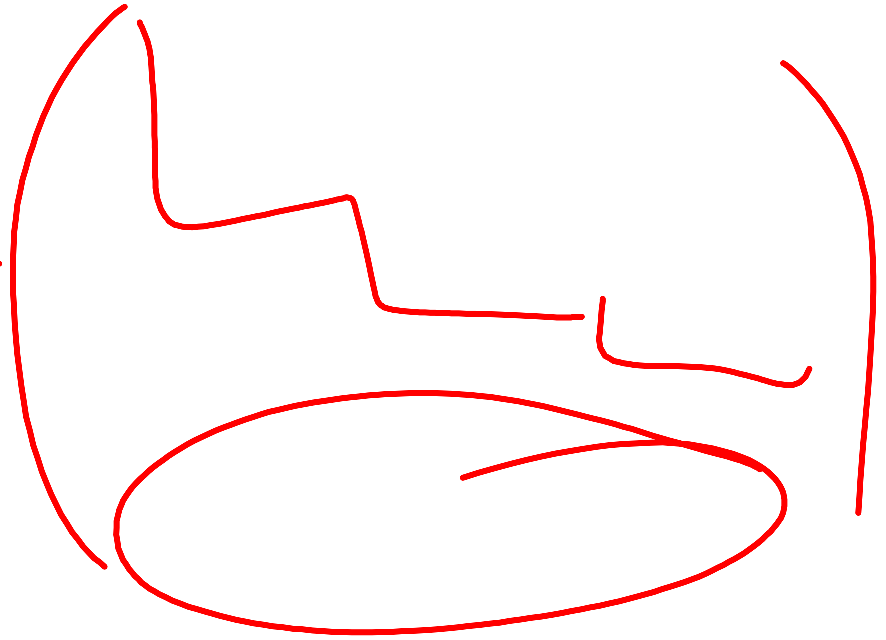
r basis V, $\quad r = (\overset{v_1}{x}, \dots\dots)$

$\varphi_x(v_1) = 1, \quad \varphi_x(v_i) = 0, \; i \neq 1 \qquad \varphi_x(x) = 1$

$A \sim B$ RST pomai el. î. grupă

$$[r_1(A), \ldots, r_m(A)] = [r_1(B), \ldots, r_m(B)] \quad (\Rightarrow r_1(A) = r_1(B)$$

$$\| \quad B = r_0(A)$$

$B =$ 

$j_1, \ldots, j_k$

mărind pe š

$$[r_1(A), \ldots, r_m(A)] = [r_{j_1}(A), \ldots, r_{j_k}(A)]$$

$$h(A) = \dim[r_1(A), \ldots, r_m(A)]$$

$$= \dim[r_1(A)^T, \ldots, r_m(A)^T] =$$

$$= \dim[r_1(A^T), \ldots, r_m(A^T)] =$$

$$= h(A^T)$$

$$A \sim \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$