Příklad. 1. Nechť $\varphi : \mathbb{R}_3[x] \to \mathbb{R}^4[x]$ je lineární zobrazení zadané předpisem

$$\varphi(p) = (x^2+1)p'(x),$$

kde p' je derivace polynomu p. Najděte matici $(\varphi)_{\beta,\alpha}$ zobrazení φ v bázích $\alpha = (1, x, x^2, x^3)$ prostoru $\mathbb{R}_3[x]$ a $\beta = (x^4, x^3, x^2, x, 1)$ prostoru $\mathbb{R}_4[x]$. (Pozor na pořadí vektorů v bázi.)

MATICA ZOBRAZENIA JE DEFINOVANÁ:

$$(\varphi)_{\boxed{\beta},\boxed{\alpha}} = \left((\varphi(\underline{p_1}))_{\boxed{\beta}} \; (\varphi(\underline{p_2}))_{\boxed{\beta}} \; \dots \; (\varphi(\underline{p_n}))_{\boxed{\beta}} \right), \text{ kde } \alpha = (p_1, p_2, \dots, p_n)$$

$\uparrow$ súradnice $\varphi(p_n)$ v bázi β

$$\varphi(1) = (x^2+1)\cdot(1)' = (x^2+1)\cdot 0 = 0 = 0\cdot x^4 + 0\cdot x^3 + 0\cdot x^2 + 0\cdot x + 0\cdot 1 \Rightarrow (\varphi(1))_\beta = \begin{pmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix} \begin{matrix} x^4 \\ x^3 \\ x^2 \\ x \\ 1 \end{matrix}$$

$$\varphi(x) = (x^2+1)\cdot(x)' = (x^2+1)\cdot 1 = 0\cdot x^4 + 0\cdot x^3 + 1\cdot x^2 + 0\cdot x + 1\cdot 1 \Rightarrow (\varphi(x))_\beta = \begin{pmatrix} 0 \\ 0 \\ 1 \\ 0 \\ 1 \end{pmatrix}$$

$$\varphi(x^2) = (x^2+1)\cdot(x^2)' = (x^2+1)\cdot 2x = 0\cdot x^4 + 2\cdot x^3 + 0\cdot x^2 + 2\cdot x + 0\cdot 1 \Rightarrow (\varphi(x^2))_\beta = \begin{pmatrix} 0 \\ 2 \\ 0 \\ 2 \\ 0 \end{pmatrix}$$

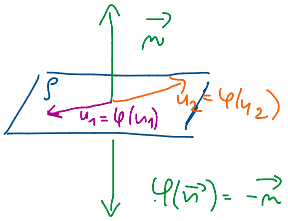
$$\varphi(x^3) = (x^2+1)\cdot(x^3)' = (x^2+1)\cdot 3x^2 = 3x^4 + 0\,x^3 + 3\cdot x^2 + 0\,x + 0\cdot 1$$

$$\Downarrow \quad (\varphi(x^3))_\beta = \begin{pmatrix} 3 \\ 0 \\ 3 \\ 0 \\ 0 \end{pmatrix}$$

$$(\varphi)_{\beta,\alpha} = \left((\varphi(1))_\beta \; (\varphi(x))_\beta \; (\varphi(x^2))_\beta \; (\varphi(x^3))_\beta \right)$$

$$= \begin{pmatrix} 0 & 0 & 0 & 3 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 3 \\ 0 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix}$$

Příklad. 2. Nechť φ je zobrazení $\mathbb{R}^3$ do sebe, které je symetrií podle roviny $x_1 + 2x_2 - x_3 = 0$. Najděte matici $(\varphi)_{\alpha,\alpha} = A$ tohoto zobrazení v bázi α složené ze dvou vektorů ležících v dané rovině a třetího vektoru kolmého k rovině. Dále najděte matici $(\varphi)_{\varepsilon,\varepsilon} = B$ ve standardní bázi prostoru $\mathbb{R}^3$.



$\rho: 1 \cdot x_1 + \boxed{2} x_2 \boxed{-} x_3 = 0 \quad \Rightarrow \quad \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} -2s+t \\ s \\ t \end{pmatrix} = t \cdot \underbrace{\begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix}}_{u_1} + s \cdot \underbrace{\begin{pmatrix} -2 \\ 1 \\ 0 \end{pmatrix}}_{u_2}$

$\vec{n} = (1, 2, -1)$

$\alpha = (u_1, u_2, \vec{n})$

$(\varphi)_{\alpha\alpha} = \left((\varphi(u_1))_\alpha \quad (\varphi(u_2))_\alpha \cdot (\varphi(u_3))_\alpha \right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -1 \end{pmatrix} = A$

$\varphi(u_1) = u_1 = 1 \cdot u_1 + 0 \cdot u_2 + 0 \cdot \vec{n}$

$\varphi(u_2) = u_2 = 0 \cdot u_1 + 1 \cdot u_2 + 0 \cdot \vec{n} \quad \Rightarrow$

$\varphi(\vec{n}) = -\vec{n} = 0 \cdot u_1 + 0 \cdot u_2 + (-1) \cdot \vec{n}$

$(\varphi(u_1))_\alpha = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$

$(\varphi(u_2))_\alpha = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$

$(\varphi(\vec{n}))_\alpha = \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix}$

$\varepsilon = (e_1, e_2, e_3)$

$B = (\varphi)_{\varepsilon,\varepsilon} = \left((\varphi(e_1))_\varepsilon \quad (\varphi(e_2))_\varepsilon \quad (\varphi(e_3))_\varepsilon \right)$ ← potrebujeme spočítať $\varphi(e_1), \varphi(e_2), \varphi(e_3)$

$$\left(\begin{array}{c|c} u_1 & \varphi(u_1) \\ u_2 & \varphi(u_2) \\ \vec{n} & \varphi(\vec{n}) \end{array} \right) = \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 1 \\ -2 & 1 & 0 & -2 & 1 & 0 \\ 1 & 2 & -1 & -1 & -2 & 1 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & 2 \\ 0 & 2 & -2 & -2 & -2 & 0 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & 2 \\ 0 & 1 & -1 & -1 & -1 & 0 \end{array} \right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 1 & 1 & 0 & 1 \\ 0 & 1 & 2 & 0 & 1 & 2 \\ 0 & 0 & 3 & 1 & 2 & 2 \end{array} \right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 2/3 & -2/3 & 1/3 \\ 0 & 1 & 0 & -2/3 & -1/3 & 2/3 \\ 0 & 0 & 1 & 1/3 & 2/3 & 2/3 \end{array} \right) = \left(\begin{array}{c|c} e_1 & \varphi(e_1) \\ e_2 & \varphi(e_2) \\ e_3 & \varphi(e_3) \end{array} \right)$$

$\underbrace{\begin{matrix} 2/3 & -2/3 & 1/3 \\ -2/3 & -1/3 & 2/3 \\ 1/3 & 2/3 & 2/3 \end{matrix}}_{B^T \text{ symetrická} = B}$

$\boxed{\text{Platí, že } (u)_\varepsilon = u \quad \forall u \in \mathbb{R}^n.}$

$\varphi(e_1) = 2/3\, e_1 + (-2/3) \cdot e_2 + (1/3) \cdot e_3 = \begin{pmatrix} 2/3 \\ -2/3 \\ 1/3 \end{pmatrix} = (\varphi(e_1))_\varepsilon.$ $\boxed{\text{Preto } B = (\varphi(e_1) \quad \varphi(e_2) \quad \varphi(e_3))}$

$$B = \frac{1}{3} \begin{pmatrix} 2 & -2 & 1 \\ -2 & -1 & 2 \\ 1 & 2 & 2 \end{pmatrix}.$$

Příklad. 3. Najděte matici $(\varphi)_{\alpha,\alpha}$ lineárního zobrazení $\varphi : \mathbb{R}^3 \to \mathbb{R}^3$ zadaného předpisem

$$\varphi(x) = \begin{pmatrix} 5 & 2 & -3 \\ 4 & 5 & -4 \\ 6 & 4 & -4 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix}$$

v bázi $\alpha = (\overset{u_1}{(1,1,2)^T}, \overset{u_2}{(1,0,1)^T}, \overset{u_3}{(1,2,2)^T})$.

$$(\varphi)_{\alpha,\alpha} = \left((\varphi(u_1))_\alpha \quad (\varphi(u_2))_\alpha \quad (\varphi(u_3)))_\alpha \right)$$

$$\varphi(u_1) = \begin{pmatrix} 5 & 2 & -3 \\ 4 & 5 & -4 \\ 6 & 4 & -4 \end{pmatrix} \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = \begin{pmatrix} 1 \\ 1 \\ 2 \end{pmatrix} = 1 \cdot u_1 + 0 \cdot u_2 + 0 \cdot u_3 \quad \Rightarrow \quad (\varphi(u_1))_\alpha = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$\varphi(u_2) = \begin{pmatrix} 5 & 2 & -3 \\ 4 & 5 & -4 \\ 6 & 4 & -4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 2 \\ 0 \\ 2 \end{pmatrix} = 0 \cdot u_1 + 2 \cdot u_2 + 0 \cdot u_3 \quad \Rightarrow \quad (\varphi(u_2))_\alpha = \begin{pmatrix} 0 \\ 2 \\ 0 \end{pmatrix}$$

$$\varphi(u_3) = \begin{pmatrix} 5 & 2 & -3 \\ 4 & 5 & -4 \\ 6 & 4 & -4 \end{pmatrix} \cdot \begin{pmatrix} 1 \\ 2 \\ 2 \end{pmatrix} = \begin{pmatrix} 3 \\ 6 \\ 6 \end{pmatrix} = 0 \cdot u_1 + 0 \cdot u_2 + 3 \cdot u_3 \quad \Rightarrow \quad (\varphi(u_3))_\alpha = \begin{pmatrix} 0 \\ 0 \\ 3 \end{pmatrix}$$

$$(\varphi)_{\alpha\alpha} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 3 \end{pmatrix}$$

Příklad. 4. Najděte matici lineárního zobrazení $\varphi : \mathbb{R}^3 \to \mathbb{R}^2$ zadaného předpisem

$$\varphi(x) = \begin{pmatrix}1 & 2 & 3\\ 2 & 1 & -1\end{pmatrix}\begin{pmatrix}x_1\\ x_2\\ x_3\end{pmatrix}$$

v bázích $\alpha = (\overset{u_1}{(1,0,1)^T}, \overset{u_2}{(1,1,2)^T}, \overset{u_3}{(1,-1,2)^T})$ a $\beta = (\underset{v_1}{(1,2)^T}, \underset{v_2}{(2,3)^T})$.

$$(\varphi)_{\beta,\alpha} = \left((\varphi(u_1))_\beta \quad (\varphi(u_2))_\beta \quad (\varphi(u_3))_\beta \right)$$

$$\Rightarrow \quad (\varphi(u_k))_\beta = \begin{pmatrix}a_{k1}\\ a_{k2}\end{pmatrix}$$

$$\left.\begin{aligned}
\varphi(u_1) &= \begin{pmatrix}1 & 2 & 3\\ 2 & 1 & -1\end{pmatrix}\cdot\begin{pmatrix}1\\ 0\\ 1\end{pmatrix} = \begin{pmatrix}4\\ 1\end{pmatrix} = \boxed{a_{11}}\cdot v_1 + \boxed{a_{12}}\cdot v_2 = a_{11}\cdot\begin{pmatrix}1\\ 2\end{pmatrix} + a_{12}\cdot\begin{pmatrix}2\\ 3\end{pmatrix}\\
\varphi(u_2) &= \begin{pmatrix}1 & 2 & 3\\ 2 & 1 & -1\end{pmatrix}\cdot\begin{pmatrix}1\\ 1\\ 2\end{pmatrix} = \begin{pmatrix}9\\ 1\end{pmatrix} = \boxed{a_{21}}\cdot v_1 + \boxed{a_{22}}\, v_2 = a_{21}\cdot\begin{pmatrix}1\\ 2\end{pmatrix} + a_{22}\cdot\begin{pmatrix}2\\ 3\end{pmatrix}\\
\varphi(u_3) &= \begin{pmatrix}1 & 2 & 3\\ 2 & 1 & -1\end{pmatrix}\cdot\begin{pmatrix}1\\ -1\\ 2\end{pmatrix} = \begin{pmatrix}5\\ -1\end{pmatrix} = \boxed{a_{31}}\cdot v_1 + \boxed{a_{32}}\, v_2 = a_{31}\cdot\begin{pmatrix}1\\ 2\end{pmatrix} + a_{32}\cdot\begin{pmatrix}2\\ 3\end{pmatrix}
\end{aligned}\right\}$$

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INÚ
PRAVÚ
STRANU

$$\underbrace{\left(\begin{array}{cc|ccc}1 & 2 & 4 & 9 & 5\\ 2 & 3 & 1 & 1 & -1\end{array}\right)}_{\left(\begin{array}{cc|ccc}v_1 & v_2 & \varphi(u_1) & \varphi(u_2) & \varphi(u_3)\end{array}\right) = \left(\begin{array}{c|c}(id)_{\varepsilon,\beta} & (\varphi)_{\varepsilon,\alpha}\end{array}\right)} \sim \text{Eko} \sim \left(\begin{array}{cc|ccc}1 & 0 & a_{11} & a_{21} & a_{31}\\ 0 & 1 & a_{12} & a_{22} & a_{32}\end{array}\right) = \left(\begin{array}{cc|ccc}\underbrace{e_1 \quad e_2}_{(id)_{\varepsilon\varepsilon}} & \multicolumn{3}{c}{\underbrace{(\varphi(u_1))_\beta \quad (\varphi(u_2))_\beta \quad (\varphi(u_3))_\beta}_{(\varphi)_{\beta,\alpha}}}\end{array}\right)$$

$$\left(\begin{array}{cc|ccc}1 & 2 & 4 & 9 & 5\\ 2 & 3 & 1 & 1 & -1\end{array}\right) \sim \left(\begin{array}{cc|ccc}1 & 2 & 4 & 9 & 5\\ 0 & -1 & -7 & -17 & -11\end{array}\right) \sim \left(\begin{array}{cc|ccc}1 & 0 & -10 & -25 & -17\\ 0 & 1 & 7 & 17 & 11\end{array}\right)$$

$$(\varphi)_{\beta,\alpha} = \begin{pmatrix}-10 & -25 & -17\\ 7 & 17 & 11\end{pmatrix}$$

Příklad. 5. Najděte matici $(\mathrm{id})_{\beta,\alpha}$ identického zobrazení zobrazení id(: $\mathbb{R}_2[x] \to \mathbb{R}_2[x]$, kde $\alpha = (\underset{p_1}{x^2+x+1}, \underset{p_2}{x+2}, \underset{p_3}{x^2-x})$ a $\beta = (\underset{q_1}{x^2+1}, \underset{q_2}{x^2-x-1}, \underset{q_3}{x+1})$. Najděte rovněž matice $(\mathrm{id})_{\alpha,\alpha}$ a $(\mathrm{id})_{\beta,\beta}$.

$$(id)_{\beta,\alpha} = \left((id(p_1))_\beta \quad (id(p_2))_\beta \quad (id(p_3))_\beta \right) = \left((p_1)_\beta \quad (p_2)_\beta \quad (p_3)_\beta \right)$$

$$p_1 = x^2+x+1 = a_{11}\cdot(x^2+1) + a_{12}\cdot(x^2-x-1) + a_{13}\cdot(x+1)$$

$$p_2 = x+2 = a_{21}\cdot(x^2+1) + a_{22}\cdot(x^2-x-1) + a_{23}\cdot(x+1) \quad \Rightarrow \quad (p_k)_\beta = \begin{pmatrix} a_{k1} \\ a_{k2} \\ a_{k3} \end{pmatrix}$$

$$p_3 = x^2-x = a_{31}\cdot(x^2+1) + a_{32}\cdot(x^2-x-1) + a_{33}\cdot(x+1)$$

$\varepsilon = (x^2, x, 1)$

$$\begin{array}{r} x^2: \\ x: \\ 1: \end{array} \underbrace{\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & -1 & 1 & 1 & 1 & -1 \\ 1 & -1 & 1 & 1 & 2 & 0 \end{array}\right)}_{\left(\underbrace{(q_1)_\varepsilon \ (q_2)_\varepsilon \ (q_3)_\varepsilon}_{(id)_{\varepsilon,\beta}} \middle| \underbrace{(p_1)_\varepsilon \ (p_2)_\varepsilon \ (p_3)_\varepsilon}_{(id)_{\varepsilon,\alpha}} \right)} \sim \ \mathrm{ERO} \ \sim \left(\underbrace{e_1 \quad e_2 \quad e_3}_{(id)_{\beta,\beta}} \middle| \underbrace{\begin{array}{ccc} \overset{(p_1)_\beta}{\overset{\parallel}{a_{11}}} & \overset{(p_2)_\beta}{\overset{\parallel}{a_{21}}} & \overset{(p_3)_\beta}{\overset{\parallel}{a_{31}}} \\ a_{12} & a_{22} & a_{32} \\ a_{13} & a_{23} & a_{33} \end{array}}_{(id)_{\beta,\alpha}} \right)$$

NÁSOBENIE Z ĽAVA $(id)_{\varepsilon,\beta}$

HĽADANÁ MATICA

$$\left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & -1 & 1 & 1 & 1 & -1 \\ 1 & -1 & 1 & 1 & 2 & 0 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & -1 & 1 & 1 & 1 & -1 \\ 0 & 2 & -1 & 0 & -2 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & -1 & -1 & -1 & 1 \\ 0 & 0 & 1 & 2 & 0 & -1 \end{array}\right)$$

$$\sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & -1 & 0 \\ 0 & 0 & 1 & \underbrace{2 \quad 0}_{(id)_{\beta\alpha}} & & -1 \end{array}\right) \qquad (id)_{\beta,\alpha} = \begin{pmatrix} 0 & 1 & 1 \\ 1 & -1 & 0 \\ 2 & 0 & -1 \end{pmatrix}$$

$$(id)_{\alpha,\alpha} = \left((id(p_1))_\alpha \quad (id(p_2))_\alpha \quad (id(p_3))_\alpha \right) = \left((p_1)_\alpha \quad (p_2)_\alpha \quad (p_3)_\alpha \right)$$

$$p_1 = 1\cdot p_1 + 0\cdot p_2 + 0\cdot p_3 \quad \Rightarrow \quad (p_1)_\alpha = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix}$$

$$p_2 = 0\cdot p_1 + 1\cdot p_2 + 0\cdot p_3 \quad \Rightarrow \quad (p_2)_\alpha = \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix}$$

$$p_3 = 0\cdot p_1 + 0\cdot p_2 + 1\cdot p_3 \quad \Rightarrow \quad (p_3)_\alpha = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix}$$

$$(id)_{\alpha,\alpha} = \left(\boxed{\begin{matrix} 1 \\ 0 \\ 0 \end{matrix}} \ \boxed{\begin{matrix} 0 \\ 1 \\ 0 \end{matrix}} \ \boxed{\begin{matrix} 0 \\ 0 \\ 1 \end{matrix}} \right)$$

Analogicky $\quad (id)_{\beta,\beta} = \left((q_1)_\beta \quad (q_2)_\beta \quad (q_3)_\beta \right) = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$

Příklad. 5. Najděte nějaké lineární zobrazení $f : \mathbb{R}^3 \to \mathbb{R}^3$ takové, že

$$\ker f = \left[\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix} \right] \quad \text{a} \quad \operatorname{im} f = \left[\begin{pmatrix}1\\0\\1\end{pmatrix} \right].$$

Zo zadania vieme, že $\overset{u_1}{\begin{pmatrix}1\\0\\0\end{pmatrix}}, \overset{u_2}{\begin{pmatrix}1\\1\\1\end{pmatrix}} \xrightarrow{f} \begin{pmatrix}0\\0\\0\end{pmatrix}$

$$? \xrightarrow{f} \begin{pmatrix}1\\0\\1\end{pmatrix}^{v}$$

Chceme $f(e_1), f(e_2), f(e_3)$.

Potrebujeme $\begin{pmatrix}1\\0\\0\end{pmatrix}, \begin{pmatrix}1\\1\\1\end{pmatrix}$ doplniť na bázu $\mathbb{R}^3$. Napr. vezmeme $e_2 = \begin{pmatrix}0\\1\\0\end{pmatrix}$.

$$\left(\begin{array}{c|c} u_1 & 0 \\ u_2 & 0 \\ e_2 & v \end{array}\right) = \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \end{array}\right) \sim \left(\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & -1 & 0 & -1 \end{array}\right)$$

$f(e_1) = \begin{pmatrix}0\\0\\0\end{pmatrix}$

$f(e_2) = \begin{pmatrix}1\\0\\1\end{pmatrix}$

$f(e_3) = \begin{pmatrix}-1\\0\\-1\end{pmatrix}$

$$f(x) = \begin{pmatrix} f(e_1) & f(e_2) & f(e_3) \end{pmatrix} \cdot x = \begin{pmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix} \cdot \begin{pmatrix}x_1\\x_2\\x_3\end{pmatrix}$$

Skúška $\begin{pmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix}1\\0\\0\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$ ✓

$\begin{pmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix} \begin{pmatrix}1\\1\\1\end{pmatrix} = \begin{pmatrix}0\\0\\0\end{pmatrix}$ ✓

$\begin{pmatrix} 0 & 1 & -1 \\ 0 & 0 & 0 \\ 0 & 1 & -1 \end{pmatrix} \cdot \begin{pmatrix}0\\1\\0\end{pmatrix} = \begin{pmatrix}1\\0\\1\end{pmatrix}$ ✓