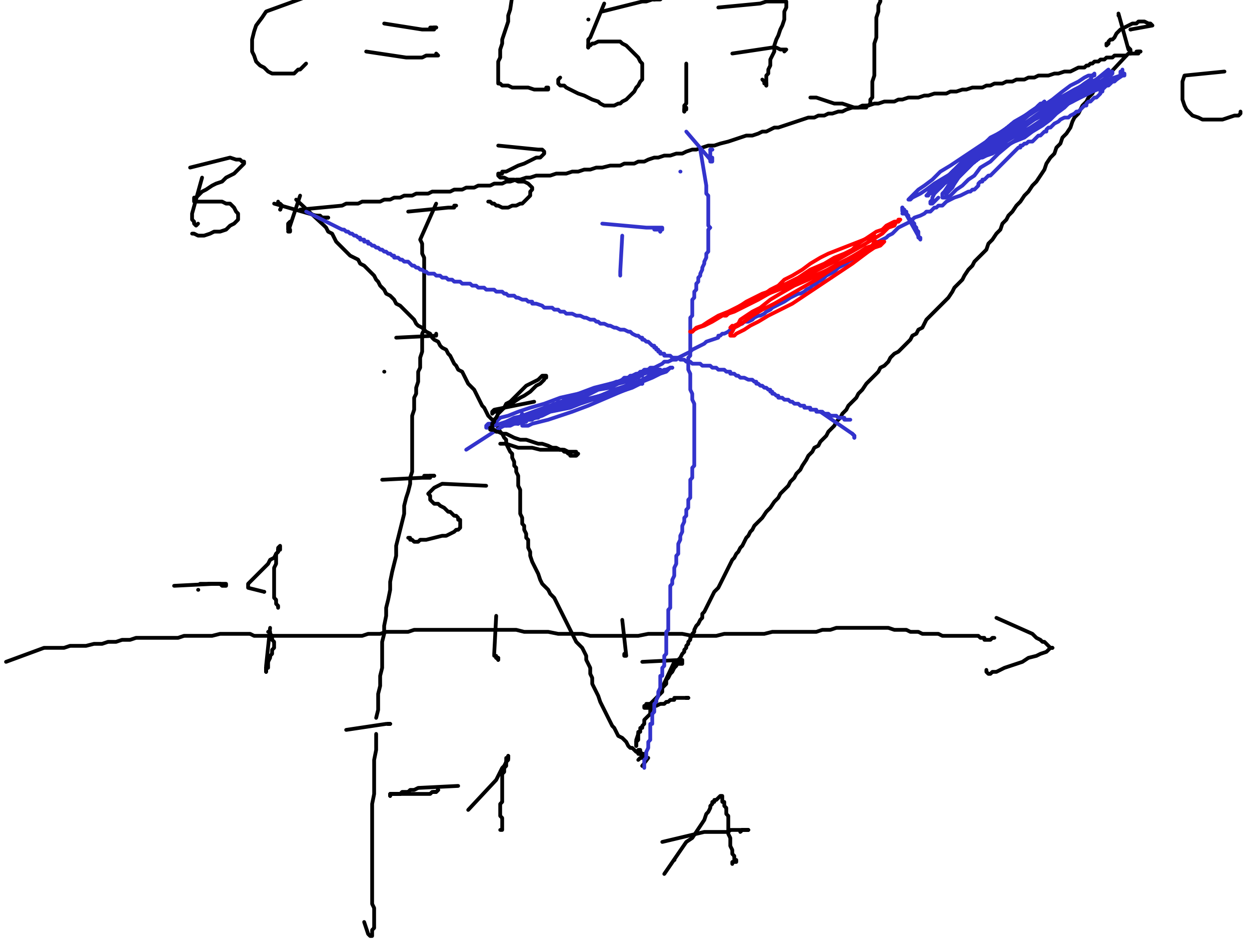
2.1) $A = [2, -1]$ $\quad$ $T = [r, s]$

$B = [-1, 3]$ $\quad$ těžiště

$C = [5, 7]$



$$S = \left[\frac{1}{2}(2-1), \frac{1}{2}(-1,3)\right]$$

$$= \left[\frac{1}{2}, 1\right]$$

$$\overrightarrow{CS} = S - C = \left(\frac{1}{2} - 5, 1 - 7\right)$$

$$= \left(-\frac{9}{2}, -6\right)$$

$$T = C + \frac{2}{3}\overrightarrow{CS} =$$

$$= [5, 7] + \frac{2}{3}\left(-\frac{9}{2}, -6\right)$$

$$= [5, 7] + (-3, -4)$$

$$= \underline{\underline{[2, 3]}}$$

<u>Jinak</u>

$$T = \frac{A + B + C}{3} = \left[\frac{6}{3}, \frac{9}{3}\right] =$$

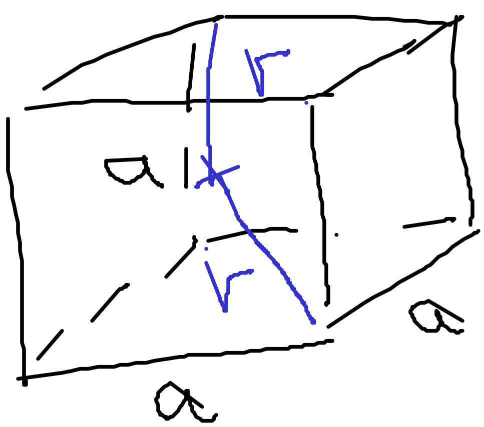
$$= [2, 3]$$

22.

Krychle vepsaná do kulové plochy

Povrch krychle = 72 cm²

$r = ?$



Povrch $= 6a^2$

$$6a^2 = 72$$

$$a^2 = 12$$

$$a = 2\sqrt{3}$$

r = polovina tělesové úhlopříčky

Úhlopříčka podstavy

$= \sqrt{a^2 + a^2} = \sqrt{2a^2}$

Tělesová úhlopříčka.

$= \sqrt{a^2 + a^2 + a^2} = \sqrt{3a^2}$

$= \sqrt{3 \cdot 12} = 2 \cdot 3 = 6$

$\Rightarrow r = 3$

2.3 $\quad |2x+1| < x+3$

1 $\quad x \le -\frac{1}{2}$

$-(2x+1) < x+3$

$-4 < 3x$

$-\frac{4}{3} < x$

$\Downarrow$

$\cancel{x > -\frac{4}{3}}$ $\quad -\frac{4}{3} < x \le -\frac{1}{2}$

$x \ge -\frac{1}{2}$

$2x+1 < x+3$

$x < 2$

$\Downarrow$

$-\frac{1}{2} \le x < 2$

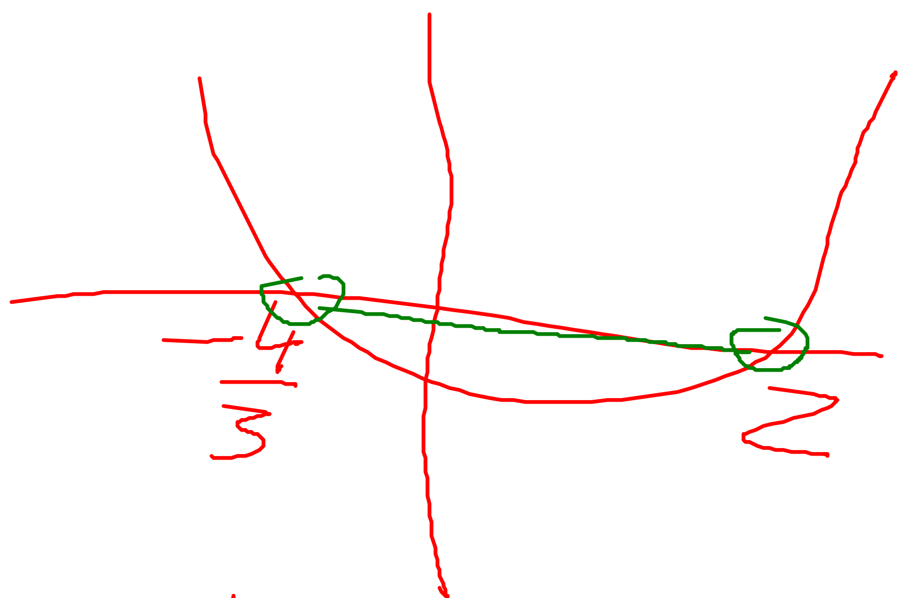
Zaključak: $\quad x \in \left[-\frac{4}{3}, 2\right)$

$$② \quad |2x+1| < x+3 \quad /(\ )^2$$

$$(2x+1)^2 < (x+3)^2$$

$$4x^2+4x+1 < x^2+6x+9$$

$$3x^2-2x-8<0$$



$$x_{1,2} = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$$

$$ax^2+bx+c=0,$$

$$3x^2 - 2x - 8 = 0$$

$$x_{1,2} = \frac{2 \pm \sqrt{4 - 4 \cdot 3 \cdot (-8)}}{6}$$

$$= \frac{2 \pm \sqrt{4(1+24)}}{6}$$

$$= \frac{2 \pm 10}{6} = \boxed{\left(-\frac{4}{3}, 2\right)}$$

Podmínka $x + 3 \geq 0$

$$x \geq -3$$

2.4 $z+|z| = 5+(2+i)^2$

Odredite z^2.

$z = a + ib, \quad a, b \in \mathbb{R}$

$|z| = \sqrt{a^2+b^2} \qquad i^2 = -1$

$a + ib + \sqrt{a^2+b^2} = 5 + 4 + 4i - 1$

$\underline{\left(a+\sqrt{a^2+b^2}\right)} + i\underline{b} = \underline{8} + \underline{4i}$

$b = 4$

$a + \sqrt{a^2+16} = 8$

$\sqrt{a^2+16} = 8 - a \quad (\;)^2$

$\rightsquigarrow$ podmínka $8 - a \geq 0$

$$a^2 + 16 = 64 - 16a + a^2$$

$$16a = 48$$

$$a = 3$$

Tedy $\boxed{z = 3 + 4i}$

$$z^2 = 9 + 24i - 16$$

$$= -7 + 24i$$

2.5 $\quad x^{2\log x + 3{,}5} = 100\sqrt{x}$

$a, b \in \mathbb{R}$ rešenj, $a < b$

$k = a^{b^2}$

$\log x = \log_{10} x$

$x^{2\log x + 3{,}5} = 100\sqrt{x} \quad / x^{-\frac{1}{2}}$

$x^{2\log x + 3} = 100 \quad / \log$

$\log y^r = r \log y$

$\underbrace{(2\log x + 3)}_{w} \underbrace{\log x}_{w} = 2$

$$w = \log x$$

$$(2w+3)w = 2$$

$$2w^2 + 3w - 2 = 0$$

$$w_{1,2} = \frac{-3 \pm \sqrt{9 - 4 \cdot 2 \cdot (-2)}}{4}$$

$$= \frac{-3 \pm \sqrt{9+16}}{4}$$

$$= \frac{-3 \pm 5}{4} = \begin{cases} -2 \\ \frac{1}{2} \end{cases}$$

$$w = \log x$$

$$-2 = \log \frac{1}{100}$$

2.6 c = součet řešení rovnic o

$$\boxed{\sin x + \cos x = \sqrt{2}}$$

v intervalu $[0, 2\pi]$

$\sin x, \cos x > 0$

$\Rightarrow$ 1. kv.

$\Rightarrow$ uhodneme $x = \frac{\pi}{4}$

$$\sin x + \cos x = \sqrt{2} \quad / (\ )^2$$

ekvivalentní úprava

$$\frac{1}{2} = \log\sqrt{10}$$

$$x_{1,2} = \left\langle \begin{array}{l} \frac{1}{100} = a \\ \sqrt{10} = b \end{array} \right.$$

$$k = ab^2 = \frac{1}{100} \cdot 10 = \frac{1}{10}$$

z. 6 pa kvadratom

$$\sin x + \cos x = \sqrt{2} \quad /(\ )^2$$

$$\underline{\sin^2 x} + \underbrace{2\sin x\cos x} + \underline{\cos^2 x} = 2$$

$$1 + \sin 2x = 2$$

$$\sin 2x = 1$$

$$2x = \frac{\pi}{2} + 2k\pi$$

$$k = 0, 1$$

$\underline{k=0}$: $2x = \frac{\pi}{2}$

$$x = \frac{\pi}{4}$$

$\underline{k=1}$: $2x = \frac{5\pi}{2} \Rightarrow x = \frac{5\pi}{4}$

není v ↑ 1. kv.

Zápis:

$x = \frac{\pi}{4} = c$

↳ jediné řešení

2.7 Počet lichých pěticiferných čísel, která neobsahují cifru 9

_ _ _ _ _

8 9 4

$8 \cdot 9 \cdot 9 \cdot 9 \cdot 4$

2.8 Kwiekosech.

$$3x^2 + 5y^2 + 6x - 2y + 8 = 0$$

a, b dilly polaas

$$c = a^2 + b^2$$

Uprowne na ctweroc $= 0$

$$3(x^2 + 2x) + 5\left(y^2 - \frac{2}{5}y\right) + 8$$

$$3\left[(x+1)^2 - 1\right] + 5\left[\left(y - \frac{1}{5}\right)^2 - \frac{1}{25}\right] + 8 = 0$$

$$3(x+1)^2 + 5\left(y - \frac{1}{5}\right)^2 - 3 - \frac{1}{5} + 8 = 0$$

$$3(x+1)^2 + 5\left(y - \frac{1}{5}\right)^2 + \frac{24}{5} = 0$$

⇒ tvá zadání: ...

Správné zadání:

$$3(x^2+2x)+5(y^2-4y)+8=0$$

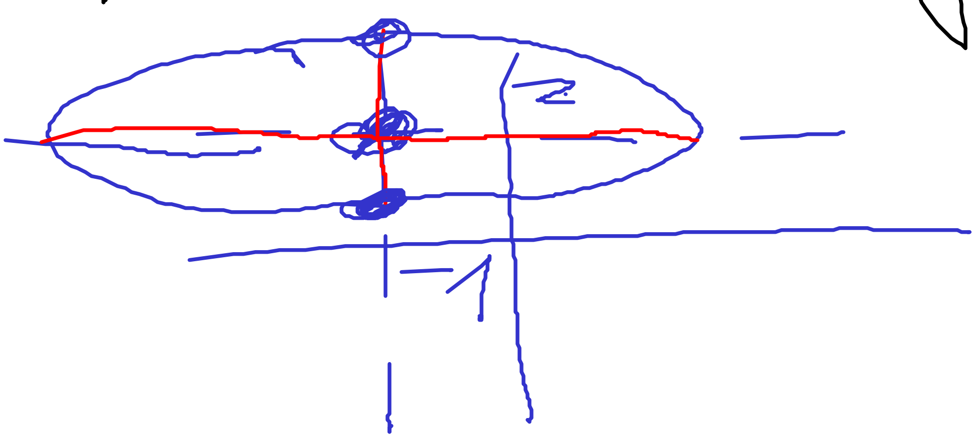
$$3[(x+1)^2-1]+5[(y-2)^2-4]+8=0$$

$$3(x+1)^2+5(y-2)^2-3-20+8=0$$

$$3(x+1)^2+5(y-2)^2-15=0$$

poloosy $\parallel$ s přímky

$x=-1$ $\qquad$ $y=2$



$\underline{x = -1} \Rightarrow 5(y-2)^2 = 15$

$(y-2)^2 = 3$

$y - 2 = \pm\sqrt{3}$

$y = 2 \pm \sqrt{3}$

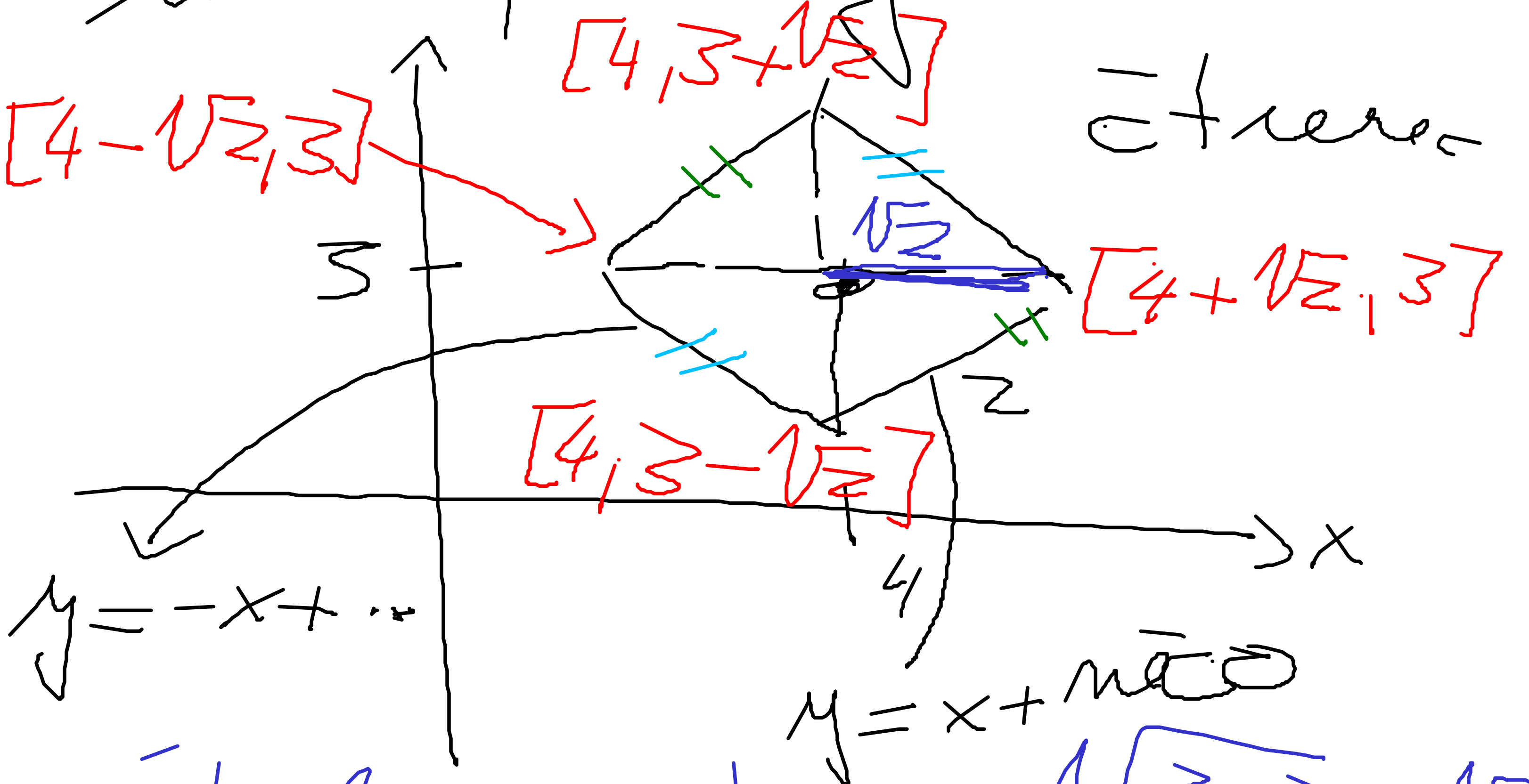
$\Rightarrow$ dělka poloosy $= \sqrt{3}$

$\underline{y = 2} \Rightarrow$ poloosa $\sqrt{5}$

<u>Závěr</u>: $c^2 = a^2 + b^2$

$= 3 + 5 = \underline{\underline{8}}$

1.7 čtverec se
středem [4,3],
stranou délky 2
úhlopříčky || s osami



Úhlopříčka $= \sqrt{2^2+2^2} = 2\sqrt{2}$

Rovnostranná $x+y=d$

$x-y=d$

$y = x + d$ bod $[4, 3-\sqrt{2}]$

$$\boxed{y = x - 1 - \sqrt{2}}$$

bod $[4, 3+\sqrt{2}]$

$$\boxed{y = x - 1 + \sqrt{2}}$$

$$\boxed{\begin{array}{l} y \cancel{\geq} > x - 1 - \sqrt{2} \\ y \cancel{\leq} < x - 1 + \sqrt{2} \end{array}}$$

+ další dvě nerovnice.