1. (i) $$\sum_{n=1}^{\infty} \underbrace{\frac{5^{4n-1}}{4^{5n+1}}}_{a_n} \quad \Bigg\| \quad \lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \lim_{n\to\infty} \frac{5^{4(n+1)-1}}{4^{5\cdot(n+1)+1}} \cdot$$

$$\frac{4^{5n+1}}{5^{4n-1}} = \lim_{n\to\infty} \frac{5^{\cancel{4n}+3} \cdot 4^{\cancel{5n}+1}}{5^{\cancel{4n}-1} \cdot 4^{\cancel{5n}+6}} = \lim_{n\to\infty} \frac{5^4}{4^5} =$$

$$= \frac{5^4}{4^5} = \frac{625}{1024} < 1 \Rightarrow \text{(K)}$$

(ii) $x_0 = -3$, $a = \frac{1}{R} = \lim_{n\to\infty} \sqrt[n]{\left|\left(1+\frac{1}{n}\right)^{n^2}\right|} =$

$= \lim_{n\to\infty} \left(1+\frac{1}{n}\right)^n = e \Rightarrow R = \frac{1}{e}$

$I = \left(-3-\frac{1}{e}, -3+\frac{1}{e}\right)$

(iii) $x_0 = 5$, $a = \frac{1}{R} = \lim_{n\to\infty} \frac{(n^4+3n^2+2)\cdot 5^n}{((n+1)^4+3\cdot(n+1)^2+2)\cdot 5^{n+1}}$

$= \lim_{n\to\infty} \frac{n^4}{5\cdot n^4} = \frac{1}{5} \Rightarrow R = 5$

$\frac{a_{n+1}}{a_n}$

$I = (0, 10)$

$$x = 0 \Rightarrow \sum_{n=1}^{\infty} \frac{(-5)^n}{(n^4+3n^2+2)\cdot 5^n} = \sum_{n=1}^{\infty} (-1)^n \cdot \frac{1}{n^4+3n^2+2}$$

$\searrow 0$

ALT. Ř. & $|a_n|$ MONOT. DO 0

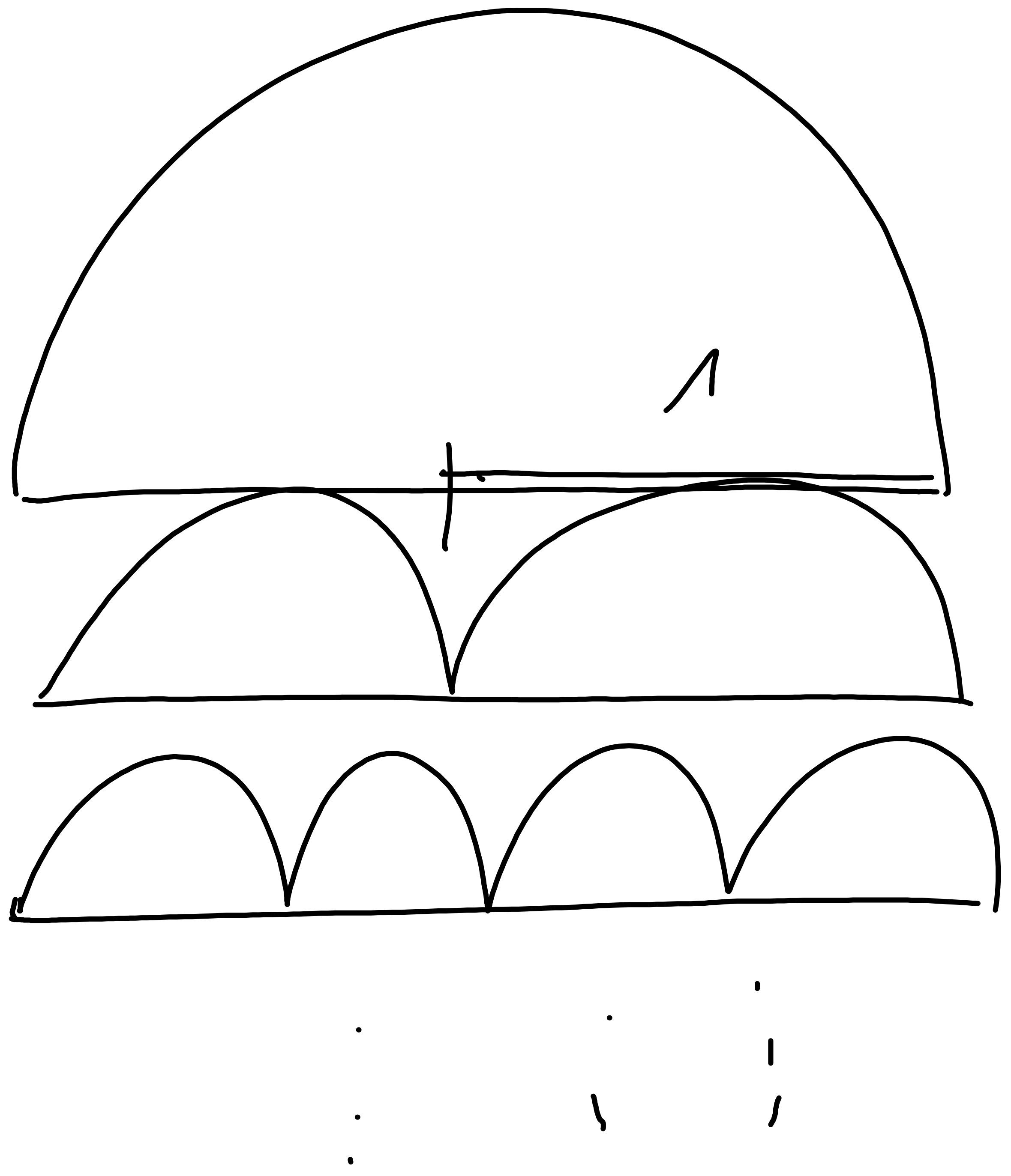
$\Rightarrow$ DLE LEIB. KRIT. (K)

$$x = 10 \Rightarrow \sum_{n=1}^{\infty} \frac{5^n}{(n^4+3n^2+2)\cdot 5^n} = \sum_{n=1}^{\infty} \frac{1}{n^4+3n^2+2} \leq \sum_{n=1}^{\infty} \frac{1}{n^4}$$

$\sum_{n=1}^{\infty} \frac{1}{n^4+3n^2+2}$ → K ⇐ K ← $\sum_{n=1}^{\infty} \frac{1}{n^4}$

⇒ ABS. K PRO $x \in [0, 10]$

(2.)



POČET v n-tém ř. $= 2^{n-1}$

POLOMĚR -- " -- $= \frac{1}{2^{n-1}}$

$$S = \frac{1}{2} \cdot \pi r^2$$

ŘÁDEK	POLOM.	POČET	OBSAH CELKEM π.
1.	$\frac{1}{2^0}$	2^0	$1 \cdot \frac{1}{2} \cdot \pi \cdot 1 = \frac{\pi}{2}$
2.	$\frac{1}{2^1}$	2^1	$2 \cdot \frac{1}{2} \cdot \pi \cdot \frac{1}{2^2} = \frac{1}{2^2} \cdot \pi$
3.	$\frac{1}{2^2}$	2^2	$2^2 \cdot \frac{1}{2} \cdot \pi \cdot \frac{1}{2^4} = \frac{1}{2^3} \cdot \pi$
⋮			
n.	$\frac{1}{2^{n-1}}$	2^{n-1}	$2^{n-1} \cdot \frac{1}{2} \cdot \pi \cdot \frac{1}{2^{2n-2}} = \boxed{\frac{1}{2^n}\pi}$

$$\sum_{n=1}^{\infty} \frac{\pi}{2^n} = \pi \cdot \sum_{n=1}^{\infty} \left(\frac{1}{2}\right)^n = \pi \cdot \frac{\frac{1}{2}}{1-\frac{1}{2}} = \underline{\underline{\pi}}$$

③ $\sum_{n=0}^{\infty} x^n \cdot (n+2)\cdot(n+1)$, $x_0 = 0$, $R = 1$

$\Rightarrow I = (-1, 1)$

$x = -\frac{1}{4}$

$-\frac{1}{4} \in I$ ✓

$n^2 + 3n + 2 = (n+2)\cdot(n+1)$

$$\sum_{n=0}^{\infty} x^n = \frac{1}{1-x} \quad \Big/ \cdot x^2$$

$$\sum_{n=0}^{\infty} x^{n+2} = \frac{x^2}{1-x} \quad \Big/ \frac{d}{dx}$$

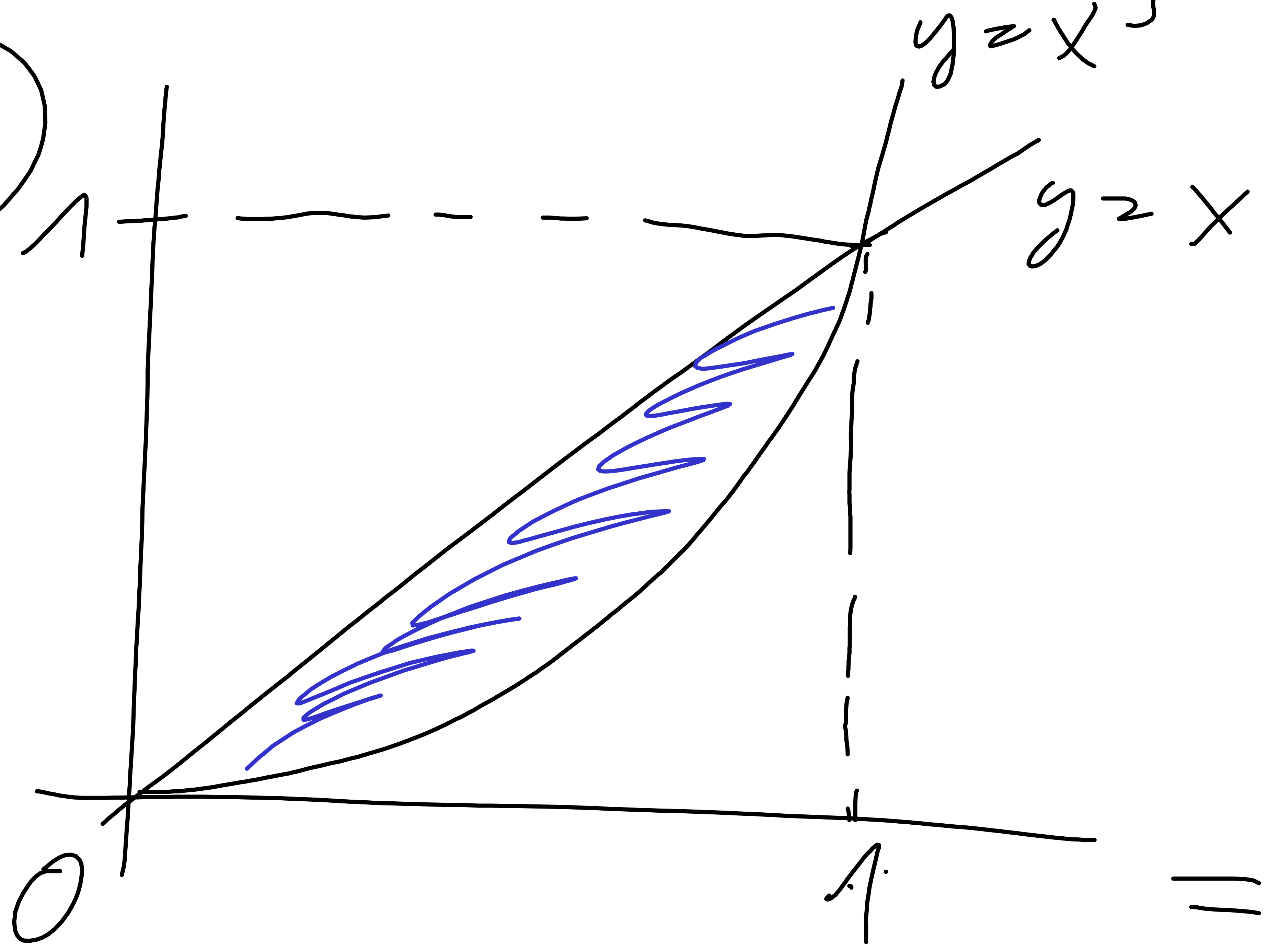
$$\sum_{n=0}^{\infty} (n+2) \cdot x^{n+1} = \frac{2x-x^2}{(1-x)^2} \quad \Big/ \frac{d}{dx}$$

$$\sum_{n=0}^{\infty} (n+2)\cdot(n+1) \cdot x^n = \frac{2}{(1-x)^3}$$

$$\sum_{n=0}^{\infty} (-1)^n \cdot \frac{n^2+3n+2}{4^n} = \frac{2}{\left(1+\frac{1}{4}\right)^3} = \frac{2}{\left(\frac{5}{4}\right)^3} = \frac{128}{125}$$

(4.) (5.) VIZ PREZ.

6.



$$\int_0^1 \left[\int_{x^3}^{x} x^3 y \, dy \right] dx =$$

$$= \int_0^1 x^3 \cdot \left[\frac{y^2}{2} \right]_{x^3}^{x} dx = \frac{1}{2} \int_0^1 x^3 \cdot (x^2 - x^6) \, dx$$

$$= \frac{1}{2} \int_0^1 x^5 - x^9 \, dx = \frac{1}{2} \cdot \left[\frac{x^6}{6} - \frac{x^{10}}{10} \right]_0^1 = \frac{1}{2} \cdot \left(\frac{1}{6} - \frac{1}{10} \right) =$$

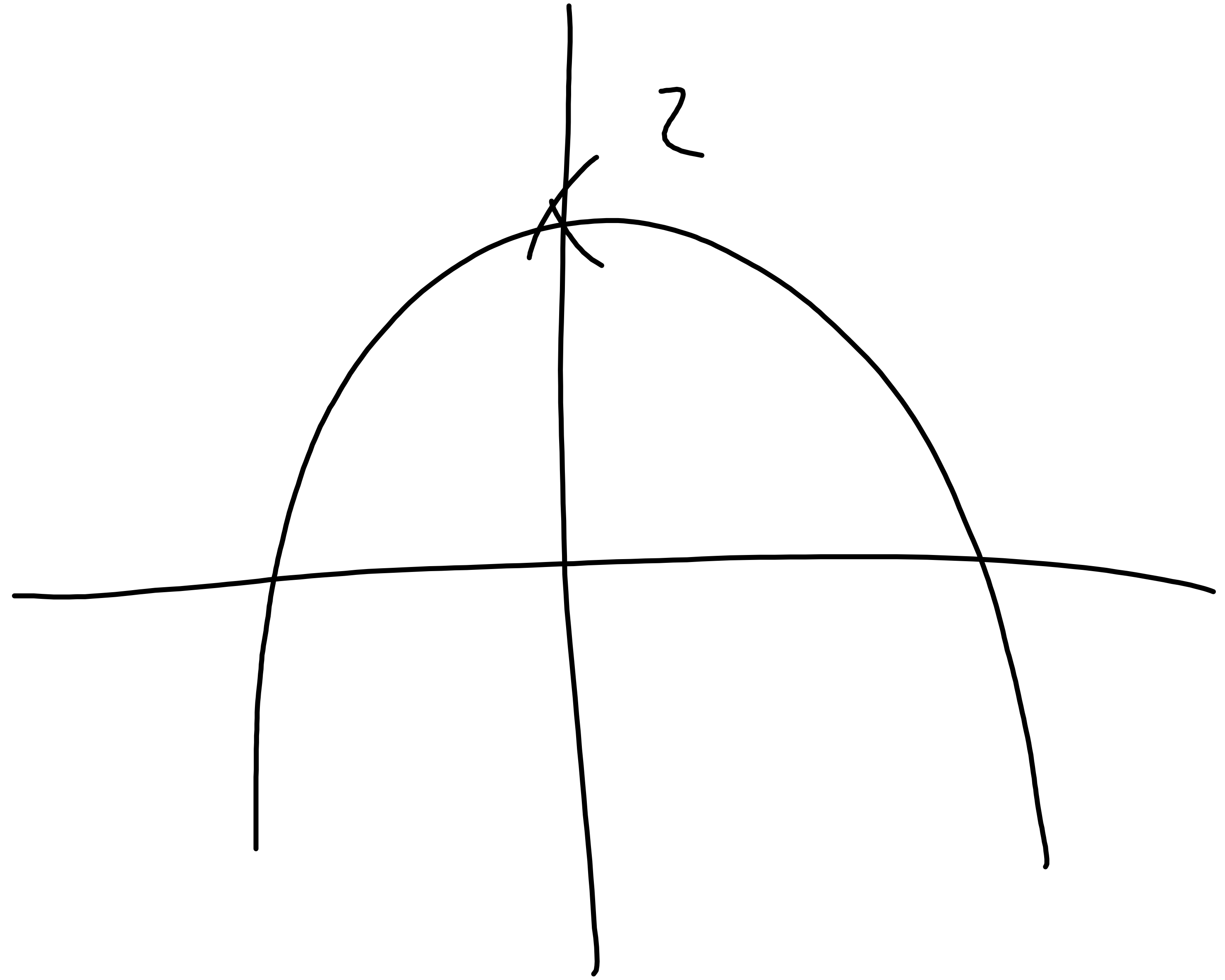
$$= \frac{1}{30}$$

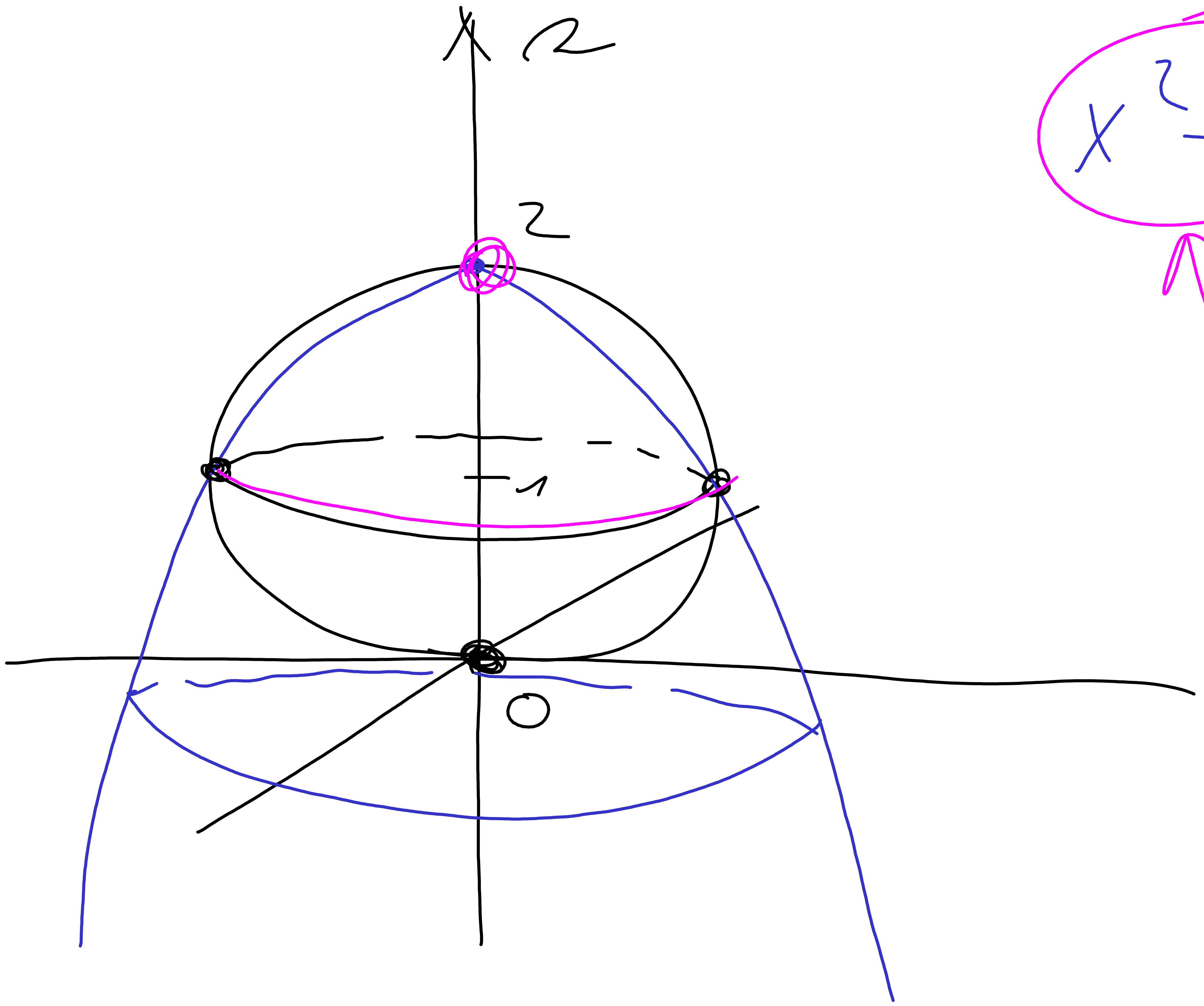
$$\int_0^1 \left(\int_y^{\sqrt[3]{y}} x^3 y \, dx \right) dy$$

⑦ $x^2+y^2+z^2 \le 2z$, $z \le 2-x^2-y^2$

$x^2+y^2+z^2-2z \le 0$

$x^2+y^2+(z-1)^2 \le 1$





$x^2+y^2+z^2=2z \,,\quad z=2-x^2-y^2$

$\Downarrow$

$x^2+y^2=2-z$

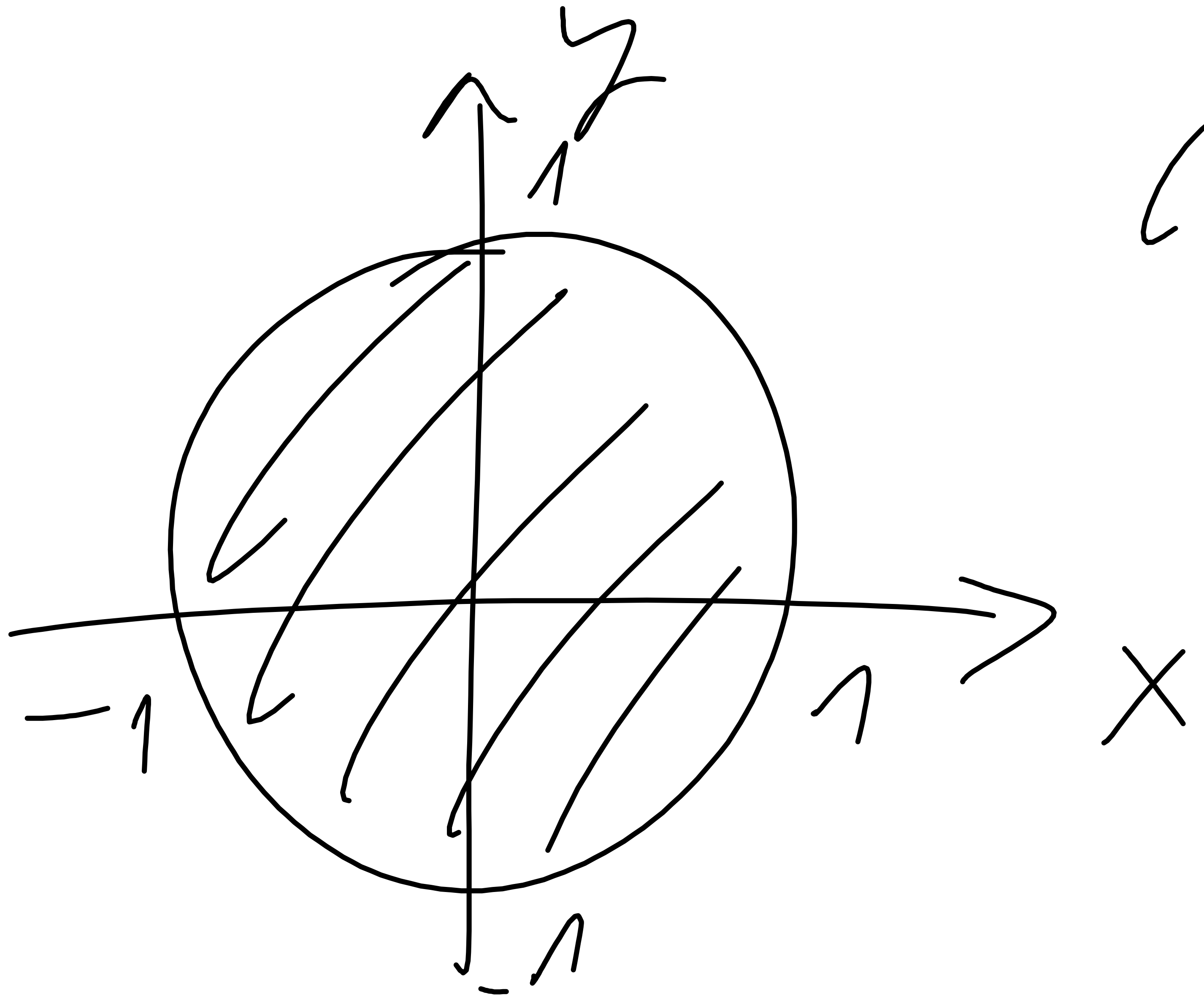
$2-z+z^2=2z$

$z^2-3z+2=0$

$$x^2 - 3x + 2 = 0, \quad D = 9 - 8 = 1$$

$$x_{1,2} = \frac{3 \pm 1}{2} = \begin{cases} = 2 \\ = 1 \end{cases}$$

$\Rightarrow$



vácc. somž.

$$\Rightarrow \quad \rho \in [0, 1]$$

$$\varphi \in [0, 2\pi]$$

$$z \in \left[1-\sqrt{1-\rho^2},\ 2-\rho^2\right]$$

$$z = 2 - x^2 - y^2$$

$$(z-1)^2 = 1 - x^2 - y^2$$

$$z - 1 = \pm\sqrt{1 - x^2 - y^2}$$

$$x = \rho \cdot \cos\varphi$$

$$y = \rho \cdot \sin\varphi$$

$$z = z$$

$$|J| = \rho$$

$$V = \iiint_T 1 \, dx\,dy\,dz = \int_0^{2\pi} \left[\int_0^1 \int_{1-\sqrt{1-\rho^2}}^{2-\rho^2} 1 \cdot \rho \; dz\,d\rho \right] d\varphi$$

$$= 2\pi \cdot \int_0^1 \rho \cdot \left[z \right]_{1-\sqrt{1-\rho^2}}^{2-\rho^2} d\rho =$$

$$= 2\pi \cdot \int_0^1 \rho \Big(\underbrace{2-\rho^2-1}_{=1-\rho^2} + \sqrt{1-\rho^2} \Big) d\rho =$$

$$t = 1-\rho^2$$

$$dt = -2\rho \, d\rho$$

$$\rho = 1 \Rightarrow t = 0$$

$$\rho = 0 \Rightarrow t = 1$$

$$= -\pi \cdot \int_1^0 t + \sqrt{t} \, dt = \pi \cdot \int_0^1 t + t^{\frac{1}{2}} \, dt =$$

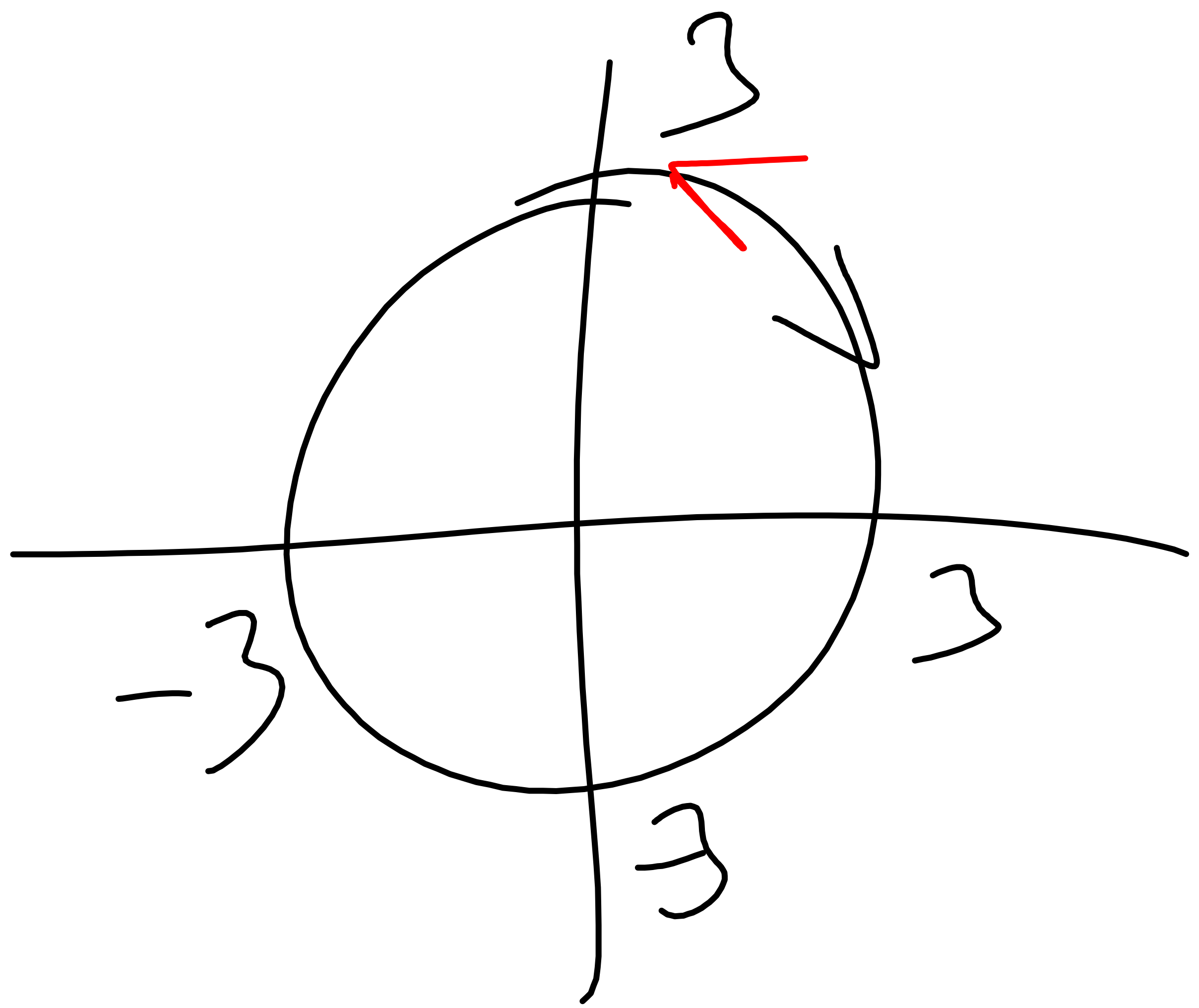
$$= \pi \cdot \left[\frac{t^2}{2} + \frac{t^{\frac{3}{2}}}{\frac{3}{2}} \right]_0^1 = \pi \left(\frac{1}{2} + \frac{2}{3} \right) = \frac{7}{6} \pi$$

POZN.:

$$V = \frac{1}{2}\cdot\left(\frac{4}{3}\pi r^{3}\right) + \int_{0}^{2\pi}\int_{0}^{1}\int_{1}^{2-\rho^{2}} \rho \, dz \, d\rho \, d\varphi$$

$$r = 1$$

⑧ $x^2+y^2=9$



$$\begin{cases} x = 3\cdot\cos\varphi \\ y = 3\cdot\sin\varphi \end{cases}, \quad \varphi\in[0,2\pi]$$

$$dx = -3\sin\varphi\, d\varphi$$

$$dy = 3\cdot\cos\varphi\, d\varphi$$

$$\oint_C \ldots = +\int_0^{2\pi} \frac{3\cdot\cos\varphi + 3\cdot\sin\varphi}{9\cdot(1)}\left(+3\cdot\sin\varphi\right) +$$

$$+\ \frac{3\cdot\cos\varphi - 3\cdot\sin\varphi}{9\cdot(1)}\cdot 3\cos\varphi \; d\varphi =$$

$$= \int_0^{2\pi} \sin\varphi \cdot \cos\varphi + \sin^2\varphi + \cos^2\varphi - \sin\varphi \cdot \cos\varphi \, d\varphi$$

$$\sin^2\varphi + \cos^2\varphi = 1$$

$$= \int_0^{2\pi} 1 \, d\varphi = 2\pi$$