Essay on Symplectic Geometry

The essay should not be longer than 10 pages and should be typed on computer. Moreover, it should contain the following:

1. Define what a symplectic manifold is and give examples. In particular, show that the cotangent space T^*M of any manifold M is in a natural way a symplectic manifold.

2. State and prove the Theorem of Darboux (which says that any symplectic manifold of dimension $2n$ is locally isomorphic to the symplectic manifold $T^*\mathbb{R}^n = \mathbb{R}^n \times (\mathbb{R}^n)^*$.)

3. Define what a Lagrangian submanifold of a symplectic manifold is. Give examples of Langrangian submanifolds of T^*M: When is the image of a section of T^*M a Lagrangian submanifold of T^*M? What about the (so-called) conormal bundle inside T^*M of a submanifold $X \subset M$?

4. What is the Poisson bracket of a symplectic manifold?

References

[1] Agrikola, I. and Friedrich, T.: *Global analysis : differential forms in analysis, geometry and physics*, Chapter 7, Transl. by A. Nestke. American Mathematical Society, 2002.

[2] Cannas da Silva, A.: *Lectures on Symplectic Geometry*, Springer, 2008. Also available at: https://people.math.ethz.ch/ acannas/Papers/lsg.pdf

[3] Michor, P.: *Topics in Differential Geometry*, Chapter VII ,American Mathematical Society, 2008.