HOMEWORK 1

Exercise 1. For the short exact sequence od chain complexes

$$0 \longrightarrow A_* \xrightarrow{f} B_* \xrightarrow{g} C_* \longrightarrow 0$$

there is a long exact sequence of homology groups

$$\ldots \longrightarrow H_{n+1}(C_*) \xrightarrow{\partial_*} H_n(A_*) \xrightarrow{f_*} H_n(B_*) \xrightarrow{g_*} H_n(C_*) \xrightarrow{\partial_*} H_{n-1}(A_*) \longrightarrow \ldots$$

with the connecting homomorphism ∂_* defined by the prescription

$$\partial_*([c]) = [a], \quad \text{where} \quad \partial c = 0, \ f(a) = \partial b, \ g(b) = c.$$

(1) Prove the exactness in $H_n(A_*)$.
(2) Prove the exactness in $H_n(B_*)$.