Diskrétní deterministické modely

Kvalitativní vlastnosti řešení nelineárního autonomního systému

Zdeněk Pospíšil
707@mail.muni.cz

Masarykova univerzita

8. prosince 2022

Model dravec-kořist Johna Maynarda Smithe

$N = N(t)$ ... velikost populace kořisti (producenta)
$P = P(t)$ ... velikost populace dravce (konzumenta)

Předpoklady:

- Velikost populace kořisti bez přítomnosti dravce se vyvíjí podle logistické rovnice.
- Dravec za časovou jednotku zlikviduje množství kořisti úměrné jejímu množství.
- Růstový koeficient dravce je úměrný množství kořisti.

$$N(t+1) = N(t)\left(r - \frac{r-1}{K}N(t)\right) - \alpha N(t)P(t),$$
$$P(t+1) = \beta N(t)P(t),$$

$r > 1$, $K > 0$, $\alpha > 0$, $\beta > 0$.

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$$P(t+1) = \beta N(t)P(t),$$

$r > 1$, $K > 0$, $\alpha > 0$, $\beta > 0$.

Změna měřítka: $x = \dfrac{1}{N}K$, $y = \dfrac{\alpha}{r}P$, $\gamma = \beta K$:

$$x(t+1) = rx(t)\left(1 - \frac{r-1}{r}x(t) - y(t)\right),$$
$$y(t+1) = \gamma x(t)y(t).$$

Model dravec-kořist Johna Maynarda Smithe

Stabilita koexistenční rovnováhy

$$
\begin{aligned}
x(t+1) &= rx(t)\left(1-\frac{r-1}{r}x(t)-y(t)\right),\\
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Stacionární bod (x^*, y^*) takový, že $x^* > 0$, $y^* > 0$ existuje $\Leftrightarrow r > 1, \gamma > 1$,

$$
(x^*, y^*) = \left(\frac{1}{\gamma}, \frac{(r-1)(\gamma-1)}{r\gamma}\right),
$$

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$$

variační matice

$$
\mathrm{J}(x^*, y^*) = \begin{pmatrix} 1-\dfrac{1-1}{\gamma} & -\dfrac{r}{\gamma} \\ \dfrac{(r-1)(\gamma-1)}{r} & 0 \end{pmatrix}.
$$

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Dostatečné podmínky asymptotické stability:

$$
3\frac{r-1}{r+3} < \gamma < 2.
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Dostatečné podmínky asymptotické stability:

$$3\frac{r-1}{r+3} < \gamma < 2.$$

Dostatečné podmínky monotonnosti v okolí stacionárního bodu

$$\tfrac{1}{2}(r-1) < \gamma < \tfrac{1}{2}(\sqrt{r}+1).$$

Model dravec-kořist Johna Maynarda Smithe

Stabilita koexistenční rovnováhy

$$x(t+1) = rx(t)\left(1 - \frac{r-1}{r}x(t) - y(t)\right),$$
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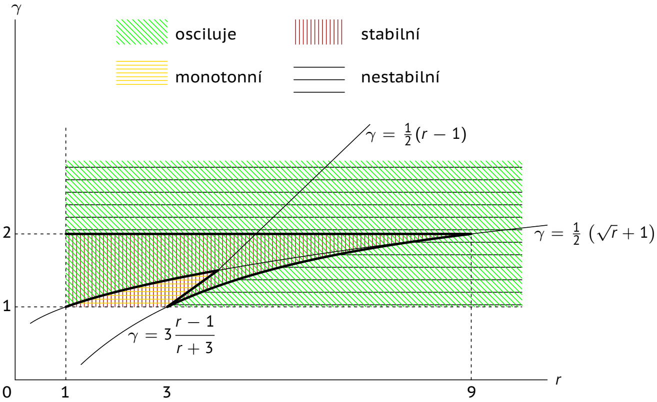

Populace strukturovaná podle plodnosti

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \quad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{aligned} \lambda = \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \\ = \tfrac{1}{2}\left(\sigma_1(1-\gamma) + \sigma_2 + \sqrt{\big(\sigma_1(1-\gamma) - \sigma_2\big)^2 + 4\sigma_1\gamma\varphi}\right) \end{aligned}$$

$$\lambda > 1 \iff \sigma_1\gamma\varphi \geq (1-\sigma_2)\big(1 - (\sigma_1(1-\gamma))\big)$$

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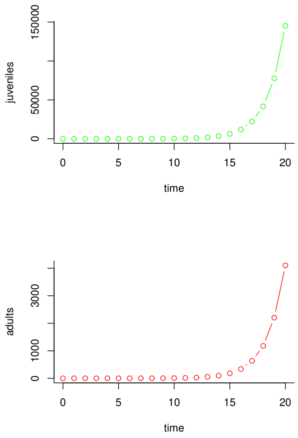
$$\lambda > 1 \;\Leftrightarrow\; \sigma_1\gamma\varphi \geq (1-\sigma_2)\big(1-(\sigma_1(1-\gamma))\big)$$

$\sigma_1 = 0.5$, $\sigma_2 = 0.1$, $\gamma = 0.1$, $\varphi = 50$

$\lambda = 1.8658$



Populace strukturovaná podle plodnosti

Parametry závislé na velikosti tříd

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \quad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\begin{aligned} \sigma_1 &= \sigma_1(n_1, n_2) = \Sigma_1 \mathrm{e}^{-s_{11}n_1 - s_{12}n_2} \\ \sigma_2 &= \sigma_1(n_1, n_2) = \Sigma_2 \mathrm{e}^{-s_{21}n_1 - s_{22}n_2} \\ \gamma &= \gamma(n_1, n_2) = \Gamma \mathrm{e}^{-g_1 n_1 - g_2 n_2} \\ \varphi &= \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2} \end{aligned}$$

$$\lambda = \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \lambda(n_1, n_2) = \lambda(\boldsymbol{n})$$

$$\begin{aligned} \lambda_0 &= \lambda(\boldsymbol{o}) \\ \lambda_\infty &= \lim_{\|\boldsymbol{n}\| \to \infty} \lambda(\boldsymbol{n}) \end{aligned}$$

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\gamma &= \gamma(n_1, n_2) = \Gamma \mathrm{e}^{-g_1 n_1 - g_2 n_2} \\
\varphi &= \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}
\end{aligned}$$

$$\lambda = \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \lambda(n_1, n_2) = \lambda(\boldsymbol{n})$$

$$\begin{aligned}
\lambda_0 &= \lambda(\boldsymbol{o}) \\
\lambda_\infty &= \lim_{\|\boldsymbol{n}\| \to \infty} \lambda(\boldsymbol{n})
\end{aligned}$$

$$\begin{aligned}
&\lim_{\sigma_1 \to 0} \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \sigma_2 \\
&\lim_{\sigma_2 \to 0} \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \tfrac{1}{2}\left(\sigma_1(1-\gamma) + \sqrt{\sigma_1^2(1-\gamma)^2 + 4\sigma_1\gamma\varphi}\right) \\
&\lim_{\gamma \to 0} \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \sigma_1 \\
&\lim_{\varphi \to 0} \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \sigma_1(1-\gamma)
\end{aligned}$$

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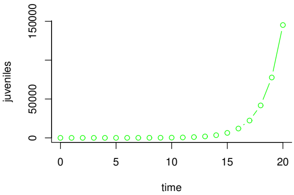
$$\varphi = \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}$$

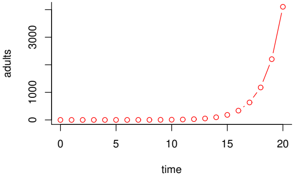
$$\lambda = \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \lambda(n_1, n_2) = \lambda(\boldsymbol{n})$$

$$\lambda_0 = \lambda(\boldsymbol{o}) = 1.8658$$
$$\lambda_\infty = \lim_{\|\boldsymbol{n}\| \to \infty} \lambda(\boldsymbol{n}) = 1.8658$$



$$\Sigma_1 = 0.5, \Sigma_2 = 0.1, \Gamma = 0.1, \Phi = 50$$

$$s_{11} = s_{12} = 0,\ s_{11} = s_{12} = 0,\ g_1 = g_2 = 0,\ f_1 = f_2 = 0$$



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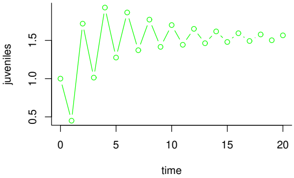
$$\varphi = \varphi(n_1,n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}$$

$$\lambda = \lambda(\sigma_1,\sigma_2,\gamma,\varphi) = \lambda(n_1,n_2) = \lambda(\boldsymbol{n})$$

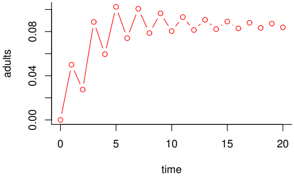
$$\lambda_0 = \lambda(\boldsymbol{o}) = 1.8658$$
$$\lambda_\infty = \lim_{\|\boldsymbol{n}\|\to\infty} \lambda(\boldsymbol{n}) = 0.45$$



$$\Sigma_1 = 0.5,\ \Sigma_2 = 0.1,\ \Gamma = 0.1,\ \Phi = 50$$

$$s_{11} = s_{12} = 0,\ s_{11} = s_{12} = 0,\ g_1 = g_2 = 0,\ f_1 = f_2 = 1$$

Stabilizace populace omezením plodnosti



Populace strukturovaná podle plodnosti

Parametry závislé na velikosti tříd

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \qquad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

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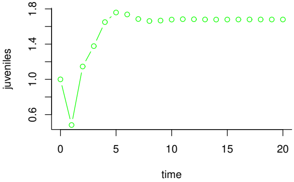
$$\varphi = \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}$$

$$\lambda = \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \lambda(n_1, n_2) = \lambda(\boldsymbol{n})$$

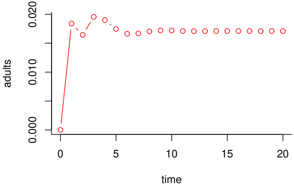
$$\lambda_0 = \lambda(\boldsymbol{o}) = 1.8658$$
$$\lambda_\infty = \lim_{\|\boldsymbol{n}\| \to \infty} \lambda(\boldsymbol{n}) = 0.5$$



$$\Sigma_1 = 0.5,\ \Sigma_2 = 0.1,\ \Gamma = 0.1,\ \Phi = 50$$

$$s_{11} = s_{12} = 0,\ s_{11} = s_{12} = 0,\ g_1 = g_2 = 1,\ f_1 = f_2 = 0$$

Stabilizace populace odložením reprodukce



Populace strukturovaná podle plodnosti

Parametry závislé na velikosti tříd

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \quad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

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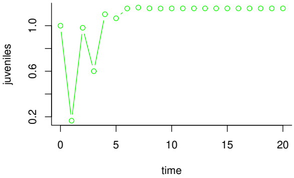
$$\gamma = \gamma(n_1, n_2) = \Gamma \mathrm{e}^{-g_1 n_1 - g_2 n_2}$$
$$\varphi = \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}$$

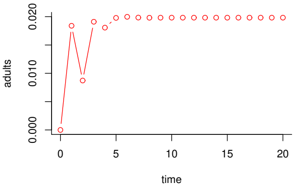
$$\lambda = \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \lambda(n_1, n_2) = \lambda(\boldsymbol{n})$$

$$\lambda_0 = \lambda(\boldsymbol{o}) = 1.8658$$
$$\lambda_\infty = \lim_{\|\boldsymbol{n}\| \to \infty} \lambda(\boldsymbol{n}) = 0.1$$



$$\Sigma_1 = 0.5, \Sigma_2 = 0.1, \Gamma = 0.1, \Phi = 50$$

$$s_{11} = s_{12} = 1, \; s_{11} = s_{12} = 0, \; g_1 = g_2 = 0, \; f_1 = f_2 = 0$$



Stabilizace populace zvětšením úmrtnosti juvenilních jedinců (infanticidou)

Populace strukturovaná podle plodnosti

Parametry závislé na velikosti tříd

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \qquad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

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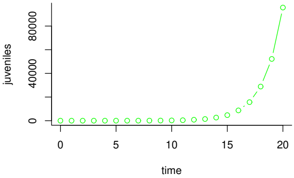
$$\sigma_2 = \sigma_1(n_1, n_2) = \Sigma_2 \mathrm{e}^{-s_{21}n_1 - s_{22}n_2}$$
$$\gamma = \gamma(n_1, n_2) = \Gamma \mathrm{e}^{-g_1 n_1 - g_2 n_2}$$
$$\varphi = \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}$$

$$\lambda = \lambda(\sigma_1, \sigma_2, \gamma, \varphi) = \lambda(n_1, n_2) = \lambda(\boldsymbol{n})$$

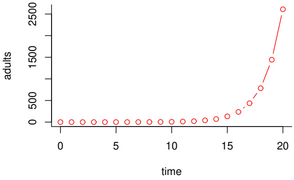
$$\lambda_0 = \lambda(\boldsymbol{o}) = 1.8658$$
$$\lambda_\infty = \lim_{\|\boldsymbol{n}\| \to \infty} \lambda(\boldsymbol{n}) = 1.8221$$



$$\Sigma_1 = 0.5,\ \Sigma_2 = 0.1,\ \Gamma = 0.1,\ \Phi = 50$$

$$s_{11} = s_{12} = 0,\ \ s_{11} = s_{12} = 1,\ \ g_1 = g_2 = 0,\ \ f_1 = f_2 = 0$$

Zpomalení růstu populace zvětšením úmrtnosti plodných jedinců (při velké plodnosti)



Populace strukturovaná podle plodnosti

Parametry závislé na velikosti tříd

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \qquad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\sigma_1 = \sigma_1(n_1,n_2) = \Sigma_1 \mathrm{e}^{-s_{11}n_1 - s_{12}n_2}$$
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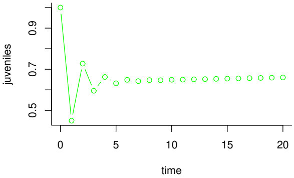
$$\gamma = \gamma(n_1,n_2) = \Gamma \mathrm{e}^{-g_1 n_1 - g_2 n_2}$$
$$\varphi = \varphi(n_1,n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}$$
$$\lambda = \lambda(\sigma_1,\sigma_2,\gamma,\varphi) = \lambda(n_1,n_2) = \lambda(\boldsymbol{n})$$

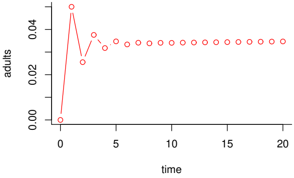
$$\lambda_0 = \lambda(\boldsymbol{o}) = 1.0204$$
$$\lambda_\infty = \lim_{\|\boldsymbol{n}\|\to\infty} \lambda(\boldsymbol{n}) = 0.9837$$



$\Sigma_1 = 0.5,\ \Sigma_2 = 0.1,\ \Gamma = 0.1,\ \Phi = 10.5$

$s_{11} = s_{12} = 0,\ s_{11} = s_{12} = 1,\ g_1 = g_2 = 0,\ f_1 = f_2 = 0$

Stabilizace populace zvětšením úmrtnosti plodných jedinců (při malé plodnosti)



Populace strukturovaná podle plodnosti

Trajektorie a atraktory

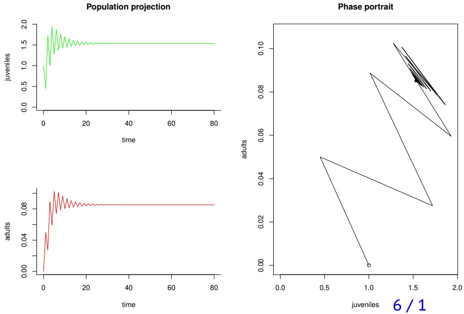
$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \qquad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$\sigma_1 = \sigma_1(n_1, n_2) = \Sigma_1 \mathrm{e}^{-s_{11}n_1 - s_{12}n_2}$ $\qquad$ $\sigma_2 = \sigma_1(n_1, n_2) = \Sigma_2 \mathrm{e}^{-s_{21}n_1 - s_{22}n_2}$

$\gamma = \gamma(n_1, n_2) = \Gamma \mathrm{e}^{-g_1 n_1 - g_2 n_2}$ $\qquad$ $\varphi = \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}$

$\Sigma_1 = 0.5,\ \Sigma_2 = 0.1,\ \Gamma = 0.1,\ \Phi = 50,$

$s_{11} = s_{12} = s_{21} = s_{22} = 0,\ g_1 = g_2 = 0,\ f_1 = f_2 = 1$



Populace strukturovaná podle plodnosti

Trajektorie a atraktory

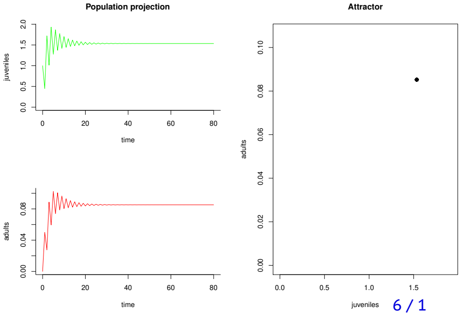
$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \quad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\sigma_1 = \sigma_1(n_1, n_2) = \Sigma_1 \mathrm{e}^{-s_{11}n_1 - s_{12}n_2} \qquad \sigma_2 = \sigma_1(n_1, n_2) = \Sigma_2 \mathrm{e}^{-s_{21}n_1 - s_{22}n_2}$$

$$\gamma = \gamma(n_1, n_2) = \Gamma \mathrm{e}^{-g_1 n_1 - g_2 n_2} \qquad \varphi = \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}$$

$$\Sigma_1 = 0.5,\ \Sigma_2 = 0.1,\ \Gamma = 0.1,\ \Phi = 50,$$

$$s_{11} = s_{12} = s_{21} = s_{22} = 0,\ g_1 = g_2 = 0,\ f_1 = f_2 = 1$$



Rovnovážný bod

Populace strukturovaná podle plodnosti

Trajektorie a atraktory

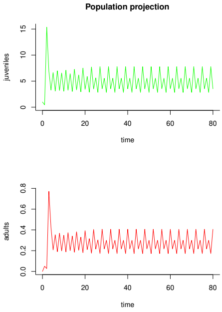
$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix}\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \quad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

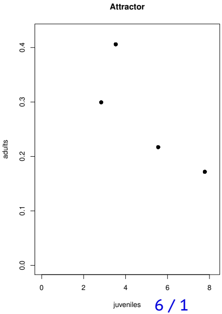
$\sigma_1 = \sigma_1(n_1, n_2) = \Sigma_1 \mathrm{e}^{-s_{11}n_1 - s_{12}n_2}$ $\qquad$ $\sigma_2 = \sigma_1(n_1, n_2) = \Sigma_2 \mathrm{e}^{-s_{21}n_1 - s_{22}n_2}$

$\gamma = \gamma(n_1, n_2) = \Gamma \mathrm{e}^{-g_1 n_1 - g_2 n_2}$ $\qquad$ $\varphi = \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}$

$\Sigma_1 = 0.5,\ \Sigma_2 = 0.1,\ \Gamma = 0.1,\ \Phi = 500,$

$s_{11} = s_{12} = s_{21} = s_{22} = 0,\ g_1 = g_2 = 0,\ f_1 = f_2 = 1$





Cyklus délky 4

Populace strukturovaná podle plodnosti

Trajektorie a atraktory

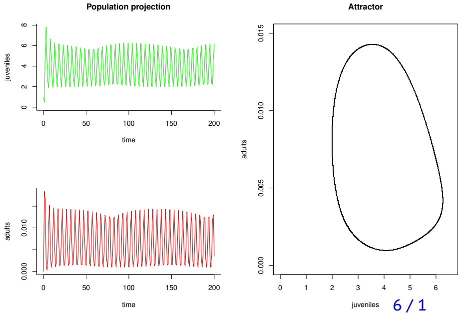
$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \qquad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$\sigma_1 = \sigma_1(n_1, n_2) = \Sigma_1 \mathrm{e}^{-s_{11}n_1 - s_{12}n_2}$ $\qquad \sigma_2 = \sigma_1(n_1, n_2) = \Sigma_2 \mathrm{e}^{-s_{21}n_1 - s_{22}n_2}$

$\gamma = \gamma(n_1, n_2) = \Gamma \mathrm{e}^{-g_1 n_1 - g_2 n_2}$ $\qquad \varphi = \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}$

$\Sigma_1 = 0.5,\ \Sigma_2 = 0.1,\ \Gamma = 0.1,\ \Phi = 300,$

$s_{11} = s_{12} = s_{21} = s_{22} = 0,\ g_1 = g_2 = 1,\ f_1 = f_2 = 0$



Invariantní smyčka

Populace strukturovaná podle plodnosti

Trajektorie a atraktory

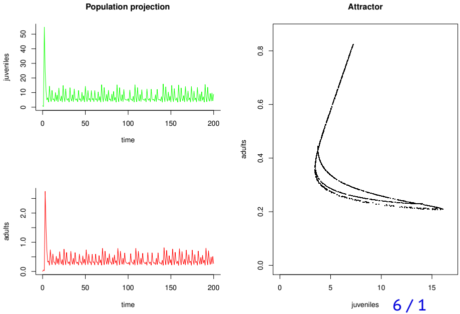
$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \qquad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\sigma_1 = \sigma_1(n_1,n_2) = \Sigma_1 \mathrm{e}^{-s_{11}n_1 - s_{12}n_2} \qquad \sigma_2 = \sigma_1(n_1,n_2) = \Sigma_2 \mathrm{e}^{-s_{21}n_1 - s_{22}n_2}$$

$$\gamma = \gamma(n_1,n_2) = \Gamma \mathrm{e}^{-g_1n_1 - g_2n_2} \qquad \varphi = \varphi(n_1,n_2) = \Phi \mathrm{e}^{-f_1n_1 - f_2n_2}$$

$$\Sigma_1 = 0.5,\ \Sigma_2 = 0.1,\ \Gamma = 0.1,\ \Phi = 1\,800,$$

$$s_{11} = s_{12} = s_{21} = s_{22} = 0,\ g_1 = g_2 = 0,\ f_1 = f_2 = 1$$



Podivný atraktor

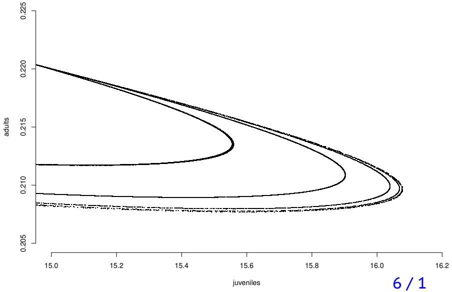
Populace strukturovaná podle plodnosti

Trajektorie a atraktory

$$\begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t+1) = \begin{pmatrix} \sigma_1(1-\gamma) & \varphi \\ \sigma_1\gamma & \sigma_2 \end{pmatrix} \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(t), \quad \begin{pmatrix} n_1 \\ n_2 \end{pmatrix}(0) = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$$

$$\sigma_1 = \sigma_1(n_1, n_2) = \Sigma_1 \mathrm{e}^{-s_{11}n_1 - s_{12}n_2} \qquad \sigma_2 = \sigma_1(n_1, n_2) = \Sigma_2 \mathrm{e}^{-s_{21}n_1 - s_{22}n_2}$$
$$\gamma = \gamma(n_1, n_2) = \Gamma \mathrm{e}^{-g_1 n_1 - g_2 n_2} \qquad \varphi = \varphi(n_1, n_2) = \Phi \mathrm{e}^{-f_1 n_1 - f_2 n_2}$$

$\Sigma_1 = 0.5,\ \Sigma_2 = 0.1,\ \Gamma = 0.1,\ \Phi = 1\,800,$
$s_{11} = s_{12} = s_{21} = s_{22} = 0,\ g_1 = g_2 = 0,\ f_1 = f_2 = 1$



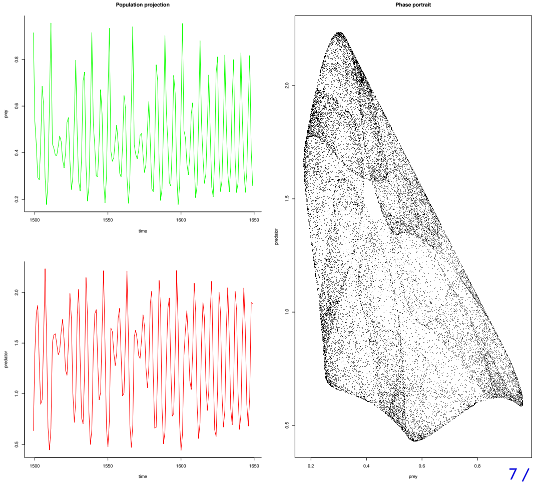
Podivný atraktor

Model dravec-kořist Johna Maynarda Smithe

Atraktor

$$x(t+1) = rx(t)\left(1 - \frac{r-1}{r}x(t) - y(t)\right),$$
$$y(t+1) = \gamma x(t)y(t),$$

$r = 3.6, \gamma = 2.4.$



MASARYKOVA
UNIVERZITA